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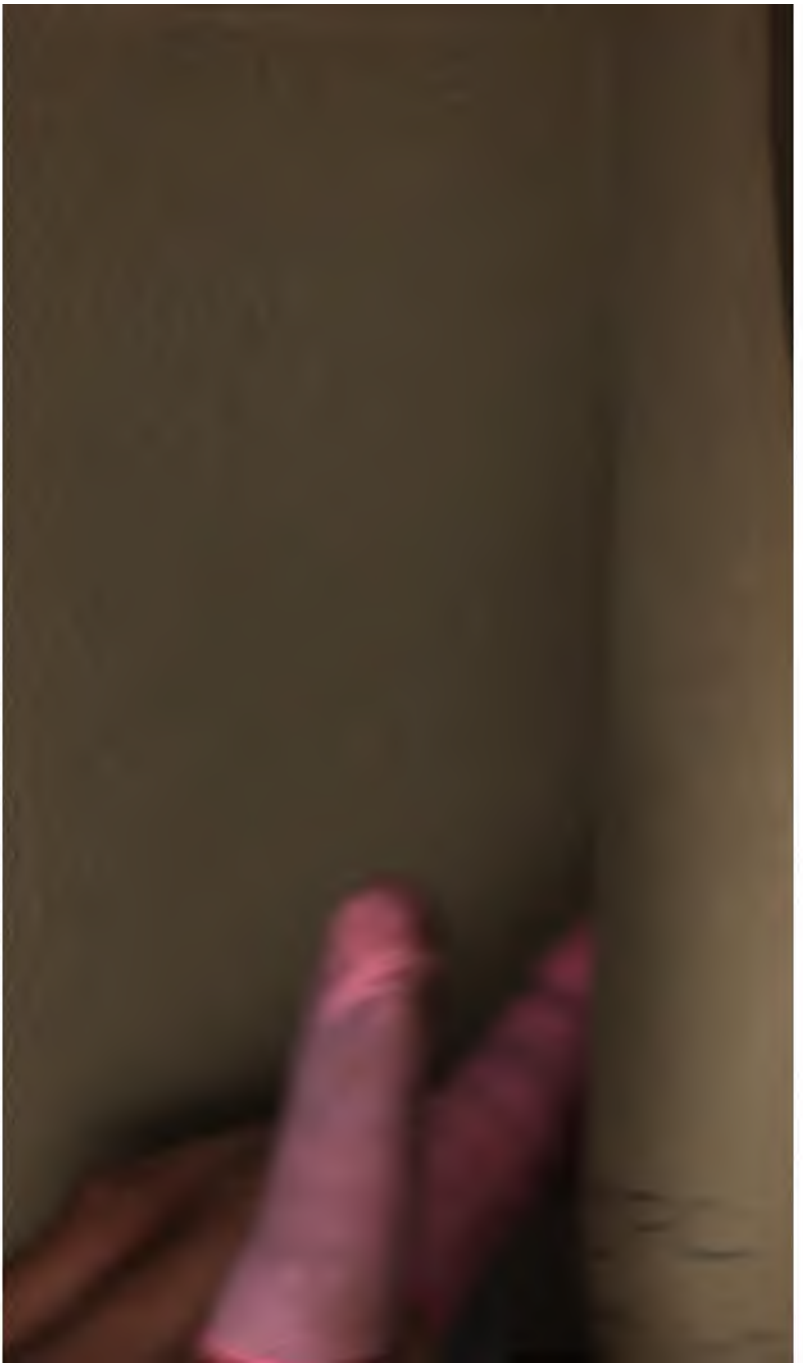

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MODERN BUSINESS ARITHMETIC

A NEW AND COMPLETE TREATISE ON
COMMERCIAL ARITHMETIC

By
U. CURTIS

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INDIANAPOLIS, U. S. A.

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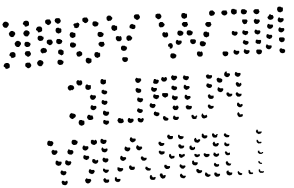

PREFACE

This work is intended to furnish complete information concerning the subject of which it treats, and forms one number of a well graded and progressive series of Arithmetics now in preparation. It has been the design of the author, in dealing with the different subjects of this volume, to furnish a scientific treatise relating to numbers and their applications, adapted to the wants and necessities of the commercial school, and of that class of high schools whose boards of education have wisely decided that the *true science* of numbers cannot be successfully comprehended by the immature minds of pupils belonging to the grades.

The aim has been to make the work, from the very beginning, purely analytical, to teach the close relations existing between associated facts, and to aid the thinking mind in the deduction of such additional facts as are the natural result of these relations. A careful study of the "Properties and Relations of Numbers," as treated in this volume, will be found invaluable to the student, and will have a tendency to elevate his thinking powers to that degree of eminence, whereby he will discern that *reason*, instead of *memory*, is the mental faculty to be developed by mathematical study.

The work will be found clear and accurate in definitions and logical in explanations, and we believe that a critical examination by thorough business scholars who are conversant with, and know the demands of the present commercial age, will result in an endorsement of the methods of solution concerning the different subjects herein presented. In nearly all of our solutions short methods have been introduced, but, in every case, the principles underly-

ing the work of "Rapid Calculation" have been thoroughly demonstrated; hence, the student will not be compelled to grope his way in darkness, but, by careful study, can be made to comprehend every successive step of his work.

Much care has been exercised to obtain correct information pertaining to commercial transactions; the latest statutes have been examined, and nearly every department of business—the bank, the insurance office, the mercantile emporium, different manufacturing industries, etc.—have contributed valuable information; and through the courtesy of government officials, we have been able to furnish correct information concerning standard units of weights and measures, the value, weight, and purity of coins, both foreign and domestic, and many other facts statistical in character, and necessary to a commercial work. As a result of this thoroughness of investigation, many facts, herein presented, will differ from statements found in many commercial arithmetics which have been gathered from works published a quarter of a century ago.

Much that is new in methods of instruction will be found in the several subjects presented. Prominence has been given to Arithmetical Analysis, Fractions, Percentage and its numerous applications, especially to Interest, Stocks, Insurance, Averaging Accounts, Domestic and Foreign Exchange, Partnership, and several other subjects necessary to qualify students for commercial pursuits.

We have made an earnest effort to prepare a work suited to the needs of teachers and students of advanced schools, and we believe that young men and young women who will study the work thoroughly, and with a determination to succeed, will be able to enter upon Life's duties with an assurance of success.

U. C.

September, 1908.

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MODERN BUSINESS ARITHMETIC

INTRODUCTION

1. Quantity is the amount, extent or magnitude of anything that can be separated into parts.

2. Number is quantity considered with special reference to the parts of which it is composed.

3. Quantity and number, although generally regarded as synonymous terms, differ somewhat in meaning. Any aggregation thought of, regardless of the parts constituting or composing it, is a quantity; but when such aggregation is considered with special reference to these parts, it becomes a number. An invoice of grain is a quantity, but when we think of it as a thousand bushels, the idea is suggestive of number. *Quantity* furnishes an answer to the question "*How much?*" and *number* expresses "*How many?*"

4. Mathematics is the science which has for its object an investigation of the properties and relations belonging to quantity.

5. Arithmetic is that branch of mathematics which treats of the properties and relations of finite numbers, and of the laws governing numerical operations.

6. A **Unit** is one.

7. The **Unit of a Number** is one of the constituent parts forming it. Thus, the unit of 24 is 1; of 16 books, 1 book.

8. Units may be classified as *Integral*, *Fractional* and *Collective*.

9. An **Integral Unit** is a unit applied to a whole: as 1 apple, 1 desk, 1 house.

10. A **Fractional Unit** is a unit applied to a piece or a part of a whole: as 1 half, 1 tenth of a dollar, 1 sixteenth of an acre of land.

11. A **Collective Unit** is a unit applied to a collection of whole things regarded as one group: as 1 dozen, 1 score, 1 ten, 1 hundred.

12. Numbers are classified to correspond to the character of the units composing them, as shown by the following definitions:

13. An **Integral Number** is a number composed of integral units: as 5, 7 pounds, 12 men.

14. A **Fractional Number** is a number composed of fractional units: as 3 pieces, 5 sevenths, 7 twelfths.

15. An **Abstract Number** is a number whose units are not applied: as 10, 15, 36.

16. A **Concrete Number** is a number whose units are applied: as 20 acres, 18 dollars, 13 books.

17. **Similar Numbers** are numbers whose units are alike in kind, but unlike in size: as 3-fourths of an orange and 7-tenths of an orange; or 2 bushels of corn and 3 pecks of corn.

18. **Dissimilar Numbers** are numbers whose units differ in kind. They may or may not be unlike in size: as 3-fifths of a melon and 3-fifths of a dollar.

19. **Like Numbers** are numbers whose units are alike in character and also in size: as 5-eighths of a cord of wood, and 7-eighths of a cord of wood. All abstract numbers having units of the same size are like numbers; if their units differ in magnitude they are only similar, but may *be made like* by reduction.

20. Unlike Numbers are numbers whose units are of different kinds: as 32 hats, 40 sheep.

21. A clearer understanding of number classification may be obtained by an inspection of the following diagram:

Numbers
- Based upon Unit Magnitude
  - Integral
  - Fractional
- Based upon Unit Application
  - Abstract
  - Concrete
- Based upon Unit Similarity
  - Similar
  - Dissimilar
- Based upon Unit Equality
  - Like
  - Unlike

NOTE. It should be observed that the distinction between the different classes of numbers exhibited in the above diagram relates more particularly to the things to which the numbers apply than to the numbers themselves.

22. A **Problem** is a question or proposition requiring a solution.

23. A **Solution** is a process by which a required result is obtained.

24. A **Demonstration** is a process of reasoning whereby, from a knowledge of two or more related facts, one or more additional facts may be deduced.

25. A **Rule** is a direction for performing the successive steps of a solution.

26. Arithmetical Operations are of two kinds, *Reductions* and *Computations*.

27. Reductions change the forms of numbers without changing their values.

28. Computations change the forms, and also the values of numbers.

NOTE. Compute means to reckon, hence multiplications and divisions by unity cannot properly be termed computations.

29. The computations of arithmetic are Addition, Subtraction, Multiplication, and Division.

30. An **Axiom** is a truth so plain and simple that proof is unnecessary.

NOTATION AND NUMERATION.

31. Notation teaches the principles to be observed in expressing numbers by means of characters.

32. Numeration teaches the principles to be observed in reading numbers that are expressed by characters.

33. Two systems of notation are in general use—the *Roman*, and the *Arabic* or *Decimal*. The former is not well adapted to the work of computation, but is chiefly used for purposes of numbering; thus: Section VII, Chapter XXVI.

THE ROMAN NOTATION

34. Employs the following seven letters, each of which expresses the value written underneath:

I	V	X	L	C	D	M
One,	Five,	Ten,	Fifty,	One hundred,	Five hundred,	One thousand.

35. The Roman notation is based upon the following principles:

1. *Whenever a letter is repeated or placed after one representing greater value, the sum of the entire group is expressed. Thus, III represents three; XII, twelve; LXXVI, seventy-six.*

2. *Whenever a letter is placed before one representing greater value, the difference between the values is expressed. Thus, IV represents four; IX, nine; XC, ninety.*

3. *Whenever a letter is placed between two letters, each of greater value, its value is to be taken from the united values of the other two. Thus, XIV represents fourteen; LXIV, sixty-four.*

4. *A dash placed over a letter increases its value a thousand-fold. Thus, $\overline{M}$ represents one million.*

36. A further illustration of these principles is contained in the following:

ROMAN NOTATION TABLE.

I is One	XV is Fifteen	CC is Two hundred
II is Two	XVI is Sixteen	CCC is Three hundred
III is Three	XVII is Seventeen	CCCC is Four hundred
IV is Four	XVIII is Eighteen	D is Five hundred
V is Five	XIX is Nineteen	DC is Six hundred
VI is Six	XX is Twenty	DCC is Seven hundred
VII is Seven	XXX is Thirty	DCCC is Eight hundred
VIII is Eight	XL is Forty	DCCCC is Nine hundred
IX is Nine	L is Fifty	M is One thousand
X is Ten	LX is Sixty	MD is Fifteen hundred
XI is Eleven	LXX is Seventy	MM is Two thousand
XII is Twelve	LXXX is Eighty	MDC is Sixteen hundred
XIII is Thirteen	XC is Ninety	$\overline{V}$ is Five thousand
XIV is Fourteen	C is One hundred	$\overline{M}$ is One million

1. Write 2397 by the Roman method.

EXPLANATION. First write the highest order (thousands), thus, MM; then write the next highest order (hundreds), thus, MMCCC; then the tens, thus MMCCCXC; then the units, thus, MMCCCXCVII.

RULE. *Write the highest order, after which write each order in succession.*

EXAMPLES FOR PRACTICE.

Express the following numbers by the Roman notation:

2. Thirteen.
3. Nineteen.
4. Thirty-seven.
5. Sixty-nine.
6. Ninety-nine.
7. One hundred eight.
8. Three hundred fifty.
9. Seven hundred thirteen.
10. One thousand six hundred.
11. One million five thousand.

THE ARABIC NOTATION

37. Employs the following ten figures to represent numbers, each of which is named to correspond to the number of units it represents:

1	2	3	4	5	6	7	8	9	0
One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Naught

38. The first nine of these characters are termed significant figures, since each signifies value when standing alone. They are also called digits.

39. The last character does not express value, but when written in connection with other figures it produces a modifying effect upon them according to certain principles hereafter explained. This character is frequently called a *cipher*, and sometimes *zero*.

40. Since nine is the greatest number that can be expressed by a single figure, it is evident that all numbers in excess of nine are expressed by combining these ten characters according to certain principles characteristic of the notation. Thus, in the number 2435, the 5 units in the first or right hand place represent 5 single things, and are termed *units of the first order*, or simply *units;* the 3 units in the second place, counting toward the left, represent 3 groups of ten single things each, and are termed *units of the second order*, or *tens;* the 4 units in the third place represent 4 groups, each of which contains 10 groups of the former order (one hundred single things), and are termed *units of the third order,* or *hundreds;* the two units in the fourth place represent 2 groups, each of which contains ten groups of the third order (one thousand single things), and are termed *units of the fourth order*, or *thousands*, etc. This method of grouping by tens is the leading feature of the decimal notation, and, when thoroughly understood, the *computations* of arithmetic are made exceedingly simple.

41. The cipher is employed to fill vacant orders and thus secure to the significant figures their correct positions. Thus, 1 unit of the third order and 7 units of the first order are written 107. The cipher fills the vacant order and causes the figure 1 to occupy a position in the third place, as it evidently should.

42. From the foregoing explanations, we deduce the following principles:

1. *The relative or local value of any figure depends upon its location. Thus, 3 written in the first place is 3; when written in the second place, 30; when in the third place, 300, etc.*

2. *The different orders of units increase in value from right to left, and decrease from left to right, in a tenfold ratio.*

3. *Every removal of a figure one place to the left increases its local value tenfold, and every removal of a figure one place toward the right diminishes its local value tenfold.*

43. For convenience in reading, numbers are usually separated into periods of three figures each, as shown by the following table:

of octillions	of septillions	of sextillions	of quintillions	of quadrillions	of trillions	of billions	of millions	of thousands	of units
tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units	hundreds tens units
3 4	5 2 7	5 0 6	1 3 8	9 0 0	7 3 0	0 0 0	0 1 6	0 0 9	3 8 0
tenth period	ninth period	eighth period	seventh period	sixth period	fifth period	fourth period	third period	second period	first period

44. From the foregoing analysis of principles we deduce the following rules:

RULE FOR NOTATION.

I. *Begin at the left and write the figures belonging to the highest period.*

II. *Write the hundreds, tens, and units of each successive period in their order, placing a cipher wherever an order is omitted.*

RULE FOR NUMERATION.

I. *Separate the number into periods of three figures each, counting from the right.*

II. *Begin at the left and read each period in regular order as if it were an independent number, adding the name of each period except that of units.*

Express, by means of figures, the following numbers:

1. Two thousand five hundred sixty-eight.

2. Nine thousand seven hundred thirty-one.

3. Fifteen thousand two hundred eighty.

4. One hundred thirty-two thousand five hundred twenty-one.

5. Seven thousand fifty-six.

6. Ten thousand twenty.

7. Two hundred nine thousand two hundred nine.

8. Sixty million sixty.

9. Ninety million ninety thousand ninety.

10. Eighteen billion sixteen thousand four.

11. Thirty-two trillion five million eighteen.

12. Seventy-two septillion fifteen quadrillion one hundred five million ten thousand seven hundred twenty.

Express, by written words, the following numbers:

1.	38	*7.*	52180	*13.*	61001300408
2.	104	*8.*	76043	*14.*	204007000096
3.	720	*9.*	20028	*15.*	100570000025
4.	1008	*10.*	751460	*16.*	780030050000
5.	10205	*11.*	302009	*17.*	12804003000710
6.	*20008*	*12.*	12408063	*18.*	1750063090

NOTE. In expressing a single number in words, the conjunction *and* should be used only to connect integral and fractional parts, as five and two-thirds. The expression one hundred and nine implies two separate numbers, one hundred being one, and nine the other.

ADDITION.

45. Addition is the process of finding a number that contains as many units as two or more given numbers.

46. An **Addend** is a number to be added.

47. The **Sum** or **Amount** is the result obtained by addition.

48. The **Sign of Addition** is a perpendicular cross, made thus, (+). It is read *plus* and signifies that the numbers connected by it are to be added; as 3+2+5, read 3 plus 2 plus 5.

49. The **Sign of Equality** is a couplet of short horizontal lines, (=). It indicates that the numbers or combination of numbers connected by it are equal; as 5+7=12, read 5 plus 7 equals 12.

50. The principles of Addition are:

1. *Only like numbers can be added.*
2. *Only like orders of units can be added.*
3. *The sum will not be changed by changing the order of the addends.*
4. *The sum of the orders of two or more numbers is equal to the sum of the numbers.*
5. *The addends and sum are numbers of like units.*

TO THE TEACHER: In Commercial Arithmetic, there is no more important subject than addition. From an experience of more than twenty years in commercial teaching, the author has learned that students, generally, who enter our business colleges,—not only those who come directly from the grammar grades of our public schools, but those from higher institutions of learning as well,—possess little more than a theoretical knowledge of the subject. To be quick and accurate in addition is not only an accomplishment, but a necessity to every one in business life; hence constant drills in

harmony with correct methods should demand our attention as commercial educators. We trust that the methods herein presented and the urgent request to give this subject the attention which its importance demands, will not be disregarded by the teacher.

EXAMPLES FOR PRACTICE.

The following exercise should be made a class drill and continued until each pupil can name the result of each combination as quickly as he can name a single figure when pointed out.

	1	5	2	7	3	6	9	4	8
1.	2	2	2	2	2	2	2	2	2
	5	1	4	7	2	6	9	3	8
2.	3	3	3	3	3	3	3	3	3
	4	7	1	5	8	2	6	3	9
3.	4	4	4	4	4	4	4	4	4
	6	3	8	1	7	2	5	9	4
4.	5	5	5	5	5	5	5	5	5
	4	1	7	2	6	3	9	8	5
5.	6	6	6	6	6	6	6	6	6
	3	1	9	5	2	7	4	8	6
6.	7	7	7	7	7	7	7	7	7
	2	8	4	1	6	3	7	9	5
7.	8	8	8	8	8	8	8	8	8
	3	7	1	5	9	4	8	2	6
8.	9	9	9	9	9	9	9	9	9

Each of the following combinations makes 10. Learn

them thoroughly, so that whenever you see one of these combinations in adding, you will instantly know the result.

9.	9	8	7	6	5
	1	2	3	4	5
	10	10	10	10	10

10.	0	1	0	0	1	0	1	2	0	1	2
	1	1	2	3	2	4	3	2	5	4	3
	9	8	8	7	7	6	6	6	5	5	5
	10	10	10	10	10	10	10	10	10	10	10

REMARK. Knowing that 6 and 7 are 13, you should as readily know that 16 and 7 are 23; that 56 and 7 are 63, etc.

1. Knowing that 8 and 5 are 13, how many are 48 and 5?

2. How many are 17 and 8?

3. How many are 32 and 7?

4. How many are 75 and 6?

5. How many are 59 and 8?

6. How many are 23, 9, and 1? (23 and 10.)

7. How many are 47, 6, and 4?

8. How many are 28, 6, 2, and 2?

ADDITION OF SINGLE COLUMNS

51. *1.* Find the sum of 4, 2, 4, 7, 3, 1, 8 and 6.

First operation.

4
2
4
7
3
1
8
6
35

EXPLANATION. We first write the numbers in a vertical column and begin with the lowest in position. We name the first figure, 6, and then, passing up the column, we name the result of each successive addition; thus, 6, 14, 15, 18, 25, 29, 31, 35. We finish the work by writing the sum underneath.

Second operation.

4
2 } 10
4

7
3 } 10

1
8 } 15
6

35

The foregoing explanation conforms to the common method, and will do for beginners; but, to the eye of the practical accountant, these numbers would reduce to 15, 10, and 10, as shown by the second operation, and he would therefore be able to name the result nearly as quickly as he could pronounce a word containing the same number of letters.

SUGGESTION. In the addition of columns of figures, be on the constant lookout for combinations, especially those whose sum is 10, 9, or 11. The number 10 is as easily added as 1. The addition of 9 increases the tens figures of the number to which it is added by 1, and diminishes the unit figure by 1, in all cases, except when the unit figure is a cipher. The addition of 11 increases the tens figure, and also the unit figure, by 1, except when the unit figure is 9.

52. PROOF. Different methods of proof have been devised for addition, but the most practical is to add the columns in the opposite direction. If the same result is obtained as in the first addition, the work is presumably correct.

EXAMPLES FOR PRACTICE.

DIRECTION. Add from the bottom upward, then add from the top downward for proof. Take advantage of the grouping principle.

1.	*2.*	*3.*	*4.*	*5.*
4	6	8	2	7
1	2	1	6	2
3	3	7	5	8
5	5	1	1	5
2	4	3	3	3
6	2	4	2	4
4	6	5	4	1
8	3	7	8	5
2	1	2	2	2

6.	*7.*	*8.*	*9.*	*10.*
5	8	5	2	5
8	7	4	9	2
3	5	8	1	8
4	4	2	4	7
6	6	1	8	6
2	3	7	7	9
5	2	6	6	4
3	1	2	2	3
7	8	5	5	8
8	2	9	7	5

11.	*12.*	*13.*	*14.*	*15.*
3	7	5	9	8
8	3	7	2	7
2	8	8	7	6
7	4	9	6	4
2	6	6	5	3
1	9	4	8	9
6	2	3	1	5
5	5	2	2	2
8	3	1	7	1
7	1	9	6	8
3	7	8	5	7
9	2	7	4	6
1	9	8	8	4

NUMBERS OF TWO OR MORE ORDERS.

53. *1.* Find the sum of 4286, 3234, 5182, 1284, 7672, 1805, 3216, 2176, 1473, and 1731.

Operation.

4286	
3234	
5182	39 ✓
1284	55 ✓
7672	40 ✓
1805	32 ✓
3216	
2176	
1473	
1731	
32059	

EXPLANATION. We write the numbers so that units of the same order shall stand in the same column. We then begin with the lowest order of units and add upward; thus, 10, 21, 29, 39, and place the result at the right of the column. Since 39 contains 3 units of the next higher order, these units must be added to the next column; thus, 13, 21, 36, 47, 55. We place this result under the result obtained by adding the first column, and carrying the 5 hundreds to the hundreds column, we proceed in the same manner, continuing the work until each column is added. For proof we add each column downward, and, the same result being obtained, we note the fact by a check mark as shown above. The work proving correct, we write the amount underneath.

REMARK. In adding columns in blank books (except in cases where few addends appear) the results obtained should be written on a slip of paper and the accuracy of the addition proven before writing the amounts underneath the columns. By this plan books will not be defaced by the correction of errors. In the addition of short columns, where there is but slight liability of error, results may be written underneath the columns at once. In adding extremely long columns, where results reach 200 or more, there is less liability of error by dividing into sections and adding each section separately. A combination of these results will, of course, make the entire sum.

54. From the foregoing principles and illustrations we deduce the following:

RULE I. *Write the numbers to be added so that units of the same order shall stand in the same column.*

II. *Commence with the lowest order of units and add each column separately, remembering to carry one for every ten.*

III. *Write the entire sum of the last column.*

EXAMPLES FOR PRACTICE.

1.	2.	3.	4.
5876	487	421768	158
3218	1324	35420	2344
1540	5762	89386	87
6872	18930	4235	136
3147	42768	5762	13542
8974	34876	184	2918
1856	12918	75	483

5. 324+512+15+32+875=how many?

6. 5284+12672+720+1275=how many?

7. What is the sum of twenty-four thousand two hundred five, eight thousand seventeen, and three hundred six thousand five hundred four? *Ans.* 338726.

8. Add one hundred thirty-five million seven hundred nine, sixty million two hundred nine thousand five hundred, and three million thirty thousand three hundred. *Ans.* 198240509.

9. How many square miles of territory has New England, there being in Maine 31764, in New Hampshire 9285, in Vermont 10198, in Massachusetts 7810, in Rhode Island 1305, and in Connecticut 4677?

10. According to the census returns of 1900, New York City had a population of 3437202; Chicago, 1698575; Philadelphia, 1293697; St. Louis, 575238; Boston, 560892; Baltimore, 508957; Cleveland, 381768; Buffalo, 352387; San Francisco, 342782; Cincinnati, 325902: Pittsburg, 321616; New Orleans, 287104; Detroit, 285704; Milwaukee, 285315; Washington, 278718. What was the total population of these 15 cities at that time?

11. What was the wheat crop of the world at the date of the last census, the number of bushels grown in the United States being 547304000, in Italy 137912000, in

Russia 393823000, in Germany 141369000, in Austria 180342000, in Argentina 92176000, in India 232585000, in France 366079000, and in all the other countries 633826000?

12. A man bequeathed his estate as follows: to each of his two sons, $8575; to each of his three daughters, $6250; to his wife $325 more than to both the sons, and the remainder, which was $500 more than he gave to all his daughters, he gave to benevolent institutions. What was the whole amount of his property?

13. A man paid $4275 for a farm and expended $1240 in improvements; taxes and insurance cost him $420. By selling at a gain equal to the sum paid for taxes and insurance, what amount will he receive?

14. Five persons formed a co-partnership; A's investment of capital was $4500; B's, $3250; C's, $5000; D's was equal to the sum of A's and C's; E's was equal to the sum of B's and C's. What was the capital of the firm?

15. B's age is twice A's age; the sum of A's and B's ages is equal to C's age, and D's age is twice B's age. If the unit 1 represents A's age, what number of units will represent the sum of their ages?

16. If the collective unit 1 in the above example is equal to 12 years, what are the respective ages of B, C, and D?

Addition of two or more columns at one operation.

55. *1.* What is the sum of 2642, 1584, 3125, and 1763?

Operation.

2642
1584
3125
1763
9114

EXPLANATION. Beginning with the units and tens of the lowest number in position, we add first the tens of the number above, then the units, thus: 63, 83, 88, 168, 172, 212, 214. Of this sum, we write the 14 under the columns added, and carry the 2 to the next columns, thus: 19, 49, 50, 60, 65, 85, 91, which we write in its place, and we have the whole amount, 9114. After a little practice, it will be found unnecessary to separate the tens from the units as illustrated above, as the addition can be per-

formed by combining the entire sum, thus, 63, 88, 172, 214. Persistent practice will enable the average student to add three, or even four columns at a single operation; mistakes, however, are more likely to occur by this plan of addition, and we doubt that it possesses very great advantages over the single column method as a time saver.

Addition of numbers in horizontal lines.

56. It is frequently found necessary in office work to add numbers occupying positions in horizontal lines, hence practice along this line of work is a valuable drill to the student.

Add the following numbers horizontally:

1. 1246+128+3142+64+137+15+3219=7951.

EXPLANATION. We commence with the 6 units in the left-hand number and add toward the right, thus: 6, 14, 16, 20, 27, 32, 41. We write the 1 in units' place and carry the 4 tens to the second order, thus: 4, 8, 10, 14, 20, 23, 24, 25. We write the 5 in tens' place and carry the 2 to the third order, and continue the work in the same manner until all the orders are added.

Caution. In passing from any number to the next in order, be careful not to change to a wrong order of units.

EXAMPLES FOR PRACTICE.

Find the sum of the numbers in each of the following examples by adding horizontally:

2. 1573+84+183+17+127=how many?

3. 172+10348+8+1765=how many?

4. 32768+4236+92+584=how many?

5. 142+5671+13+5716=how many?

6. 3214+182+6+52915=how many?

7. 14+321+106+8+14+7=how many?

8. A grain dealer's receipts from the sale of different cereals during one year are shown by the following tabu-

lated statement. Extend the columns for each month and find the totals:

Months.	Rec'ved for Wheat.	Rec'ved for Corn.	Rec'ved for Barley.	Rec'ved for Rye.	Totals.
January	$1315	$740	$974	$85	****
February	832	385	216	40	****
March	850	418	194	76	****
April	482	372	156	....	****
May	570	463	225	15	****
June	848	384	187	24	****
July	386	....	184	....	***
August	425	....	...	32	***
September	1845	474	586	112	****
October	2137	983	1240	96	****
November	1138	872	764	38	****
December	1424	615	552	20	****
Totals	*****	****	****	***	*****

SUBTRACTION.

57. Subtraction is the process of finding the difference in unit value between two numbers of the same kind.

58. The **Minuend** is the greater number, and is the one to be diminished by the operation of subtraction.

59. The **Subtrahend** is the number to be subtracted from the minuend.

60. The **Difference** or **Remainder** is the result obtained by subtracting one number from another.

61. The **Sign of Subtraction** is a short horizontal line made thus, (—). It is read *minus*, and signifies that the number written after it is to be subtracted from the number before it; thus, 12—5, read 12 minus 5.

62. When the given numbers contain more than one figure each, the computation of subtraction is based upon *the following* principles:

1. If the units of each order in the subtrahend be taken

from units of like order in the minuend, the sum of the differences must be equal to the entire difference between the minuend and the subtrahend. (Axiom).

2. *If both minuend and subtrahend be equally increased, the remainder will not be changed.* (Axiom).

1. From 837 take 284.

Operation.	
Minuend	837
Subtrahend	284
Remainder	553

EXPLANATION. We first write the subtrahend under the minuend with units of the same order in the same column. We then begin with the lowest order and subtract 4 units from 7 units and obtain 3 units, which we write underneath as a partial remainder. The 8 units of the second order (tens) cannot be taken from 3, hence we add 10 to the 3, (10 being the radix of the decimal system) making 13 tens; then 8 tens from 13 tens leave 5 tens, the second partial remainder. Since we added 10 tens to the minuend, we add 1 hundred to the subtrahend, and the remainder will not be changed. (Prin. 2.) By the addition of 1 hundred, the next subtrahend figure becomes 3, which we subtract from 8 (hundreds) obtaining 5 (hundreds) for the result; and we have for the total remainder 553.

NOTE. Some would prefer to consider the minuend in the above example reduced to 7 hundreds, 13 tens, and 7 units, and to make no change in the subtrahend; either method of analysis is logical, and we advise students to adopt the plan to which they are most accustomed.

63. From the foregoing principles and illustrations we deduce the following:

RULE I. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

II. *Beginning with the lowest order, subtract each order of the subtrahend from the order above it, and write the result underneath.*

III. *If any figure in the subtrahend exceeds in unit value the figure above it, add 10 to the upper figure and 1 to the next figure of the subtrahend; or, if preferable, consider the next figure of the minuend one less.*

64. PROOF. Since the remainder is what is left of the minuend after taking therefrom the units of the subtrahend,

it is evident that the subtrahend and the remainder combined contain the same number of units as the minuend; hence addition and subtraction are opposites, and each will prove the other. Two practical methods of proof for subtraction are the following:

1. Add the remainder to the subtrahend; the sum will be equal to the minuend.

2. Subtract the remainder from the minuend; the difference will be equal to the subtrahend.

EXAMPLES FOR PRACTICE.

	1.	*2.*	*3.*	*4.*
From	4873	35704	3120	321487
Take	1394	7281	196	321399
Rem.	3479	28423	2924	88

5. From 1703421 take 648394.

6. 54032187—29874185=how many?

7. 3870042—998745=how many?

8. 100000000—1990=how many?

9. The population of a certain city in 1890 was 237845; in 1900 it was 248763. What was the increase during the ten intervening years?

10. The bonded indebtedness of a certain city in 1901 was $154850; during the next two years the amount was reduced $72155. What was the indebtedness in 1903?

11. During a certain year the imports of the United States amounted to $634185350, and the exports to $826324575. How much did the exports exceed the imports?

12. The population of Chicago, in 1900, was 1698575, and that of St. Louis, 575238. How many more inhabitants did Chicago contain at that time than St. Louis?

13. *The resources* of a manufacturing firm amount to

$84275, and their liabilities are $16725. What is the net capital of the firm?

14. A merchant's sales during a given time amounted to $38742. Assuming that the cost of the goods sold was $35984, what was his gain?

15. A has $1500 less than C, and $3240 more than B; C has $18750. If D's money is equal to the difference between C's and B's, how much has he?

16. The native born inhabitants of a certain city number 124872; the German population numbers 57860; the Scandinavian, 32548; the Irish, 5216; the French, 4817. The foreigners from the West India Islands are as many as the difference between the Irish and the French population. How much does the native born exceed in number the difference between the Scandinavians and the West India Islanders?

EXAMPLES IN ADDITION AND SUBTRACTION.

65. Two operations may frequently be combined in one, as shown by the following illustrative example:

1. What number added to the sum of 584, 392, and 765 will make 2138?

Operation.

584
392
765
397
2138

EXPLANATION. This example naturally suggests two operations; first, the addition of the three addends, and second, a subtraction of the sum thus obtained from the given amount; however, one computation can be made to serve the purpose. We write the numbers as here shown, leaving a space above the footline for the addend sought. Commencing at the top of the unit column, we add downward, thus; 6, 11, and 7 (supplied in the addend sought) are 18; then carrying the 1 ten, we add the tens column downward, thus: 9, 18, 24, and 9 (which we supply) are 33; carrying the 3 to the next column and adding, thus; 8, 18, and 3 (which we supply) are 21. The number 397, written in full-faced figures, is the addend sought.

A practical application of the method of combining two operations in one is very convenient to the bookkeeper in

balancing ledger accounts, as illustrated by the following example:

2. What number will balance the following account?

CASH.

June	5		12	800		June	6		13	432	
"	6		13	254	25	"	"		"	176	50
"	"		"	525		"	8		14	38	20
"	8		14	87	40	"	12		17	215	45
"	"		"	63	75	"	"		"	380	
"	10		15	210		"	13		"	243	10
"	11		16	472	35	"	15		18	184	
"	12		"	117	40						
"	14		17	12		"	"	Balance		***	**
"	15		"	120							
				2662	15					2662	15

NOTE. In the solution of ledger examples like the one above, the following directions should be observed: first, add the side representing the greater amount; second, place this amount under the foot-line of each column; third, find the balance number by a single operation according to the method previously explained.

Find the wanting addend in each of the following examples:

3.	*4.*	*5.*	*6.*	*7.*
4287	3281	1285	327	762
3182	1342	1624	1640	1340
4963	786	398	872	3821
2145	124	245	398	76
1760	1320	7821	146	385
2134	5214	3216	327	19724
7216	817	4218	2482	3286
1354	1292	79	1342	1548
2133	1760	132	820	3724
1654	85	1260	412	916
30951	*21000*	24885	9584	51392

8. A fruit vendor paid 48 cents for apples; he sold part of this stock for 33 cents, and had left a number which cost him 27 cents. How much did he gain?

SUGGESTION. The value received from the sale, plus the value (at cost) of the quantity on hand, represents the full production of value from the speculation; if this amount exceeds cost, he has realized a profit; otherwise, he has sustained a loss.

9. A merchant purchased goods during the first month in which he was in trade, amounting to $6875; his sales, during the same month, amounted to $3115. If the goods on hand at the end of the month were worth, at cost, $4235, did he gain or lose, and how much?

10. A dealer had on hand, at the beginning of a certain month, merchandise valued, at cost, $12534; during the month his purchases were $3275; his sales, during the given time, amounted to $4136; the goods on hand, at the close of the month, were worth, at cost, $10680. What amount did he gain or lose?

11. The debit side of a merchandise account, at the close of the year, amounted to $46845, *cost*, the credit side amounted to $28146, *sales*; the inventory of goods on hand exhibited a cost value of $26852. How much did he gain or lose?

12. A, B, C, and D form a partnership. A's investment is $12500: B's is $825 more than A's; C's is $2275 less than B's and the sum of A's and C's minus B's is equal to D's. What is the capital of the firm?

MULTIPLICATION.

66. Multiplication is the process of repeating the unit value of a number any given number of times, by a shorter operation than addition.

67. The **Multiplicand** is the number whose unit value is to be repeated.

68. The **Multiplier** is the number that shows how many repetitions of the multiplicand are required.

69. The **Product** is the result obtained by the process of multiplication.

70. The **Sign of Multiplication** is an oblique cross, made thus, $(\times)$. It signifies that the number before it is to be multiplied by the number after it; thus, $12\times5=60$; read 12 multiplied by 5 equals 60.

NOTE. A few authors of arithmetic place the multiplier before the sign and the multiplicand after it; this plan, however, is inconsistent with the general principle, which requires that the number to be operated upon, in all computations, should be written before the sign, and the instrument by which the operation is affected after it. The incorrect rendering of the sign, *times* in place of the correct expression *multiplied by*, is the cause of the inconsistency. Such expressions as *5 times 6*, and *7 times 8*, are allowable, but the expression $4\frac{2}{3}$ *times 9* is meaningless. We can express $\frac{2}{3}$ of 9, but we cannot repeat 9 $\frac{2}{3}$ of a time. Argument may possibly be advanced favoring the use of such an expression as $4\frac{2}{3}$ times 9, or $5\frac{1}{4}$ times 8, as a conventionalism.

71. Continued Multiplication is an operation containing more than one multiplier; as $5\times3\times2=30$; read 5 multiplied by 3 multiplied by 2 equals 30.

72. The **Factors** are the multiplicand and multiplier, or multipliers.

73. The **Index of an Order** is the number which marks its place from units. The index of 3 in the number 328 is 2, since it occupies the 2d place from units. Indices

when designated, are usually marked by accent marks, thus, 4″3′6. The index of units of the first order is 0.

REMARKS. 1. In all computations in commercial affairs requiring multiplication, one of the given numbers, and *only one*, is the logical or true multiplicand; but for the simple purpose of finding the number of units in the product, either number in the operation may be taken as the multiplicand, since the number of units in the product will be the same regardless of the order in which the factors are multiplied.

2. The multiplier is always an *abstract* number. Such expressions as "5 dollars times 8," and "3 bushels times 7," are not only meaningless but vulgar.

3. The product is of the same name as the multiplicand, since it is the multiplicand repeated.

74. The computations of multiplication depend upon the following principles:

1. *If units be multiplied by units, the product will be units; if tens be multiplied by units, or units by tens, the product will be tens; and generally, any order of units in the product has an index equal to the sum of the indices of the orders multiplied together to produce it. Units whose index is 4, multiplied by units whose index is 3, will give an order in the product whose index is 7.*

2. *If the units of the multiplicand be taken separately as many times as there are units in each order of the multiplier, the sum of the products will be equal to the entire product of the given numbers.* (Axiom).

3. *Adding any number to either factor, increases the product by as many times the other factor as there are units in the number added; and subtracting any number from either factor diminishes the product by as many times the other factor as there are units in the number subtracted.*

4. *Multiplying either factor by any number multiplies the product by the same number, and dividing either factor by any number divides the product by the same number.*

75. Multiplication table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	**4**	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	**9**	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	**16**	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	**25**	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	**36**	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	**49**	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	**64**	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	**81**	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	**100**	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	**121**	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	**144**	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	**169**	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	**196**	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	**225**	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	**256**	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	**289**	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	**324**	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	**361**	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	**400**	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	**441**	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	**484**	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	**529**	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	**576**	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	**625**
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

NOTE. We advise every student who is fitting himself for mercantile pursuits, to memorize the whole of the above table. In any event, he should memorize the squares of the 25 numbers, which are printed in full-faced figures diagonally across the table.

1. Multiply 283 by 7.

Operation.

Multiplicand	283
Multiplier	7
Product	1981

EXPLANATION. We repeat the unit value of the multiplicand 7 times according to the requirements of the example. This is accomplished by repeating the unit value of each order 7 times, (Prin. 2), and carrying as in addition. We write each figure in the place of its respective order, and obtain for the correct result 1981.

Multiply 635 by 234.

)n. EXPLANATION. In this example we multiply the multipli-
5 cand by each figure of the multiplier, separately. Multiplying
4 635 by 4 units gives 2540 units (Prin. 1); 635 multiplied by 3
0 tens gives 1905 tens (Prin. 1), which we write with the lowest
order in tens' place, or directly under the figure by which we
multiplied; 635 multiplied by 2 hundreds furnishes a result of
1270 hundreds (Prin. 1), which we write with the lowest order
0 in hundreds' place. Since the sum of these partial products
must be the entire product of the given factors (Prin. 2), we
n together and obtain 148590, the correct result.

From the foregoing principles and illustrations we) the following:

LE I. *Write the multiplier under the multiplicand, plac-* ; *right hand significant figure thereof under the right ignificant figure of the multiplicand.*

Multiply the multiplicand by each significant figure of ltiplier, beginning with the lowest order, and write the ure of each partial product directly under the figure mul- by, carrying as in addition.

. *If there are partial products, combine them by addi- nd the sum will be the complete product.*

ARK. If there are ciphers on the right of one or both factors, the mber of ciphers must be annexed to the product to preserve the units.

There are two practical methods of proving multi- ion:

By changing the order of the factors; that is, by)lying the multiplier by the multiplicand; if the product is obtained as by the first multiplication, the is presumably correct.

By division.

vide the product by one of the factors; if the quo-)btained is equal to the other factor, the work may sumed to be correct.

NOTE. To those who have never before studied arithmetic, this method of proof should not be presented until they have read the chapter pertaining to division.

EXAMPLES FOR PRACTICE.

3. Multiply 587 by 8.

4. Multiply 2472 by 23.

5. Multiply 8765 by 76.

6. Multiply 52163 by 357.

7. Multiply 51243 by 215.

8. Multiply 32076 by 685.

9. Multiply 15734 by 8970.

10. Multiply 245342 by 2407.

11. Multiply 305740 by 10042.

12. What is the product of 5431×527?

13. What is the product of 15432×8046?

14. What is the product of 6040328×105407?

15. What will it cost to build 195 miles of railroad at $58750 per mile?

16. A fruit dealer bought 7 car loads of apples, each containing 195 barrels, at $3 per barrel; what was the entire cost?

17. Two men start from the same place, and travel in opposite directions, one at the rate of 24 miles a day, the other at the rate of 31 miles a day; how far apart will they be at the end of 8 days?

18. If 15 men can do a piece of work in 80 days, how long will it take 1 man to do the same work?

19. If a carriage wheel makes 328 revolutions in going 1 mile, how many revolutions will it make in going 93 miles?

20. Of what number are 15, 56, 35, and 112 the factors?

21. *If, in a certain state, there are 3825 miles of railroad in operation, and the average cost of construction was*

$1423 per mile, what was the total cost of the railroads f that state?

22. A bundle of paper contains 5 reams, each ream ontains 20 quires, and each quire 24 sheets; how many heets of paper in 6 bundles?

23. A government township contains 36 sections of 640 cres each; how many acres of land in a county containng 22 townships?

POWERS OF NUMBERS.

78. A **Power** is a product of equal factors.

79. The **First Power** of a number is the number itself.

80. The **Second Power** is the product of two equal factors; thus, $3\times3=9$. 9 is the second power of 3.

81. The **Third Power** is the product of three equal factors; thus, $5\times5\times5=125$. 125 is the third power of 5.

NOTE. In the higher mathematics, the 1st power is called the *root;* the 2d power the *square*, and the 3d power the *cube* of the number expressed by one of the factors.

82. The **Index** or **Exponent** is a small number placed at the right of a quantity expressing the number of times it occurs as a factor; thus, $7^3=7\times7\times7=343$.

83. The following illustrative example will render the above definitions and explanations more clear.

1. What is the fourth power of 15?

Operation.

$\overline{15}^4=15\times15\times15\times15=50625$.

The operation is self-explanatory.

EXAMPLES FOR PRACTICE.

2. What is the square (2d power) of 35?

3. What is the cube (3d power) of 18?

4. *What is the fifth power* of 6?

5. Find the powers indicated by the expressions $\overline{31}^4$, $\overline{123}^2$.

6. What is the product of $5^3 \times 8^2$?

7. What is the product of $12^3 \times 3^5$?

CONTRACTIONS IN MULTIPLICATION.

CASE I.

84. When the multiplier is a unit of any order.

Annexing a cipher to the multiplicand removes each figure one place to the left; consequently the value of the entire number is increased tenfold (42, 3). Annexing two ciphers increases the value one hundredfold, etc. Hence the following:

RULE. *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

EXAMPLES FOR PRACTICE.

1. Multiply 874 by 100.

2. Multiply 1487 by 1000.

3. What is the cost of 1000 barrels of flour at $6 a barrel?

4. What will it cost to build 10000 miles of railroad at $45280 a mile?

5. Multiply one million by ten thousand.

CASE II.

85. When the multiplier is a composite number.

86. A **Composite Number** is one that may be produced by multiplying together two or more numbers. Thus, 15 is a composite number, since $5 \times 3 = 15$.

1. Multiply 256 by 21.

Operation.

256
3
768
7
5376

EXPLANATION. The factors of 21 are 3 and 7. We multiply the multiplicand by 3, and the result thus obtained by 7, and obtain 5376. The multiplication by 3 gives 3 times the multiplicand, and 7 times 3 times the multiplicand gives 21 times the multiplicand. Hence the following:

RULE I. *Separate the multiplier into two or more factors.*

II. *Multiply the multiplicand by one of these factors, and that product by another, and so continue until all the factors have been used successively; the last product will be the required result.*

NOTE. The factors may be used in any order desired.

EXAMPLES FOR PRACTICE.

2. Multiply 1284 by 42.

3. Multiply 523 by 35.

4. Multiply 728 by 30 (3×10).

5. Multiply 2435 by 72.

6. What will it cost to construct 84 miles of railroad at \$57162 per mile?

CASE III.

87. When there are ciphers on the right of one or both factors.

1. Multiply 3200 by 30.

Operation.

3200
30
96000

EXPLANATION. The factors of the multiplicand are 32×100; the factors of the multiplier are 3×10; hence, according to the former cases, we have $32 \times 3 \times 100 \times 10$. We first multiply 32 by 3, and the product thus obtained by 100 by annexing two ciphers, and this product by 10 by annexing one cipher. Hence the following:

RULE. *Multiply the significant figures of the factors together, and to the product annex as many ciphers as there are ciphers on the right of both factors.*

EXAMPLES FOR PRACTICE.

2. Multiply 840 by 70.
3. Multiply 26000 by 500.
4. Multiply 104200 by 7000.
5. Multiply 1456 by 123000.
6. Multiply 1300 by 503.

CASE IV.

88. When the multiplier contains but two significant figures, the left hand figure being 1.

1. Multiply 12547 by 14.

Operation.

12547
50188
175658

EXPLANATION. Multiply the multiplicand by the unit figure of the multiplier, and place the product as many places to the right as there are orders of units in the multiplier, less one. In the example here given we place the product *one* place to the right; had the multiplier been 104, we should have placed it *two* places to the right. This product added to the multiplicand gives the required result. By the operation, the multiplicand is carried one place to the left and is therefore multiplied by 10, and 10 times the multiplicand plus 4 times the multiplicand, equals 14 times the multiplicand.

CASE V.

89. When the multiplier contains but two significant figures, the unit figure being 1.

1. Multiply 2542 by 301.

Operation.

2542
7626
765142

EXPLANATION. The method is the same as in the former case, except that the product is carried to the left. Multiplying by 3 and placing the product two places to the left multiplies by 300, and 300 times the multiplicand plus the multiplicand equals 301 times the multiplicand.

EXAMPLES FOR PRACTICE.

2. Multiply 3524 by 17. (Case IV.)
3. *Multiply* 18752 by 1004.
4. *Multiply 5876* by 61. (Case V.)

5. Multiply 512876 by 3001.

6. Multiply 38542 by 501.

CASE VI.

90. When each figure of the multiplier is 9.

1. Multiply 2854 by 999.

Operation.

2854000
2854
2851146

EXPLANATION. We annex three ciphers to the multiplicand, thus multiplying by 1000; this makes the product too large by once the multiplicand (74 4); therefore, the multiplicand must be subtracted. Hence the following:

RULE. *Annex as many ciphers to the multiplicand as there are nines in the multiplier, and subtract the multiplicand from the result.*

EXAMPLES FOR PRACTICE.

2. Multiply 574 by 99.

3. Multiply 1872 by 9999.

4. Multiply 32851 by 999.

5. Multiply 7543 by 99999.

CASE VII.

91. When one part of the multiplier is a factor of another part.

1. Multiply 1542 by 279.

Operation.

1542
279
13878 Prod. by 9
41634 Prod. by 270
430218

EXPLANATION. In this example 9, the unit figure of the multiplier, is a factor of 27, the other part. We first multiply by 9 as in the ordinary manner; we next multiply this product by 3 and write the first figure in tens' place. Now it is evident that the second partial product is obtained in the following manner: Multiplicand $\times 9 \times 3 \times 10$, which is the same as to multiply the multiplicand by 270, and the multiplication by 9 plus the multiplication by 270 equals the multiplication by 279.

2. Multiply 253416 by 48246.

Operation.		
253416		
48246		
1520496	Prod. by	6
6081984	Prod. by	240
12163968	Prod. by	48000
12226308336	Prod. by	48246

EXPLANATION. In this example the unit figure is a factor of 24, and 24 is a factor of 48, which stands to the left. We obtain the first two partial products as in the former example; we then multiply this second product by 2 and place the first figure two places to the left, or in thousands' place. This last product is the result of the following operation: Multiplicand $\times 6 \times 4 \times 10 \times 200$, or, what is the same thing, the multiplicand multiplied by 48000; therefore, the sum of these partial products must be the true product.

3. Multiply 52174 by 21735.

Operation.		
52174		
21735		
365218	Prod. by	700
1826090	Prod. by	35
1095654	Prod. by	21000
1134001890	Prod. by	21735

EXPLANATION. In this example the figure 7 of the multiplier is a factor of 35, which stands to the right, and of 21 on the left. We first multiply by 7 and write the first figure in the place of hundreds; we next multiply this product by 5 and write the first figure in units' place; we next multiply the same product by 3 and write the first figure in thousands' place. A careful analysis will reveal the following computations: multiplicand $\times 7 \times 100$ equals first partial product. Multiplicand $\times 7 \times 5$ equals second partial product. Multiplicand $\times 7 \times 100 \times 3 \times 10$ equals third partial product. Therefore, the sum of these partial products must be the product by 21735.

NOTE. A careful study of the above explanations should enable the student to formulate a rule.

CASE VIII.

92. When the multiplier is 11.

1. Multiply 387254 by 11.

Operation.	
387254	Multiplicand.
4259794	Product.

EXPLANATION. It will be readily observed that if any intergral number be multiplied by 11 in the ordinary manner, the partial products (each of which is the multiplicand), *will be so arranged* in position that, when added, the unit figure of the *product will be the* unit figure of the multiplicand; that the tens' figure of *the product is* equal to the units' digit plus the tens' digit of the multipli-

cand; that the hundreds' figure of the product is equal to the tens' digit plus the hundreds' digit; and generally any figure belonging to the product if found by adding to the corresponding order in the multiplicand the figure immediately preceding it, carrying 1 for each 10 as in any other case in addition. Observing these principles, we perform the above example by first writing the unit figure 4, then 4 and 5 are 9, which we write for the second place; 5 and 2 are 7, the third figure; 2 and 7 are 9, the fourth figure; 7 and 8 are 15; writing the 5 we carry the 1 to the sum of 8 and 3, making 12, carrying the 1 to 3 gives 4, the last figure.

NOTE. The rule is clearly set forth in the above explanation.

2. Multiply 3842 by 11.

3. Multiply 53076 by 11.

4. What is the product of 58764 by 11?

5. At 11 cents a pound what is the cost of an invoice of bacon weighing 3472 pounds?

CASE IX.

93. When the sum of the units of the two factors is equal to 10 and the remaining parts of the factors are equal.

1. Multiply 57 by 53.

Operation.

57
53
3021

EXPLANATION. The 7 units multiplied by 3 units give 21 units (74, 1), which we write for the first two places of the product; the 5 tens multiplied by 3 plus the 5 tens multiplied by 7 is equal to the 5 tens multiplied by 1 ten, therefore the 5 tens of the multiplicand are actually multiplied by a number of tens one more than is expressed by the figure in the multiplier. In consideration of this fact we multiply 5 tens by 6 tens, obtaining 30 hundred, which we write in its place, and we have the complete product, 3021.

94. The foregoing explanation of principles suggests the following:

RULE I. *Multiply the units of the multiplicand by the units of the multiplier and write the entire result underneath.*

II. *Multiply the remaining part of the multiplicand by the remaining part of the multiplier increased by 1, and write the entire result to the left of what was obtained by the first multiplication.*

NOTE. *If the product of the first multiplication contains but one figure, place a cipher in tens' place.*

EXAMPLES FOR PRACTICE.

2. Multiply 75 by 75.
3. Multiply 89 by 81.
4. Multiply 114 by 116.
5. Multiply 78 by 72.
6. Multiply 993 by 997.
7. Multiply 194 by 196.

CASE X.

95. When the difference between the factors is exactly divisible by 2.

It will be necessary, before commencing a study of the principles employed in this case, to comprehend the following:

THEOREM.

The product of the sum and difference of two numbers is equal to the difference of their squares.

PROOF. Let the two numbers be 8 and 5; then, 8+5 will represent the sum, and 8—5 their difference. Now it is evident that the product obtained by multiplying 8+5 by 8, minus the product obtained by multiplying 8+5 by 5, will be equal to the product obtained by multiplying 8+5 by 3 (the difference between 8 and 5). The product of 8+5 by 8 is 8^2+40, and the product of 8+5 by 5 is $40+5^2$. Subtracting the last product from the first gives 8^2-5^2, which proves the truth of the theorem.

1. What is the product of 21×19?

Operation.

$20^2=400$
$1^2=\ 1$
399

EXPLANATION. These factors may be written 20+1 and 20—1, and since they represent the sum of 20 and 1, and the difference of 20 and 1, their product must be $20^2-1^2=400-1=399$.

NOTE. This contraction is not practical in cases where the numbers *are so large that* their squares cannot be determined mentally. The *student, however, should* memorize the squares of all numbers to 25.

96. From the principles embodied in the above theorem and analysis, we deduce the following:

RULE I. *Select a number half way between the two factors in unit value.*

II. *From the square of this number subtract the square of half the difference of the factors.*

EXAMPLES FOR PRACTICE.

2. Multiply 32 by 28. (30+2) (30—2.)
3. Multiply 22 by 26.
4. Multiply 97 by 103.
5. Multiply 44 by 36.
6. Multiply 17 by 19.

CASE XI.

97. To square a number whose digits are all 9.

RULE. *Begin at the left and write as many 9's, less one, as are in the given number, then write 8, then as many ciphers as 9's, and finally the unit 1.*

NOTE. An inspection of a product obtained by actual multiplication will prove the truth of the rule.

EXAMPLES FOR PRACTICE.

1. What is the square of 999? Ans. 998001.
2. What is the square of 99999?
3. What is the square of 99?
4. What is the square of 99999999?

CASE XII.

98. To find the product by a single multiplication.

1. Multiply 72 by 26.

Operation.

72
26
1872

EXPLANATION. By referring to (74, 1) it will be observed that units multiplied by units give units, that tens multiplied by units, and units multiplied by tens, give tens, and that, in all cases, the sum of the indices of any two orders multiplied together *will* determine the order to which such product belongs. *Taking advantage of this* principle, we multiply 2 units by 6 units

and obtain 12 units (1 ten 2 units); we write the 2 units and carry the 1 ten, and find the next order thus; $1+(7\times6)+(2\times2)=47$ (tens)=(4 hundreds 7 tens); we write the 7 tens and proceed in the following manner to obtain the next place: $4+(7\times2)=18$ (hundreds), which we write, thus obtaining the complete product, 1872.

2. Multiply 426 by 32.

Operation.

$$\begin{array}{r} 4''2'6^\circ \\ 3'2^\circ \\ \hline 1^{\text{IV}}3'''6''3'2^\circ \end{array}$$

EXPLANATION. Notice that the sum of the indices of orders multiplied together to make units must be 0, that the sum of the indices to make tens must be 1, to make hundreds, 2, etc. The different orders in the product are found in the following manner: $6^\circ\times2^\circ=1'2^\circ$; $1'+(2'\times2^\circ)+(6^\circ\times3')=2''3'$; $2''+(4''\times2^\circ)+(2'\times3')=1'''6''$; $1'''+(4''\times3')=1^{\text{IV}}3'''$.

NOTE. In most examples solved by this method mistakes will be almost sure to occur unless extreme caution is exercised; therefore, we do not consider it practical in application. A study of the plan, however, will prove beneficial to the student by fixing in mind the relations of different orders of units.

EXAMPLES FOR PRACTICE.

3. Multiply 542 by 235.

4. Multiply 384 by 27.

5. Multiply 3285 by 24.

DIVISION.

99. *1.* If a number contains 12 units, how many times can 4 be subtracted and leave nothing remaining?

Answer, 3 times.

2. How many times can 5 apples be taken from a basket containing 17 apples?

Answer, 3 times, and 2 apples will remain.

NOTE. The number $3\frac{2}{5}$ does not furnish a correct answer to the above question.

3. If 2 melons be divided equally among 4 boys, how much will each boy receive?

A study of the first question given above might lead us to believe that *division is a short method of* performing subtraction in cases where the

subtrahends are equal; or, that it is a process of finding how many times one number is contained in another. Both of these definitions appear in books of different authors, but it will be seen, by a careful investigation of the second and third questions above, that these definitions are true only in *special* cases. We want a definition sufficiently broad to cover *every* case.

100. Division is the process of finding a wanting factor when the product and the other factor or factors are given.

101. The **Dividend** is the number to be divided. It corresponds to the product in multiplication.

102. The **Divisor** is the number by which the division is performed. It corresponds to one of the factors in multiplication.

103. The **Quotient** is the result obtained by division. It, like the divisor, corresponds to one of the factors in multiplication.

104. The **Remainder** is that part of the dividend remaining when the division is not exact.

NOTE. The division is said to be exact when the divisor is contained in the dividend an exact number of times, without a remainder.

105. The **Sign of Division** is a short horizontal line between two dots, made thus: (÷). It is read "divided by," and signifies that the number before it is to be divided by the number after it.

NOTE. The operation of division is signified also by the dots (colon), without the line, and also by writing the dividend above, and the divisor below, a horizontal line. Thus, $\frac{24}{8}$ signifies that 24 is to be divided by 8.

106. The computations of division depend upon the following principles:

1. *If each of the separate parts of a number be divided, the entire number will be divided.*

2. *Since the remainder resulting from the division of any order is less than the divisor, it must be reduced to a lower order; hence.*

3. *The operation must commence with the units of the highest order.*

1. Divide 6396 by 3.

Operation.

3)6396
2132

EXPLANATION. We place the divisor to the left of the dividend, and commence by dividing 6 (thousands) by the divisor 3; we find that it is contained 2 (thousands) times; writing the result in thousands' place, we advance to the next place and find that the divisor is contained in 3 (hundreds) 1 (hundred) times; we write the 1 in hundreds' place and divide the 9 (tens) and obtain for a result 3 (tens), which we write in its order, and then divide the last order, 6 units, and obtain 2 units, which we write in units' place.

2. Divide 1725 by 5.

Operation.

5)1725
345

EXPLANATION. In this example we find that the left-hand figure will not contain the divisor, hence we begin by dividing 17 (hundreds) by 5; we obtain 3 with a remainder of 2 (hundreds); reducing the 2 hundreds to tens and adding the 2 tens, gives 22; 5 is contained in 22 4 times, with a remainder of 2 (tens); reducing this remainder to units and adding the 5 units, gives 25 units, which will contain 5 exactly 5 times; writing this result in its place gives the complete quotient, 345.

3. Divide 18725 by 73.

Operation.

73)18725(256$\frac{37}{73}$
146
412
365
475
438
37

EXPLANATION. We take the fewest figures that will contain the divisor, and find that 73 is contained in this number (187) 2 times; we write this figure for the first figure of the quotient and multiply the divisor by it, writing the result under that part of the dividend divided; subtracting this result from 187, we have a remainder of 41 (hundreds), which we combine with the 2 tens brought down to the right; 73 is contained in this number (412) 5 times; multiplying the divisor by this quotient figure, and subtracting the result as before, we have a remainder of 47, to which we annex the next and last figure of the dividend, and, continuing the work as before, we obtain 6 for the last quotient figure; performing the work of multiplication and subtraction as formerly, we have a final remainder of 37, the division of which we indicate by writing it over the divisor at the right of the quotient.

107. From the foregoing principles and illustrations we *derive the following*:

RULE 1. Beginning at the left hand, select the fewest figures

of the dividend that will contain the divisor one or more times; find how many times the divisor is contained in this partial dividend, and write the result for the first figure of the quotient; multiply the divisor by this quotient figure and subtract the product from that part of the dividend divided.

II. *To the remainder bring down the next figure of the dividend, and proceed as before, and thus continue until all the figures of the dividend have been brought down and divided.*

III. *If, at any time, the remainder, increased by the figure brought down, is less than the divisor, place a cipher in the quotient, bring down the next figure of the dividend and proceed as before, remembering that each figure brought down will add a figure to the quotient.*

IV. *If the division be not exact, place the final remainder over the divisor at the right of the integral part of the quotient.*

108. PROOF. There are two practical methods of proving division.

1. By multiplication.

Multiply the divisor and quotient together, and to the product add the remainder, if any; if the result is equal to the dividend, the work is presumably correct.

2. By division.

Subtract the remainder, if any, from the dividend, and divide the result by the quotient; if a number equal to the divisor results from the division, the work is presumed to be correct.

EXAMPLES FOR PRACTICE.

4. Divide 15848 by 4.
5. Divide 25398 by 6.
6. Divide 487315 by 5.
7. Divide 347886 by 9.
8. Divide 124864 by 32.
9. Divide 34804 by 234.

10. Divide 2587476 by 1542.

11. Divide 387640 by 2431.

12. Divide 58764329 by 34287.

13. Divide 27482635 by 284320.

14. Divide 8463218 by 24372.

15. Divide 17826432 by 1523264.

16. What is the value of $873250 \div 125$?

17. What is the value of $6215668 \div 2342$?

18. What is the value of $125 \div 1125$?

Operation.

$$1125)\,125(0\tfrac{125}{1125}$$
$$\underline{0}$$
$$125$$

EXPLANATION. In this example, in which the dividend is less than the divisor, it will be seen that the principles already presented are sufficient for our purpose. Since the dividend will not contain all the units of the divisor, we place a cipher in the quotient. Multiplying the divisor by the quotient figure gives *nothing* for a product, and nothing subtracted from 125 gives 125 for a remainder; this sum placed over the divisor gives $\frac{125}{1125}$, the correct quotient.

NOTE. It may seem to the student who is not far advanced in the science of numbers, that the less of two numbers cannot be divided by the greater, but careful thought will remove the absurdity of such an idea; for surely 2 apples can be divided equally among 8 boys. From the operation above it will be seen that the division of a less number by a greater is affected by placing the less over the greater.

19. What is the value of $2 \div 4$? *Answer.* $\frac{2}{4}$.

20. Divide 1 by 8. *Answer.* $\frac{1}{8}$.

21. Divide 13 by 57.

22. $135 \div 431$ = how many?

23. $431 \div 135$ = how many?

24. A section of land containing 640 acres was sold for $26880. What was the price per acre?

25. A man paid $3825 for 45 horses. What was the average price per head?

26. If a railroad 232 miles in length cost, for construction and equipment, $10496376, what was the average cost per mile?

GENERAL PRINCIPLES OF DIVISION.

109. The quotient in division depends upon the relative values of the dividend and divisor; hence any change made upon either dividend or divisor will affect this relation, and, consequently, will change the value of the quotient; however, certain changes may be made upon both dividend and divisor, at the same time, in such a manner as not to change the quotient.

If any divisor is contained in a given dividend a certain number of times, it is evident that it will be contained in twice the same dividend twice as many times; in three times that dividend, three times as many times, etc.

If any divisor is contained in a given dividend a certain number of times, it is contained in one-half the same dividend one-half as many times; in one-third the dividend, one-third as many times, etc.

If a given divisor is contained in any dividend a certain number of times, it is evident that twice that divisor will be contained in the same dividend one-half as many times; three times the divisor one-third as many times, etc.

If a given divisor is contained in any dividend a certain number of times, one-half that divisor is contained twice as many times; one-third that divisor three times as many times, etc.

If a given divisor is contained in a given dividend a certain number of times, twice the divisor will be contained in twice the dividend the same number of times; three times the divisor will be contained in three times the dividend the same number of times, etc.

If a given divisor is contained in a given dividend a certain number of times, one-half the divisor will be contained in one-half the dividend the same number of times; one-third the divisor in one-third the dividend the same number of times, etc. Hence the following principles:

1. *Multiplying the dividend by any number multiplies the quotient by the same number.*

2. *Dividing the dividend by any number divides the quotient by the same number.*

3. *Multiplying the divisor by any number divides the quotient by the same number.*

4. *Dividing the divisor by any number multiplies the quotient by the same number.*

5. *Multiplying both dividend and divisor by the same number does not change the value of the quotient.*

6. *Dividing both dividend and divisor by the same number does not change the value of the quotient.*

7. *Numbers having the same relations, respectively, as the dividend and divisor, may be added to those terms without changing the value of the quotient.*

8. *Numbers having the same relations, respectively, as the dividend and divisor, may be subtracted from those terms without changing the value of the quotient.*

We present the following in illustration of the above cases:

	Dividend.		Divisor.		Quotient.	
	32	÷	8	=	4	
1.	64	÷	8	=	8	Multiplying the dividend by 2 gives twice as large a quotient.
2.	16	÷	8	=	2	Dividing the dividend by 2 gives one-half as large a quotient.
3.	32	÷	16	=	2	Multiplying the divisor by 2 gives one-half as large a quotient.
4.	32	÷	4	=	8	Dividing the divisor by 2 gives twice as large a quotient.
5.	64	÷	16	=	4	Multiplying both dividend and divisor by 2 gives the same quotient.
6.	16	÷	4	=	4	Dividing both dividend and divisor by 2 gives the same quotient.
7.	36	÷	9	=	4	Adding 4 to the dividend and 1 to the divisor (they having the same relation as 32 and 8) does not change the quotient.
8.	28	÷	7	=	4	Subtracting 4 from the dividend and 1 from the divisor does not change the value of the quotient.

110. The above principles may be embraced in the following:

GENERAL LAW.

A change in the dividend produces a like change in the quotient; but a change in the divisor produces an opposite change in the quotient.

CONTRACTIONS IN DIVISIONS.

CASE I.

111. When the divisor is a composite number.

1. Divide 11445 by 35.

Operation.

5)11445
7)2289
327

EXPLANATION. The factors of 35 are 5 and 7. We divide the dividend by 5, and the resulting quotient by 7, and obtain for the final result, 327, which is equal to the quotient of the dividend by 5 times 7, or 35. Had we first divided by 7, and then by 5, it is evident that the same result would have been obtained. Hence the following:

RULE. *Divide the dividend by one of the factors, and the quotient thus obtained by another, and so proceed until each factor has been made a divisor. The last quotient will be the quotient required.*

TO FIND THE TRUE REMAINDER.

112. If remainders occur by the successive divisions, it must not be supposed that their sum will be the true remainder. We trust that the following explanation will render this statement clear:

2. Divide 5876 by 105, using the factors 3, 5, and 7.

Operation.

3)5876
5)1958 2
7)391 $3\times3=9$
55 $6\times5\times3=90$
True remainder 101

EXPLANATION. The remainder, 2, resulting from the first division, being directly a part of the dividend, is a part of the true remainder. The second remainder, 3, results from dividing a *third* of the dividend by 5, hence we must multiply this remainder by 3, as each of the other thirds divided would give 3. The third remainder results from dividing one-*fifteenth* of the dividend by 7, hence we must

multiply this remainder, 6, by 15 (3×5), the preceding divisors. These products and the first remainder added together make the true remainder.

113. From this explanation we derive the following:

RULE I. *Multiply each partial remainder by all the preceding divisors.*

II. *To the sum of these products add the first remainder; the sum will be the true remainder.*

EXAMPLES FOR PRACTICE.

NOTE. In each of the following examples you are required to find both quotient and remainder.

3. Divide 645 by 15 (3×5).
4. Divide 10626 by 42 (6×7).
5. Divide 8344 by 56 (2×4×7).
6. Divide 7682 by 35 (5×7). *Remainder*, 17.
7. Divide 25817 by 84 (2×2×3×7). *Remainder*, *29*.
8. Divide 18543 by 126 (7×3×6).
9. Divide 453284 by 135 (3×3×3×5).
10. Divide 763840 by 504 (2×2×2×3×3×7).
11. Divide 58761 by 180 (5×9×4).
12. Divide 38763 by 420 (4×3×5×7).

CASE II.

114. When the divisor is a unit of any order.

If the right hand figure of a number be cut off or removed, the remaining figures are removed one place toward the right, and consequently diminished in value tenfold, or divided by 10 (42, 3). For a similar reason, the cutting off of two figures will divide by 100; by cutting off three figures we divide by 1000, etc. The figures cut off will constitute the remainder. Hence the

RULE. *Cut off as many figures from the right of the dividend as there are ciphers in the divisor. Under the figures cut off place the divisor, and the whole will* form the quotient.

EXAMPLES FOR PRACTICE.

1. Divide 140 by 10.
2. Divide 238 by 100. *Answer*, $2\frac{38}{100}$.
3. Divide 54823 by 1000.
4. Divide 34876 by 10000. *Answer*, $3\frac{4876}{10000}$.
5. Divide 4003702 by 1000.

CASE III.

115. When the right hand figure, or figures, of the divisor are ciphers.

1. Divide 38726 by 400.

Operation.

4|00)387|26
96 Rem. 3.

EXPLANATION. By cutting off the two ciphers in the divisor we separate it into the factors 100 and 4. Dividing by 100 as in the former case, we have 387 for a quotient, and a remainder of 26. Dividing 387 by the other factor, 4, gives 96 for a quotient and a second remainder of 3. Multiplying this remainder by the former divisor and adding the first remainder, 26, gives for the true remainder 326. By a careful analysis it will be seen that the figures cut off from the dividend annexed to the last remainder will be the true remainder. From the explanation here given, we deduce the following:

RULE I. *Cut off the ciphers of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor for the final quotient.*

III. *Annex the figures cut off from the dividend to the last remainder for the true remainder.*

EXAMPLES FOR PRACTICE.

2. Divide 3842 by 50.
3. Divide 58460 by 300.
4. Divide 28764 by 7000.
5. Divide 15876218 by 400000.

CASE IV.

116. When the divisor is one of the equal parts into which *10, 100, or 1000 may* be divided.

1. Divide 2934 by 25.

Operation.
2934
4
117|36

EXPLANATION. The divisor being the fourth part of 100, we multiply the dividend by 4; we then cut off the two right hand figures of this product, thereby dividing by 100, which is 4 times the true divisor. The quotient must be correct, since both dividend and divisor have been multiplied by the same number (109, Prin. 5). The complete quotient is $117\frac{36}{100}$. Since the dividend was multiplied by 4, the number cut off, 36, is 4 times the true remainder.

NOTE. A careful study of the above explanation will give the student a clear understanding of the following general directions:

1. To divide by $2\frac{1}{2}$, multiply the dividend by 4 and cut off one place.

2. To divide by $3\frac{1}{3}$, multiply the dividend by 3 and cut off one place.

3. To divide by $16\frac{2}{3}$, multiply the dividend by 6 and cut off two places.

4. To divide by 25, multiply the dividend by 4 and cut off two places.

5. To divide by $33\frac{1}{3}$, multiply the dividend by 3 and cut off two places.

6. To divide by $166\frac{2}{3}$, multiply the dividend by 6 and cut off three places.

7. To divide by 250, multiply the dividend by 4 and cut off three places.

8. To divide by $333\frac{1}{3}$, mutiply the dividend by 3 and cut off three places.

NOTE. This case will apply to other divisors than those here given.

EXAMPLES FOR PRACTICE.

2. Divide 537 by $3\frac{1}{3}$.
3. Divide 1462 by $16\frac{2}{3}$.
4. Divide 5846 by $33\frac{1}{3}$.
5. Divide 54821 by $166\frac{2}{3}$.
6. Divide 3748 by $333\frac{1}{3}$.
7. *Divide* 78425 by 250.
8. *Divide* 4826 by $12\frac{1}{2}$.
9. Divide 5163 by 25.
10. Divide 3417 by 50.
11. Divide 18742 by $33\frac{1}{3}$
12. Divide 365 by $16\frac{2}{3}$.
13. Divide 1524 by $12\frac{1}{2}$.
14. Divide 8730 by 5.
15. Divide 1579 by 125.

117. To multiply by the numbers referred to in the above case, it will be necessary to reverse the order of the work. Where we have employed multiplication, we must perform division, and in cases where we have made use of division, we must substitute multiplication.

1. Multiply 254 by $33\frac{1}{3}$.

Operation.

$$3\overline{)25400}$$
$$8466\frac{2}{3}$$

EXPLANATION. The multiplier being one-third of 100, we annex two ciphers, thus multiplying by 100. Since we have multiplied by a number 3 times as large as the true multiplier, the product thus obtained is 3 times the true product (74, 4); hence we must divide by 3.

EXAMPLES FOR PRACTICE.

2. Multiply 5372 by $12\frac{1}{2}$.
3. Multiply 3462 by 25.
4. Multiply 1756 by 125.
5. Multiply 375 by $166\frac{2}{3}$.
6. Multiply 1860 by $16\frac{2}{3}$.
7. Multiply 5490 by $33\frac{1}{3}$.
8. Multiply 2742 by $333\frac{1}{3}$.
9. Multiply 13240 by $12\frac{1}{2}$.

RELATIONS OF APPLIED NUMBERS.

118. The following facts in relation to the character of numbers employed in business affairs, will be found of great value in analytical operations:

1. The addends of an example in addition are like numbers and concrete.

2. The sum is of the same kind as the addends combined to produce it, and is concrete.

3. The minuend, subtrahend, and remainder, occurring in the same example, are like numbers and concrete.

4. The multiplicand is a concrete number.

5. The multiplier is *always* an abstract number.

6. The product is a concrete number, and of the same name as the multiplicand.

7. The *dividend is a* concrete number.

8. The divisor, when it characterizes the multiplicand, is concrete and of the same name as the dividend. When it characterizes the multiplier, it is an abstract number.

9. The quotient is an abstract number when it characterizes the multiplier. When it characterizes the multiplicand, it is concrete and of the same name as the dividend.

10. A remainder left from division, and not regarded as divided, is a part of the dividend; hence it is concrete, and of the same name as the dividend.

119. An analysis of any arithmetical computation will reveal the following elements: *Number of units*, *Unit value*, and *Aggregate value*. By a careful study of the relations of these elements to one another, and to examples in which they are employed, it will become a comparatively easy task to ascertain a wanting element when the other elements are given.

Aggregate value is found either by addition or multiplication,—by addition when unit values are unequal, and by multiplication when unit values are equal.

Unit value is found by subtraction or division,—by subtraction when the unit values are unequal, and by division when all unit values are equal.

Number of units is found by subtraction when unit values are unequal, and by division when unit values are equal.

120. The above facts suggest the following formulas:

1. (The sum of the units) = (aggregate value).
2. (Aggregate value) − (sum of all the units but one) = (that one).
3. (Unit value) × (No. of units) = (aggregate value).
4. (Aggregate value) ÷ (unit value) = (No. of units).
5. (Aggregate value) ÷ (No. of units) = (unit value).

COMPARISON OF TERMS.

121. Each addend represents *unit value.*

The number of addends is the *number of units.*

The sum represents the *aggregate value.*

The minuend represents the *aggregate value.*

The subtrahend represents the *value of one of the units,* or *of all except one.*

The remainder represents the *value of all units not included in the subtrahend.*

The multiplicand represents *unit value.*

The multiplier represents the *number of units.*

The product represents the *aggregate value.*

The dividend represents the *aggregate value.*

The divisor, when concrete, represents *unit value;* when abstract, it represents the *number of units.*

The quotient, when abstract, represents the *number of units;* when concrete, it represents the *unit value.*

122. Employing the terms common to the four computations, we have the following formulas:

1. An addition of the addends=the sum.
2. The less of two numbers+their difference=the greater.
3. The subtrahend+the remainder=the minuend.
4. The sum of two numbers—one of them=the other.
5. The minuend—the subtrahend=the remainder.
6. The minuend—the remainder=the subtrahend.
7. The multiplicand×the multiplier=the product.
8. Any number of factors multiplied together=their product.
9. The divisor×the quotient=the dividend.
10. The dividend÷the divisor=the quotient.
11. The dividend÷the quotient=the divisor.
12. The *product of two* factors÷the given factor=the *other factor.*

13. The product of several factors÷the product of all but one=that one.

REVIEW EXAMPLES FOR ANALYSIS.

123. The following examples are designed for a further illustration of the elements of computation.

1. Three parties form a co-partnership; A invests as capital $3200; B, $3500; and C, $4000; what is the capital of the firm?

ANALYSIS. The sum furnished by each constitutes a collective unit, (one sum of money); hence, the number of units is 3, and they are unequal in value. The aggregate value, (capital of firm) is found by addition. (119, 1.)

2. What is the value of 12 tons of coal at $7 a ton?

In this example 1 ton is the standard unit. The units are 12 in number, and they are equal in value; hence we employ multiplication to find aggregate value. (120, 3.) The analytical statement is as follows:

$7 (unit value)×12 (No. of units)=$84 (aggregate value).

3. A dealer paid $126 for hats; if he paid $3 each, how many did he buy?

ANALYSIS. $126 (aggregate value)÷$3 (unit value)=42 (No. of units).

4. B has three times as much money as A; C has four times as much as A, and D has as much as both B and C. If they all have $225, how much has A?

ANALYSIS. Let the standard unit represent A's money; then by the conditions, A has 1 unit; B, 3 units; C, 4 units; and D, 7 units; therefore the whole number of units is 15, and the aggregate value $225. Hence:

$225 (aggregate value)÷15 (No. of units)=$15 (unit value).

Write an analytical statement of each computation necessary to the solution of the following examples, and, instead of using the terms belonging to the general formulas, make use of such terms as are appropriate to the examples.

5. A man bought 320 acres of land at $18 per acre, expended $1520 in improvements, and then sold his entire purchase at $26 per acre. How much did he gain?

ANALYSIS.

$18 (purchase price per acre)×320 (No. of acres)=$5760 (purchase price).
$5760 (purchase price)+$1520 (cost of improvements)=$7280 (total cost).
$26 (selling price per acre)×320 (No. of acres)=$8320 (selling price).
$8320 (selling price)—$7280 (total cost)=$1040 (gain).

6. What is the cost of 1200 cords of wood at $4 per cord?

7. A clerk's salary is $850 a year. During the year he pays $70 for clothing; his board costs him $16 per month, and his incidental expenses during the year amount to $84. How much does he save per month?

8. A grocer's sales amounted to $72 on Monday, $68 on Tuesday, $63 on Wednesday, $70 on Thursday, $42 on Friday and $87 on Saturday. What were his average sales per day during the week?

9. A speculator bought 640 acres of land for $13440, and sold 350 acres of the same at $23 per acre, and the remainder at $25 per acre. What was his gain?

10. A drover bought of a farmer 23 cows at $26 per head, 53 sheep at $5 per head, and 16 calves at $4 each. He gave in payment a draft on a merchant, amounting to $575, and the remainder in cash. How much cash did the farmer receive?

11. A holds a note against B, the face of which is $1280. Four payments have been endorsed thereon, amounting to $214, $573, $170, and $84 respectively. How much is still due on the note?

12. A farmer sold 160 acres of land at $48 per acre; he received $4230 in cash, and a note for the remainder. What was the face of the note?

13. If $75 be paid for 25 barrels of apples, how many

barrels can be bought for $246 at the same price per barrel?

14. An agent received $1200 with which to purchase horses; after buying 12 he had $336 remaining. What was the average price per head of the number bought?

15. If 18 men can build a bridge in 45 days, how long a time will be required for 30 men to build it?

16. A and B start from the same place at the same time and travel in the same direction; A's rate of speed is 4 miles per hour, and he travels 9 hours per day; B's rate of speed is 3 miles per hour, and he travels 8 hours per day. How far apart will they be at the end of the fourth day?

17. If 12 men can perform one-fourth of a certain job of work in 10 days, how many additional men must be employed to complete the work in 30 days from the time of commencing it?

18. The product of 5 factors is 5040; four of the factors are respectively, 2, 3, 7, and 8. What is the other factor?

19. The sum of two numbers is 872, and their difference is 48. What are the numbers?

20. A house and lot valued at $2800, and four horses, worth $80 each, were given in exchange for 78 acres of land. What was the price of the land per acre?

21. The divisor is 23, the quotient 87, and the remainder is 17. What is the dividend?

22. Wishing to obtain the quotient of a certain number divided by 120, I employ the factors thereof, in the order here given, 3, 5, and 8; from the first division I have a remainder of 2, from the second division the remainder is 3, and the third division furnishes a remainder of 6. What is the true remainder?

23. A, B, C, and D together have $2400; B's money is

twice A's, C's is equal to A's and B's, and D's is twice B's. How much has A?

24. A dealer bought 3 bushels of cranberries at 8 cents a quart and sold them at 10 cents a quart. Considering a bushel 32 quarts, what was his gain?

25. A grocer bought 4 crates of strawberries for $6 and sold them for 10 cents a quart. Allowing 16 quarts to the crate, how much did he gain?

26. A coal dealer paid $875 for a quantity of coal. He sold 160 tons at $5 a ton, and the remainder at $4 a ton, thereby gaining, on the whole invoice, $45. How many tons did he buy?

27. A grocer mixes 120 pounds of Java coffee, worth 30 cents a pound, with 80 pounds of Rio, worth 15 cents a pound. At what price should he sell the mixture so as neither to gain nor lose?

28. An army consists of 2 army corps, each army corps of 3 divisions of infantry, each division of 3 brigades, each brigade of 3 regiments, each regiment of 3 battalions, each battalion of 4 companies and each company of 125 enlisted soldiers and 3 officers. The regimental field and staff consists of 14 officers and 12 enlisted men. The brigade division and corps headquarters include 9 general officers and 70 staff officers. In addition to foregoing, there are 12000 cavalry, 8000 artillery, 400 engineers, 180 officers and men in the signal corps and 280 officers and men in the medical department. What is the total strength of the army?

PROPERTIES OF NUMBERS.

EXACT DIVISION.

124. An **Exact Divisor** of a number is one that gives an integral number for a quotient. It has already been shown *that every divisor is a* factor of the dividend, hence

all divisors of a number are its factors, and all the factors of a number are its divisors.

NOTE. The number itself and unity are not to be regarded as factors or divisors in this connection.

125. An **Even Number** is a number having 2, 4, 6, 8, or 0 for its right-hand figure.

126. An **Odd Number** is a number having 1, 3, 5, 7, or 9 for its right-hand figure.

127. To show the nature of exact division, we present the following series of numbers, each of which is divisible by 5: 5, 10, 15, 20, 25, 30, etc., to any limit. Observe that the series is once 5, two times 5, three times 5, and so on indefinitely. From this series it is evident that any number of times 5 must appear, also that the sum of any two numbers of the series, and the difference of any two numbers, as well, is an exact number of 5's, and therefore must contain 5 an exact number of times. Hence we derive the following principles:

1. *An exact divisor of any number will exactly divide any number of times the number.*

2. *An exact divisor of each of two numbers is an exact divisor of their sum.*

3. *An exact divisor of each of two numbers is an exact divisor of their difference.*

128. From the foregoing principles we derive the following properties:

1. Any number terminating with 0 is exactly divisible by 10; any number terminating with 00 is exactly divisible by 100, etc. (115.)

2. A number is divisible by any factor of 10 if the right-hand figure is divisible by such factor, as the number, *exclusive* of the right-hand figure, is an exact number *of times the said* factor; therefore, 2 will exactly divide

any number terminating with 0, 2, 4, 6, or 8, and 5 will exactly divide any number terminating with 0 or 5.

3. A number is divisible by any factor of 100 if the number expressed by the two right-hand figures is divisible by such factor, since the remaining part of the number is an exact number of times said factor; therefore this test will aptly apply to 4, 25, and 50. It will also apply to $12\frac{1}{2}$, $16\frac{2}{3}$, and $33\frac{1}{3}$.

4. A number is exactly divisible by 8, or any other factor of 1000, if the number expressed by the three right-hand figures is divisible by said factor.

5. A number is divisible by 9 if the sum of its digits is divisible by 9.

For, if any number, as 3542, be separated into its parts, 3000+500+40 +2, and each part be divided by 9, the several remainders will be the digits composing the number; hence, if the sum of these digits, or remainders, be 9, or some exact number of 9's, the entire number must be an exact number of 9's, and will therefore be divisible by 9.

6. A number is divisible by 3 if the sum of its digits is divisible by 3.

For, since 3 is a factor of 9, it conforms to the same law as governs 9.

7. A number is divisible by a composite number if it is divisible, successively, by all the component factors of the composite number.

8. A number is divisible by 6 if it is even and the sum of its digits is divisible by 3.

This principle corresponds to No. 7, since 2 and 3 are the component factors of 6.

9. A number is divisible by 11 if the sum of its digits representing odd orders, plus the excesses of 11 over the digits representing even orders is divisible by 11.

For, if any number, as 3546, be separated into its parts, 3000+500+40 +6, and each part be divided by 11, those parts representing odd orders will furnish *remainders corresponding* to the digits; but those parts representing *even orders will furnish* remainders equal to the differences between

11 and such digits respectively. In illustration of the above explanation, let it be required to ascertain whether or not 11 will divide 57343. Selecting the digits of odd orders we have 5, 3, and 3. Passing to the digits of even orders we have 4 (11—7); and 7, (11—4); hence the excesses are 5+3+3+4+7=22. Since 22 is divisible by 11, the number 57343 is divisible by 11.

10. An odd number is not divisible by an even number.

For, if one factor of a number is even, the number is even.

11. An even number that is divisible by an odd number is divisible by twice that odd number. (Why?)

12. Any number having no odd factor is a power of 2.

Since 2 is the only even number that is not composite, a number having no odd factor has a succession of 2's for its factors.

PRIME NUMBERS.

129. A **Prime Number** is one that can not be separated into two or more factors.

NOTE. Every number is either prime or composite. For the definition of a composite number see (86).

130. All even numbers except 2 are composite, hence prime numbers are to be found among the odd numbers. In order to find all the prime numbers within any given limit, we write the odd numbers in their order, thus: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, etc. The numbers divisible by 3 are 9 and every third number thereafter; cancelling these numbers, we proceed in the same manner with 5; but 3 times 5 is already cancelled, hence we cancel 5 times 5 and every fifth number thereafter. We next cancel 7 times 7 and every seventh number thereafter, etc. Hence:

131. To find all the prime numbers within a given limit, we have the following:

RULE I. *Write all the odd numbers in their natural order.*

II. *Cancel the second power of 3 and every third number after it; the second power of 5 and every fifth number after it; and so on, beginning with the second power of each prime number in succession till the given limit is reached. The numbers uncancelled, together with the number 2, will be the prime numbers required.*

NOTE. This method of obtaining a list of the prime numbers originated with Eratosthenes, who was born 275 years before Christ, and is called *Eratosthenes' Sieve.*

132. Table of Prime Numbers less than 1000.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

FACTORING.

CASE I.

133. To resolve any composite number into its prime factors.

134. The process of factoring numbers depends upon the following principles:

1. *Every prime factor of a number is an exact divisor of that number.*

2. *The only exact divisors of a number* are its prime factors, or some *combination of its prime factors.*

1. What are the prime factors of 546?

Operation.

$$\begin{array}{r|l} 2 & 546 \\ 3 & 273 \\ 7 & 91 \\ & 13 \end{array}$$

EXPLANATION. We first divide the number by the smallest prime factor, 2, and continue the division, using each time the smallest prime factor, until a prime number is obtained for a quotient. The several divisors and the last quotient are the prime factors required.

NOTE. The order in which the prime factors are employed has no modifying effect upon the final result; but by using the smallest one each time they occur in better symmetrical order.

RULE. *Divide the given number by any prime factor; divide the quotient in the same manner, and continue the division until the quotient is a prime number. The several divisors together with the last quotient will be the prime factors required.*

135. PROOF. Multiply the factors together; if the work is correct the product will be the given number.

EXAMPLES FOR PRACTICE.

2. What are the prime factors of 960?
3. What are the prime factors of 720?
4. What are the prime factors of 1260?
5. What are the prime factors of 5040?
6. What are the prime factors of 8400?
7. What are the prime factors of 11284?
8. What are the prime factors of 10752?
9. What are the prime factors of 354375?
10. What are the prime factors of 999999?
11. What are the prime factors of 715?
12. What are the prime factors of 4284?
13. What are the prime factors of 2090?

CASE II.

136. To find all the exact divisors of a number.

It is evident that all the exact divisors of a number must consist of its *prime factors together* with all possible combinations of these factors.

1. What are all the exact divisors of 792?

Operation.

$$792=2\times2\times2\times3\times3\times11=2^3\times3^2\times\overline{11}^1.$$

Ans.

1, 2, 4, 8	Combinations of 1 and 2.
3, 6, 12, 24 9, 18, 36, 72	Combinations of 1, 2, and 3.
11, 22, 44, 88	Combinations of 1, 2, and 11.
33, 66, 132, 264 99, 198, 396, 792	Combinations of 1, 2, 3, and 11.

EXPLANATION. The prime factors of 792, as found by Case I, are 2, 2, 2, 3, 3, and 11. As 2 occurs three times as a factor, the different combinations of 1 and 2 are evidently the numbers appearing in the first line; namely, 1, 2, 4, and 8. Multiplying these numbers by 3 and the resulting products again by 3, (since 3 occurs twice as a factor) we have the second and third lines, which contain all combinations of 1, 2, and 3. Multiplying each of these three lines by 11, we have in the six lines all possible combinations formed from the prime factors of 792, hence this list must contain all the exact divisors of 792. Hence the following:

RULE I. *Resolve the given number into its prime factors.*

II. *Form a series having 1 for the first term, that prime factor which occurs the greatest number of times in the given number for the second term, the square of this factor for the third term, and so on till a term is reached containing this factor as many times as it occurs in the given number.*

III. *Multiply the numbers in this line by another factor, and if this factor occurs twice, multiply the resulting products also by this factor, and so on continuing this multiplication of resulting products by this multiplier as many times as it occurs as a factor.*

IV. *Multiply all combinations obtained by another factor in the same manner, and so proceed till all the prime factors have been used. All the combinations obtained will be the exact divisors sought.*

NOTE. The *number* of exact factors or divisors may be quickly obtained by increasing the exponent of each prime factor by unity and multiplying these results together. Thus, in the illustrative example given, the

exponents are 3, 2, and 1; increasing each by unity, we have 4, 3, and 2; these numbers multiplied together give 24 for a product, which is the number of exact divisors of 792.

EXAMPLES FOR PRACTICE.

2. Find all the exact divisors of 360.
3. Find all the exact divisors of 96.
4. Find all the exact divisors of 200.
5. How many exact divisors has 2520?
6. How many exact divisors has 75600?
7. Find all the exact divisors of 630.
8. Find all the exact divisors of 5040.

GREATEST COMMON DIVISOR.

137. A **Common Divisor** of two or more numbers is a number that will exactly divide each of them.

138. The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide each of them.

139. Numbers Prime to one another are such as have no common divisor greater than 1.

CASE I.

140. When the numbers can be easily factored.

1. What is the greatest common divisor of 120, 144, and 168?

Operation.

3)	120	144	168
8)	40	48	56
	5	6	7

EXPLANATION. We use 3 as our first divisor, as it will divide each of the given numbers; we next divide by 8, as it will divide each of the resulting quotients. Since the last quotients, 5, 6, and 7, are prime to one another, the division can not be continued, and the divisors 3 and 8 are the component factors of the greatest number that will divide the given numbers. Multiplying these factors together gives 24, the greatest common divisor sought.

141. From the above explanation we deduce the following:

RULE I. *Write the numbers in a horizontal line, and divide by any factor common to all the numbers.*

II. *Divide the resulting quotients in like manner, and continue the division till the quotients obtained are prime to one another.*

III. *The product of the divisors will be the greatest common divisor.*

EXAMPLES FOR PRACTICE.

2. What is the greatest common divisor of 50, 75, and 125?

3. What is the greatest common divisor of 28, 42, 56, and 84?

4. What is the greatest common divisor of 216, 720, 432, and 1296?

5. What is the greatest common divisor of 104, 156, and 364?

6. What is the greatest common divisor of 170, 255. and 595?

7. What is the greatest common divisor of 171, 228, and 285?

8. What is the greatest common divisor of 9, 36, 54, 60, and 72?

9. What is the greatest common divisor of 2520 and 1800?

10. What is the greatest common divisor of 115 and 161?

CASE II.

142. When the numbers cannot be conveniently factored.

1. What is the greatest common divisor of 248 and 403?

Operation.

403
248
155 1st Rem.
93 2d Rem.
62 3d Rem.
31 4th Rem.
31 5th Rem.

EXPLANATION. Since an exact divisor of each of two numbers is an exact divisor of their difference (127, 3.), we subtract the less number, 248, from the greater, 403, and obtain the difference, 155. Again, since the divisor sought must divide 248 and 155, it must divide their difference, 93, the second difference, and so on through the entire series of differences found by the continued subtractions. Having continued the subtraction until two remainders of the series are equal, it is evident that this number is the greatest common divisor, because each of the two last remainders of the series is divisible by it, as any number is an exact divisor of itself; and since this number, 31, will divide both the fifth and fourth remainders, it will divide their sum, 62 (127, 2.), and since it will divide the fourth and third remainders, it will divide their sum, 93; it will also divide 155, as it is the sum of the second and third remainders. It will also divide 248 and 403, the given numbers, for reasons already explained. We have now proved that 31 is a common divisor, and it must be the greatest common divisor. as no number greater than the last remainder of the series can be an exact divisor of the entire series.

143. A more convenient method of obtaining this series of differences is shown by the second operation.

Second Operation.

248)403(1
248
155)248(1
155
93)155(1
93
62)93(1
62
31)62(2
62

EXPLANATION. We divide the greater number by the less and obtain the first remainder, then dividing the last divisor by this remainder, we obtain the second remainder, 93; again dividing the last divisor by this remainder, we find the next remainder, and continue the work in the same manner, we obtain the same series of remainders as by the first operation; namely, 155, 93, 62, and 31.

144. From the foregoing explanations we deduce the following:

RULE I. *Divide the greater number by the less.*

II. Divide the less number by the remainder from the first

division, and continue the division of the last divisor by the last remainder until nothing remains. The last divisor will be the greatest common divisor sought.

III. *If more than two numbers be given, first find the greatest common divisor of two of them, and then of their divisor and one of the remaining numbers, and so on to the last; the last common divisor found will be the greatest common divisor required.*

NOTE. If a prime number occurs as a remainder at any time in the course of the division, it is the greatest common divisor, or the numbers have no common divisor greater than 1.

EXAMPLES FOR PRACTICE.

Find the greatest common divisor:

2. Of 575 and 725.

3. Of 2520 and 8740.

4. Of 1728 and 2448.

5. Of 5265 and 7119.

6. Of 528, 693, and 484

7. Of 56, 70, and 581.

8. Of 5286 and 13248.

9. Of 6840 and 7260.

10. Of 13825 and 19000.

11. Of 1022 and 1095.

12. Of 10573 and 11227.

13. Of 18464 and 19756.

14. Of 283 and 419.

15. A has $1615, B, $1955, and C, $2635, with which they agree to purchase horses, at the highest price per head that will allow each man to invest all his money; how many horses can each man purchase?

16. How many boards will be required to enclose a field 1442 feet long and 1162 feet wide, provided there are 4 boards to each panel, and that the boards are of equal length and *the longest* that can be used?

LEAST COMMON MULTIPLE.

145. A **Multiple** is a number exactly divisible by a given number: thus, 18 is a multiple of 6.

146. A **Common Multiple** of two or more numbers is a number exactly divisible by each of them; thus, 24 is a common multiple of 2, 3, 4, 6, 8, and 12.

147. The **Least Common Multiple** of two or more numbers is the least number exactly divisible by each of them; thus, 30 is the least common multiple of 5, 6, and 10.

148. It is evident that a common multiple of two or more numbers may be found by multiplying the several numbers together, since every product is exactly divisible by each of its factors; such a number, however, may not be the *least common multiple*.

149. The work employed in finding the least common multiple is based upon the following principles:

1. *A multiple of a number must contain all the prime factors of that number.*

2. *A common multiple of two or more numbers must contain all the prime factors of each of those numbers.*

3. *The least common multiple of two or more numbers must contain all the prime factors of each of the numbers and no others.*

FIRST METHOD.

150. *1.* Find the least common multiple of 8, 9, 12, and 20.

Operation.

$8=2\times2\times2$
$9=3\times3$
$12=\cancel{2}\times\cancel{2}\times\cancel{3}$
$20=\cancel{2}\times\cancel{2}\times5$
$2\times2\times2\times3\times3\times5=360.$

EXPLANATION. Since the least common multiple must contain all the prime factors of the given numbers, we resolve each of the numbers into its prime factors. Since the number 8 contains the factor 2 three times, this factor must appear three times in the least common multiple; and since this factor, 2, does *not occur more than three times* in any one of the numbers, all other 2's in *the list may be cancelled,* as they form no part of the least common mul-

tiple. The factor 3 occurs twice in 9, therefore it must be a factor twice in the least common multiple, and is not required more than twice, hence we cancel all other 3's. The factor 5 appears once in 20, hence it must appear once in the least common multiple. Since the uncancelled factors are the only ones necessary, we multiply these together, and obtain the least common multiple of the given numbers.

151. From the foregoing explanation we derive the following:

RULE I. *Resolve each of the given numbers into its prime factors.*

II. *Cancel from the list all factors not necessary to the production of each of the given numbers. The product of the uncancelled factors will be the least common multiple.*

SECOND METHOD.

152. In this method of solution the principles employed are essentially the same as those governing the first method, the factors being rejected by division instead of by cancellation. For an illustration we will make use of the same example as was selected to explain the first method.

Operation.

2)8	9	12	20
2)4	9	6	10
3)2	9	3	5
2	3	1	5

$2\times2\times3\times2\times3\times5=360$

EXPLANATION. We write the numbers in a horizontal line, and divide by any prime factor that will exactly divide two or more of them; the quotients we again divide by any prime factor that will divide two or more of them, and thus continue the division until the resulting quotients are prime to one another. In our first division by 2 we rejected the factor three times, once from 8, once from 12, and once from 20; in the second division we rejected the factor 2 again three times, from 4, from 6, and from 10, thus making 6 rejections in all; but we introduced the factor 2 twice as a divisor, hence the actual number of rejections was 4, the same as by the first method. In the third division we rejected the factor 3 twice, but introduced it as a factor once, thus making really but one rejection, hence we have rejected by division the same factors rejected by cancellation in the first method. Multiplying together the last quotients and divisors, we have 360 as before.

153. From the foregoing explanation we deduce the following:

RULE I. *Write the numbers in a horizontal line and divide by any prime factor that will exactly divide two or more of them.*

II. *Divide the resulting quotients in the same manner, and continue the division until no two of the resulting quotients can be exactly divided by the same number.*

III. *Multiply the divisors and final quotients together, and the product will be the least common multiple.*

NOTE 1. A composite number may be used for a divisor when it will exactly divide each of the numbers.

NOTE 2. A number that is an exact divisor of one of the other numbers may be omitted, as all its prime factors appear in the number of which it is a divisor.

EXAMPLES FOR PRACTICE.

Find the least common multiple:

2. Of 50, 45, and 20.

3. Of 21, 24, 30, and 36.

4. Of 18, 32, 36, and 40.

5. Of 14, 15, 18, 21, and 24.

6. Of 7, 35, 42, and 84.

7. Of 8, 12, 16, 20, and 24.

8. Of 9, 15, 18, 30, and 54.

9. Of 45, 81, 120, and 117.

10. Of 22, 33, 52, and 78.

11. I can buy calves at $4 per head, sheep at $6, cows at $30, and horses at $72. If I invest the smallest sum of money that will enable me to purchase an exact number of each kind, and with the amount buy sheep, what number will I purchase?

12. What is the smallest quantity of wine that will exactly fill 5 gallon casks, 8 gallon casks, 12 gallon casks, or 20 gallon casks.

13. A piece of machinery contains 5 cog wheels, so arranged that the first, with 41 cogs, mashes into the second, which has 42 cogs; the second mashes into the third, which has 43 cogs; the third mashes into the fourth, which has 44 cogs; and the fourth mashes into the fifth, which contains 45 cogs. If the largest wheel makes 60 revolutions a minute, how long a time from the moment they are put in motion before they will be in the same position with relation to one another as when they started?

14. A surveyor's chain was broken into three pieces, whose respective lengths are 18, 20, and 28 feet. What is the shortest distance that may be measured by each piece?

15. What is the least common multiple of the first 7 prime numbers?

CANCELLATION.

154. Cancellation is the process of rejecting equal factors from the dividend and divisor.

NOTE. It is evident that the rejection of factors common to the dividend and divisor will not affect the quotient (109, 6).

1. Divide the product of $21\times15\times4$ by the product of $7\times5\times6$.

Operation.

$$\frac{\cancel{3}\quad 3\quad 2}{\cancel{21}\times\cancel{15}\times\cancel{4}}{\cancel{7}\times\cancel{5}\times\cancel{6}}=6.$$
$$\cancel{2}$$

EXPLANATION We first indicate the division by writing the factors of the dividend above, and the factors of the divisor below a horizontal line (105, note). We cancel the factor 7 from the divisor, and also reject 7 from 21 in the dividend, which is accomplished by cancelling 21 and writing the other factor, 3, above; in like manner we cancel 5 from the divisor and reject 5 from 15 in the dividend; we also cancel 3 from the dividend and reject the same factor from 6 in the divisor; then rejecting the factor 2 from both dividend and divisor, our work is complete, since each factor of the divisor has been cancelled. The product of the uncancelled factors, 2 and 3 in the dividend, divided by 1 equals 6, the correct quotient.

155. From the explanation given we deduce the following:

RULE I. *Write the dividend or the factors thereof, above, and the divisor, or its factors, below a horizontal line.*

II. *Cancel all factors common to the dividend and divisor.*

III. *Divide the product of the uncancelled factors of the dividend by the product of the uncancelled factors of the divisor, and the result will be the quotient.*

NOTE. The cancellation of a factor is, in effect, a division by the factor; hence, when a factor is cancelled, the factor 1 still remains.

2. What is the quotient of $(5\times7\times8\times13)\div(2\times4\times5\times7\times5)$?

Some prefer to write the factors of the dividend at the right, and the factors of the divisor at the left of a vertical line. Both methods, in their application to example 2, are here given.

First Operation.

$$\frac{\cancel{4}}{\cancel{5}\times\cancel{7}\times\cancel{8}\times13}{\cancel{2}\times\cancel{4}\times\cancel{5}\times\ \cancel{7}\times\cancel{5}}=\frac{13}{5}=2\tfrac{3}{5}$$

Second Operation.

$$\begin{array}{r|l} \cancel{2} & \cancel{5} \\ \cancel{4} & \cancel{7} \\ \cancel{5} & \cancel{8}\ \cancel{2} \\ \cancel{7} & 13 \\ 5 & \end{array}$$

$13\div5=2\frac{3}{5}$

EXAMPLES FOR PRACTICE.

3. What is the quotient of the product of the numbers 16, 14, 16, 5, 40, 50, and 60 divided by the product of the numbers 7, 10, 24, 40, 20, and 50?

4. How many tons of coal worth \$8 a ton must be given for 12 tons of hay worth \$6 a ton?

5. A merchant exchanged 20 cases of eggs of 30 dozen each, for 6 sacks of coffee, each weighing 75 pounds and worth 20 cents a pound. What was the price of the eggs per dozen?

6. How many sacks of clover seed, averaging 2 bushels each, and worth 2 dollars a bushel, must be given in exchange for 8 sheep worth 5 dollars per head?

7. A grocer exchanged 15 boxes of soap, each containing 66 pounds at 12 cents a pound, for 88 barrels of pota-

toes, each containing 3 bushels; how much were the potatoes worth a bushel?

8. A farmer exchanged 240 bushels of corn worth 45 cents a bushel for one-half as many bushels of wheat. What was the wheat worth a bushel?

9. A fruit grower took 5 loads of apples to market, each load containing 12 barrels of 3 bushels each. He sold the entire quantity at 50 cents a bushel, and received in payment 6 chests of tea, each containing 25 pounds; how much was the tea worth a pound?

10. A and B, residing in towns 360 miles apart, start at the same time, and travel in a direct line toward each other; at the end of the 8th day they meet, the distance traveled by A being twice that traveled by B. If A's rate of speed is 3 miles an hour and he travels 10 hours each day, what is B's rate of speed, the time he travels each day being 6 hours?

FRACTIONS

156. Fractional numbers result from the division of integral units into parts. The same units employed in numbering whole things are also applied to pieces or parts of things, and when so applied they are called *Fractional Units* (10), and numbers formed from these units are called *Fractional Numbers* (14).

157. Two terms are necessary, according to the principles of the fractional notation, to express a fraction; one of these terms is written above, and the other below a short horizontal line; thus, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{12}$.

158. The **Numerator** is the term written above the line, and it expresses the number of units belonging to the fraction.

159. The **Denominator** is the term written below the line, and it indicates the size or value of the fractional unit, in comparison with integral unit. The denominator 3 indicates that the fractional unit is one-third of an integral unit; if the denominator is 2, the fractional unit is one-half of an integral unit. The denominator 1 indicates that the unit is integral; hence, any integral number may be written with two terms, thus representing it in fractional form.

160. The **Reciprocal** of a number is the number formed by changing the relation of its terms. Thus, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; the reciprocal of 5 is $\frac{1}{5}$. ($\frac{5}{1}$ changed to $\frac{1}{5}$.)

161. The value of the reciprocal of any number is equal to 1 divided by the number. $1 \div 8 = \frac{1}{8}$, which is the reciprocal of 8. $1 \div \frac{3}{4} = \frac{4}{3}$, the reciprocal of $\frac{3}{4}$.

162. One principle of the fractional notation, often overlooked by the student, is that the reciprocal of the number expressing the value of the fractional unit is written as the denominator, and, conversely, the reciprocal of the denominator expresses the value of the fractional unit. If the value of the fractional unit is $\frac{1}{5}$, the denominator is 5, the reciprocal of $\frac{1}{5}$. If the denominator is 7, the value of the fractional unit is $\frac{1}{7}$ of an integral unit.

163. To analyze a fraction is to name and designate the office of each of its terms. Thus, $\frac{3}{8}$ is analyzed as follows: 3 is the numerator and shows that the fraction contains 3 units. 8 is the denominator and shows that the value of each unit is $\frac{1}{8}$ of an integral unit.

164. Express with figures each fraction contained in the following:

EXERCISE.

	No. of Units.	Value of each Unit.	
1.	7	$\frac{1}{8}$.	
2.	12	$\frac{1}{5}$.	
3.	23	$\frac{1}{32}$.	
4.	17	$\frac{1}{25}$.	
5.	471	$\frac{1}{561}$.	
6.	1245	$\frac{1}{1371}$.	
7.	$4\frac{1}{2}$	$\frac{1}{12}$	*Answer*, $\frac{4\frac{1}{2}}{12}$.
8.	2	$\frac{3}{4}$	*Answer*, $\frac{2}{\frac{4}{3}} = \frac{2}{1\frac{1}{3}}$.

165. Analyze each of the following fractions:

$\frac{3}{4}$, $\frac{7}{12}$, $\frac{5}{8}$, $\frac{17}{21}$, $\frac{131}{289}$, $\frac{17}{100}$, $\frac{3}{80}$, $\frac{5}{2000}$, $\frac{1}{1\frac{1}{2}}$.

The analysis of the last fraction in the above list is as follows: 1 is the numerator and shows that the fraction contains 1 unit. $1\frac{1}{2}$ (which is equal to $\frac{3}{2}$), is the denominator and shows that the value of the fractional unit is equal to $\frac{2}{3}$ of an integral unit.

Note. *The above analysis illustrates the fact that the fractional unit*

is not always one of the *equal* parts of an integral unit, since an equal part cannot exceed $\frac{1}{2}$. We would also, in this connection, caution the student against making the statement, in fractional analysis, that "the denominator shows into how many equal parts the integral unit has been divided." In analyzing the fraction $\frac{3}{4\frac{1}{2}}$ it would be the height of absurdity to say that the denominator shows that an integral unit has been divided into $4\frac{1}{2}$ equal parts. Remember that the fractional unit may be *any* part of the integral unit. The definition commonly given of a fraction, that it is "one or more of the equal parts of an integral unit" has application to a simple fraction only.

166. Every fractional number indicates a division of its numerator by its denominator (105, Note.); hence, the numerator is a dividend, the denominator is a divisor, and the fraction (or its value) is the quotient.

167. A **Simple Fraction** is one which has each of its terms integral in form. $\frac{5}{12}$, $\frac{13}{16}$, and $\frac{18}{7}$ are simple fractions.

168. A **Complex Fraction** is one which has a fraction in one or both of its terms. $\frac{3}{5\frac{2}{3}}$, $\frac{4\frac{1}{2}}{8}$, and $\frac{\frac{8}{7}}{8\frac{2}{3}}$ are complex fractions.

169. A **Proper Fraction** is one whose numerator is less than its denominator. Its value is less than an integral unit. $\frac{5}{7}$ and $\frac{13}{20}$ are proper fractions.

170. An **Improper Fraction** is one whose numerator equals or exceeds its denominator. Its value can never be less than an integral unit. $\frac{12}{7}$, $\frac{5}{5}$, and $\frac{13}{4}$ are improper fractions.

171. A **Mixed Number** is a number that is part integral and part fractional; as $5\frac{1}{2}$, $3\frac{4}{7}$, $18\frac{9}{13}$.

NOTE. A number may be fractional in form, but integral in fact. It is customary to speak of such a number as a fraction. $\frac{8}{1}$, $\frac{4}{\frac{1}{10}}$, $\frac{3}{\frac{1}{5}}$, although written in fractional form, are in reality integral, which fact is clearly revealed by analysis. The denominator of the last of the above fractions, $\frac{1}{5}$, *indicates that the value* of the unit is 5 (a collective unit containing 5 integral units).

172. Since a fraction indicates that the numerator is to be divided by the denominator, it is evident that the principles governing division will also apply to fractions. We have simply to substitute in place of the terms, dividend, divisor, and quotient, the corresponding terms belonging to fractions.

GENERAL PRINCIPLES OF FRACTIONS.

173. 1. *Multiplying the numerator multiplies the fraction, and dividing the numerator divides the fraction.*

2. *Multiplying the denominator divides the fraction, and dividing the denominator multiplies the fraction.*

3. *Multiplying or dividing both terms of a fraction by the same number does not change its value.*

4. *Adding to, or subtracting from, both terms, numbers having the same respective relations as the numerator and the denominator does not change the value of the fraction.*

174. These principles may be summed up in one

GENERAL LAW.

A change in the numerator produces a like change in the value of the fraction; but a change in the denominator produces an opposite change in the value of the fraction.

REDUCTION.

175. Reduction of fractions is an operation whereby forms are altered without changing values. (27.)

CASE I.

176. To reduce a fraction to its lowest terms.

REMARK. **A fraction is in its lowest terms when the numerator and denominator are prime to each other.**

1. Reduce $\frac{120}{216}$ to its lowest terms.

Operation.

$$\frac{120}{216} = \frac{30}{54} = \frac{5}{9}$$

EXPLANATION. Since dividing both terms by the same number does not change the value of the fraction (173, Prin. 3), we divide both terms by 4, and both terms of the result by 6, and obtain $\frac{5}{9}$. Since these terms, 5 and 9, have no common divisor, the fraction is in its lowest terms.

NOTE. By dividing by the greatest common divisor, the result can be obtained by a single operation.

177. From the explanation given we deduce the

RULE. *Cancel or reject all factors common to both terms, or divide both terms by their greatest common divisor.*

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{126}{504}$ to its lowest terms.

3. Reduce $\frac{96}{144}$ to its lowest terms.

4. Reduce $\frac{91}{143}$ to its lowest terms.

5. Reduce $\frac{450}{1000}$ to its lowest terms.

6. Reduce $\frac{378}{1260}$ to its lowest terms.

7. Reduce $\frac{17188}{60184}$ to its lowest terms.

SUGGESTION. When divisors cannot be readily distinguished from the principles given in Article 128, find the greatest common divisor of the numerator and denominator.

8. Reduce $\frac{96691}{149977}$ to its lowest terms.

9. Reduce $\frac{76110}{83871}$ to its lowest terms.

10. Reduce $\frac{57688}{148848}$ to its lowest terms.

11. Reduce $\frac{7800}{11700}$ to its lowest terms.

12. Reduce $\frac{115885}{218446}$ to its lowest terms.

CASE II.

178. To reduce an integral number to a fraction of a given denomination.

1. Reduce 12 to a fraction whose denominator shall be 7.

Operation.

$$\frac{12}{1} = \frac{12\times 7}{1\times 7} = \frac{84}{7}$$

EXPLANATION. We first place the given number in fractional form by writing 1 for a denominator. We then multiply both terms by 7, which reduces to 7ths, and we have for a result $\frac{84}{7}$.

RULE I. *Place the integral number in fractional form by writing 1 for a denominator.*

II. *Multiply both terms by the required denominator.*

EXAMPLES FOR PRACTICE.

2. Reduce 17 to a fraction having 5 for a denominator.

3. How many 12ths in 15?

4. How many 35ths in 23?

5. Reduce 125 to 8ths.

6. Reduce 72 to 25ths.

7. If 12 apples be divided into 8ths, what fractional number will express the value?

8. What fractional number having 16 for a denominator is equal to 40?

9. Reduce 320 to 75ths, and reduce the fraction obtained to its lowest terms.

CASE III.

179. To reduce a fraction to higher terms of a required denominator.

1. Reduce $\frac{3}{4}$ of a dollar to 20ths of a dollar.

Operation. $\frac{3\times5}{4\times5}=\frac{15}{20}$

EXPLANATION. The multiplication of the denominator 4 by 5 will give 20, the required denominator, and since both terms must be multiplied by the same number to preserve the value of the fraction, we also multiply the numerator by 5. The explanation suggests the following

RULE. *Divide the required denominator by the denominator of the given fraction, and multiply both terms by the quotient.*

NOTE. The reduction of a fraction to any other fraction whose denominator is not a multiple of the denominator of the given fraction is possible, but such a reduction is impracticable. $\frac{3}{4}$ may be reduced to 10ths by multiplying both terms by $2\frac{1}{2}$, but a complex fraction would result.

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{3}{8}$ to 56ths.

3. Reduce $\frac{5}{7}$ to a fraction whose denominator is 63.

4. Reduce $\frac{11}{12}$ to a fraction whose denominator is 84.

5. Reduce $\frac{83}{91}$ to a fraction whose denominator is 455.

6. Reduce $\frac{11}{13}$ to 156ths.

7. Reduce $\frac{9}{125}$ to a fraction whose denominator is 1000.

8. Change the denomination of $\frac{29}{40}$ to 360ths.

9. Reduce $\frac{138}{1000}$ to a fraction whose denominator is 10000.

10. Reduce $\frac{5}{8}$ and $\frac{7}{9}$ to other fractions, each of which shall have 72 for a denominator.

CASE IV.

180. To reduce a mixed number to an improper fraction.

1. Reduce $5\frac{2}{3}$ to an improper fraction.

Operation.

$$\frac{5\times3}{1\times3}=\frac{15}{3}$$

$$\frac{15}{3}+\frac{2}{3}=\frac{17}{3}$$

or

$$5\tfrac{2}{3}$$
$$5$$
$$\overline{15}$$
$$2$$
$$\overline{\tfrac{17}{3}}$$

EXPLANATION. Since only the integral number requires a change to 3ds to correspond to the fraction, we write 1 for a denominator and multiply both terms by 3 as explained in Case II, and obtain $\frac{15}{3}$, which, combined with the $\frac{2}{3}$ expressed by the fraction, gives $\frac{17}{3}$. A more practical method for obtaining the result, when analysis is not required, is the following, which may be also easily explained: The denominator 3 expresses the number of 3ds in 1; multiplying this number by 5, gives the number of 3ds in 5, which is 15, and adding thereto the 2 thirds belonging to the fraction, gives $\frac{17}{3}$. Hence the

RULE. *Multiply the denominator of the fraction by the integral number; to the product add the numerator, and under the result write the denominator.*

EXAMPLES FOR PRACTICE.

Reduce to improper fractions:

2. $18\frac{2}{3}$.

3. $24\frac{5}{7}$.

4. $52\frac{3}{5}$.

5. $15\frac{3}{8}$ rds.

6. $121\frac{5}{7}$ lbs.

7. $111\frac{4}{11}$ bus.

8. $18\frac{11}{31}$.

9. $172\frac{8}{75}$.

10. $31\frac{28}{400}$.

11. $19\frac{5}{16}$ dols.

12. $12\frac{5}{19}$ miles.

13. $2\frac{156}{250}$ gals.

CASE V.

181. To reduce an improper fraction to an integer or mixed number.

1. Reduce $\frac{121}{18}$ to a mixed number.

Operation.

18)121($6\frac{13}{18}$
108
13

EXPLANATION. Since every fraction indicates a division of the numerator by the denominator, we perform the division and obtain the mixed number, $6\frac{13}{18}$, for a result. The following explanation may be regarded by some as more analytical: Since an integral unit is equal to 18 eighteenths, there are as many integral units in 121 eighteenths as 18 is contained times in 121.

RULE. *Divide the numerator by the denominator.*

EXAMPLES FOR PRACTICE.

Reduce the following to integers or mixed numbers:

2. $\frac{127}{19}$. | *5.* $\frac{427}{121}$. | *8.* $\frac{327}{12}$ lb. | *11.* $\frac{526}{17}$ A.

3. $\frac{360}{45}$. | *6.* $\frac{3872}{16}$. | *9.* $\frac{186}{14}$ bu. | *12.* $\frac{1234}{15}$ ft.

4. $\frac{1728}{37}$. | *7.* $\frac{278}{13}$. | *10.* $\$\frac{381}{14}$. | *13.* $\frac{1000}{19}$ ds.

CASE VI.

182. To reduce two or more fractions to other fractions whose denominators are common, or alike.

1. Reduce $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{2}{5}$ to fractions having a common denominator.

Operation.

$2\times4\times5=40$
$3\times4\times5=\overline{60}$
$1\times3\times5=15$
$4\times3\times5=\overline{60}$
$2\times3\times4=24$
$5\times3\times4=\overline{60}$

EXPLANATION. We multiply the terms of each of the given fractions by the denominators of the other fractions; by so doing the denominators obtained will be equal, as the same numbers are multiplied together in each case to produce them, and the new denominator is a multiple of the denominators of the given fractions, since each denominator is a factor.

RULE. *Multiply the terms of each fraction by the denominators of the other fractions.*

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{7}{8}$, $\frac{5}{11}$, and $\frac{1}{7}$ to fractions having a common denominator.

3. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{4}{5}$, and $\frac{3}{7}$ to fractions having a common denominator.

4. Reduce $\frac{5}{8}$, $\frac{4}{9}$, and $1\frac{1}{3}$ to fractions having a common denominator.

5. Reduce $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{9}$ to fractions having a common denominator.

183. The **Least Common Denominator** is the least common multiple of the given denominators.

An application of the principles above explained will give the least common denominator when the denominators of the given fractions are prime to one another; but when two or more of the denominators have a common factor, it is evident that the product of the denominators will not be their *least* common multiple.

1. Reduce $\frac{1}{6}$, $\frac{5}{8}$, and $\frac{4}{9}$ to fractions having the least common denominator.

Operation.

$1\times12=12$
$\overline{6}\times12=\overline{72}$
$5\times\ 9=45$
$\overline{8}\times\ 9=\overline{72}$
$4\times\ 8=32$
$\overline{9}\times\ 8=\overline{72}$

EXPLANATION. We first find the least common multiple of the given denominators, 6, 8, and 9 by applying principles explained in 152. This least common multiple, which is the least common denominator, is 72. Dividing this new denominator, 72, by each of the given denominators successively, we find what number each denominator has really been multiplied by to produce 72. We find by this division that the denominator of the first fraction has been multiplied by 12, hence both terms must be multiplied by 12 as shown in the operation. The denominator of the second fraction having been multiplied by 9, both terms must be multiplied by 9. Pursuing the same reasoning, we find that both terms of the third fraction must be multiplied by 8. From the explanation we deduce the following:

RULE I. *For the least common denominator take the least common multiple of the given denominators.* (152.)

II. *Reduce each fraction to the denomination expressed by the common denominator, as explained in* 179.

Reduce to fractions having the least common denominator.

2. $\frac{3}{15}, \frac{7}{10}, \frac{14}{20}$.
3. $\frac{5}{7}, \frac{9}{14}, \frac{7}{8}, \frac{17}{28}$.
4. $\frac{5}{36}, \frac{11}{12}, \frac{5}{9}, \frac{1}{8}$.
5. $\frac{19}{24}, \frac{7}{12}, \frac{9}{16}, \frac{121}{144}$.
6. $\frac{7}{15}, \frac{53}{78}, \frac{3}{14}, \frac{80}{91}$.
7. $\frac{5}{8}, \frac{31}{34}, \frac{5}{12}, \frac{16}{17}, 4\frac{7}{8}$.

NOTE. Mixed numbers should first be reduced to improper fractions.

8. Reduce the following to fractions having the least common denominator: $15\frac{1}{2}, \frac{3}{4}, 7\frac{2}{3}, 12\frac{1}{6}, 18\frac{1}{6}, 21\frac{7}{10}, 1\frac{5}{12}, 8\frac{4}{15}$.

ADDITION.

184. Addition of Fractions is the process of finding a number that shall contain as many fractional units as two or more fractional addends.

185. The addition of fractions is based upon the following principles, which are the same in substance as the principles governing the addition of integral numbers:

1. *The numerators express the units belonging to the addends.*

2. *Only like numbers can be added.* (*50, Prin. 1.*) *Like numbers are those having units equal in value, and the same in application,* (*19*); *hence, fractional numbers cannot be like numbers unless they have a common denominator.*

3. *The sum will represent units of the same denomination as the addends, therefore it will have a like denominator.*

1. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

Operation.

$\frac{2}{3} = \frac{8}{12}$
$\frac{3}{4} = \frac{9}{12}$
$\frac{5}{6} = \frac{10}{12}$
$2\frac{1}{4} = 12)\overline{27}(2$
$\underline{24}$
$\frac{3}{12} = \frac{1}{4}$

EXPLANATION. The first step necessary, according to principle 2, is a reduction to fractions having a common denominator. We readily see, (as the result of a mental process), that the least common denominator is 12, which we write above with a line underneath. We then obtain the numerators, which contain the units to be added, as explained in 179. These numerators, 8, 9, and 10, combined, furnish a sum of 27 units, (12ths), which are equal to $2\frac{1}{4}$ integral units.

2. Find the sum of $142\frac{3}{5}$, $18\frac{1}{2}$, and $137\frac{3}{4}$.

Operation.

$142\frac{3}{5}$	20	12
$18\frac{1}{2}$		10
$137\frac{3}{4}$		15
$298\frac{17}{20}$	.20)	37 (1
		20
		17

EXPLANATION. We first find the sum of the fractions as explained in the former example, which is 1 and $\frac{17}{20}$. We write the fractional part of this sum, $\frac{17}{20}$, underneath the fractional column, and carry the 1 unit to the column of units and proceed with the addition as in the case of integral numbers.

RULE. *Reduce to equivalent fractions having a common denominator; add the numerators and if the sum be less than the denominator, write the denominator underneath, otherwise, reduce to an improper fraction, carrying units of a higher order according to the principles of simple addition.*

NOTE. Proper fractions should be reduced to lowest terms.

EXAMPLES FOR PRACTICE.

Find the sum of

3. $\frac{4}{5}$, $\frac{7}{10}$, and $\frac{3}{4}$.

4. $\frac{3}{8}$, $5\frac{1}{2}$, and $12\frac{2}{3}$.

5. $\frac{4}{7}$, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{11}{14}$.

6. $123\frac{4}{7}$ and $98\frac{2}{5}$.

7. $\frac{5}{12}$, $\frac{4}{9}$, and $175\frac{5}{6}$.

8. $1248\frac{4}{9}$ and $324\frac{1}{6}$.

9. $12\frac{1}{2}$ bu., $8\frac{3}{4}$ bu. and $17\frac{1}{3}$ bu.

Find the sum of

10. $7\frac{1}{4}$ yd., $18\frac{1}{2}$ yd., $36\frac{3}{8}$ yd., and 36 yd.

11. $132\frac{3}{4}$ lb., $75\frac{3}{8}$ lb., and $92\frac{2}{3}$ lb.

12. $51\frac{3}{4}$ ft., $126\frac{5}{12}$ ft., and $170\frac{2}{3}$ ft.

13. $\frac{8}{9}$ of a ton, $5\frac{1}{6}$ tons, and $7\frac{2}{3}$ tons.

14. 134 miles, $112\frac{3}{8}$ miles, and $81\frac{1}{2}$ miles.

15. A merchant bought 8 pieces of cloth, containing respectively 38^1, 42, 38^3, 34^2, 46^1, 37, 36^3, and 48 yards. What number of yards did the eight pieces contain?

NOTE. The small figures at the right of the addends express fourths. This practice of expressing fractional parts of a yard is common among merchants.

16. Find the sum of 19^3 yards, 27^1 yards, 24 yards, 31 yards, 16^1 yards, 35^2 yards, 30 yards, 17^2 yards, and 40 yards.

17. A grocer bought 5 barrels of sugar, the net weight of each being respectively $268\frac{5}{8}$, $284\frac{3}{4}$, $291\frac{7}{8}$, 263, and $286\frac{1}{2}$ pounds. What was the total weight of the entire invoice?

18. A merchant sold $32\frac{1}{2}$ yards of cloth for $\$52\frac{1}{4}$, 37 yards for $\$65\frac{3}{8}$, $17\frac{3}{5}$ yards for $\$48\frac{1}{2}$, and $23\frac{1}{4}$ yards for $\$82\frac{7}{8}$. Required the whole number of yards sold and the total amount received.

SUBTRACTION.

186. Subtraction, as applied to fractions, is the process of determining the difference in unit value between two fractional numbers of like units.

187. Fractional subtraction is based upon the following principles:

1. *The numerators express the fractional units whose difference is to be determined.*

2. *The units of the minuend and subtrahend must be of the same kind and value, hence the fractions must have a common denominator.*

3. *The units of the remainder are of the same kind and value as those of the minuend and subtrahend.*

1. From $\frac{5}{7}$ subtract $\frac{2}{5}$.

Operation.

$\frac{5}{7}$ 35

25

$\frac{2}{5}$ 14

$\frac{11}{35}$ 11

EXPLANATION. We first find the least common denominator, (183), which we see at a glance is 35. The numerators are found by the same method explained in addition. Subtracting 14, the units of the subtrahend, from 25, the units of the minuend, we obtain 11 units, (35ths) for a remainder, which we write in its place.

NOTE. It is advisable to reduce small mixed numbers to improper fractions.

Find the difference between

2. $\frac{5}{12}$ and $\frac{1}{4}$.
3. $\frac{11}{14}$ and $\frac{3}{7}$.
4. $2\frac{1}{2}$ and $1\frac{3}{4}$.
5. $5\frac{1}{3}$ and $\frac{4}{6}$.
6. 3 and $1\frac{1}{3}$.
7. $\frac{5}{16}$ and $\frac{1}{8}$.
8. $4\frac{1}{3}$ and $2\frac{1}{7}$.
9. $\frac{1}{11}\frac{1}{2}$ of a lb. and $\frac{5}{8}$ of a lb.
10. $\$\frac{1}{11}\frac{5}{6}$ and $\$\frac{3}{4}$.

11. From $87\frac{2}{3}$ take $43\frac{2}{5}$.

Operation.

$87\frac{2}{3}$ · 15
$43\frac{2}{5}$ 10
6
$44\frac{4}{15}$ $\frac{4}{15}$

EXPLANATION. We find the common denominator and the numerators as previously explained. Subtracting the 6 units of the subtrahend from the 10 units of the minuend, we obtain 4 units (15ths); this we write under the fractional column and then subtract the integral part of the subtrahend from the integral part of the minuend.

12. Subtract $34\frac{3}{4}$ from $88\frac{1}{3}$.

Operation.

$88\frac{1}{3}$ 12
$34\frac{3}{4}$ 4+12
9
$53\frac{7}{12}$

EXPLANATION. Proceeding as in the former case, we obtain 4 (12ths) for the numerator of the minuend and 9 (12ths) for the numerator of the subtrahend. It being impossible to subtract 9 units from 4 units, we borrow one integral unit from the 8 integral units of the minuend; this integral unit ($\frac{1}{12}$), combined with the 4 twelfths makes 16 twelfths. Subtracting the 9 units of the subtrahend from the 16 units of the minuend, gives 7 units (12ths), which we write in its place; then subtracting the 34 integral units of the subtrahend from the 87 units remaining of the minuend, we have the complete remainder.

RULE. *Subtract the fractional part of the subtrahend from the fractional part of the minuend, borrowing a unit when necessary; then perform the integral subtraction and combine the results.*

13. Find the difference between $183\frac{4}{7}$ and $118\frac{1}{4}$.

14. From $1256\frac{3}{8}$ bushels subtract $864\frac{5}{8}$ bushels.

15. A farmer harvested $876\frac{1}{4}$ bushels of wheat and sold therefrom $528\frac{5}{8}$ bushels. How many bushels had he remaining?

16. A merchant's sales on Monday amounted to $\$63\frac{5}{8}$; on Tuesday they were $\$58\frac{3}{4}$; on Wednesday, $\$48\frac{7}{16}$; on

Thursday, $35\frac{1}{8}$; on Friday, $28, and on Saturday, $136\frac{1}{4}$. How much less were his Saturday's sales than the sales of the previous five days?

SPECIAL APPLICATION.

188. To find the sum or difference of two fractions when the numerators are each 1.

1. What is the sum of $\frac{1}{3}$ and $\frac{1}{4}$?

Operation.

$1\times4=4$
$3\times4=12$

$1\times3=3$
$4\times3=12$

7
12

EXPLANATION. We reduce to fractions having a common denominator as explained in (182). By referring to the operation it will be seen that the numerator of each resulting fraction is the same as the other denominator. That such will always be the case is evident from the fact that it is the product of 1 multiplied by the other denominator. It is also evident that the denominator of each of the resulting fractions is the product of the given denominators.

189. The foregoing explanation suggests the following

RULE I. *Write the product of the denominators for the denominator of either the sum or the difference of the given fractions.*

II. *Write the sum of the denominators for the numerator of the sum of the fractions, and the difference of the denominators for the numerator of the difference*

EXAMPLES FOR PRACTICE.

Write the sum of

2. $\frac{1}{3}$ and $\frac{1}{5}$.

3. $\frac{1}{7}$ and $\frac{1}{5}$.

4. $\frac{1}{3}$ and $\frac{1}{9}$.

5. $\frac{1}{9}$ and $\frac{1}{12}$.

6. $\frac{1}{3}$ and $\frac{1}{7}$.

7. $\frac{1}{6}$ and $\frac{1}{10}$.

8. $\frac{1}{11}$ and $\frac{1}{3}$.

9. $\frac{1}{9}$ and $\frac{1}{12}$.

Write the result of

10. $\frac{1}{10}$ of a bushel—$\frac{1}{18}$ of a bushel.

11. $\frac{1}{3}$ of a pound—$\frac{1}{8}$ of a pound.

12. $\frac{1}{4}$ of a mile—$\frac{1}{7}$ of a mile.

13. $\frac{1}{2}$ of a yard—$\frac{1}{8}$ of a yard.

MULTIPLICATION.

190. Fractional multiplication implies that one or both of the factors are fractions.

191. The definition given for the multiplication of integral numbers (66) applies with equal force to the multiplication of fractions.

NOTE. Since the multiplier, in fractional multiplication, may be either integral or fractional, the subject naturally presents two cases.

CASE I.

192. To multiply a fractional number by an integer:

1. Multiply $\frac{5}{6}$ by 3.

First operation. $\frac{5}{6} \times 3 = \frac{15}{6} = 2\frac{1}{2}$.

EXPLANATION. The definition for multiplication requires that we express, in this example, a threefold unit value of the multiplicand, $\frac{5}{6}$; we therefore increase the number of units from 5 to 15, and make no change in the value of the fractional unit. The operation shows that we have multiplied the number of frational units, the numerator, by the units of the multiplier and placed the result over the denominator.

Second Operation. $\frac{5}{6} \times 3 = \frac{5}{2} = 2\frac{1}{2}$.

EXPLANATION. In this case we have made no change regarding the number of units, the number being 5 in the product the same as in the multiplicand; but our product expresses a three-fold value of the multiplicand, as the operation renders each unit three times as great in value. The unit value in the multiplicand is $\frac{1}{6}$, and in the product it is $\frac{1}{2}$. This change is caused by dividing the denominator by 3.

193. From the explanations given we deduce the following

RULE. *Divide the denominator by the multiplier when it can be done without a remainder; otherwise multiply the numerator.*

EXAMPLES FOR PRACTICE.

2. Multiply $\frac{3}{8}$ by 4.

3. Multiply $\frac{5}{7}$ by 3.

4. Multiply $7\frac{2}{3}$ by 8.

5. Multiply $12\frac{3}{3}$ ft. by 7.

6. Multiply $\$8\frac{4}{5}$ by 32.

7. Multiply $15\frac{3}{5}$ lb. by 42.

NOTE. Reduce mixed numbers to improper fractions before multiplying, or multiply integral and fractional parts separately and add the results.

What is the product of

8. $\frac{12}{17}\times 6$?	12. $18\frac{5}{7}\times 16$?	16. $25\frac{13}{16}\times 16$?
9. $\frac{13}{16}\times 5$?	13. $123\frac{1}{2}\times 32$?	17. $3\frac{125}{328}\times 15$?
10. $\frac{128}{380}\times 14$?	14. $75\frac{12}{13}\times 24$?	18. $4\frac{121}{526}\times 40$?
11. $\frac{162}{175}\times 12$?	15. $\frac{123}{542}\times 130$?	19. $120\frac{12}{195}\times 13$?

CASE II.

194. To multiply a number by a fraction.

1. Multiply 5 by $\frac{2}{3}$.

Operation.

$5\times 2=10.$

$10\div 3=\frac{10}{3}=3\frac{1}{3}.$

EXPLANATION. Since the instrument of the operation (the multiplier) contains two terms, it is evident that two computations will be necessary. By referring to the definition of multiplication (66), we see that the unit value of the multiplicand must be repeated two times, there being two units in the multiplier; hence, the first computation, which gives 10. By referring to the units of the multiplier, we see that they are 3ds, but we have multiplied by 2 integral units, whose value is 3 times as much as two 3ds, hence our product, 10, is 3 times as great in value as it should be, (74, principle 4), and we therefore divide it by 3 and obtain the correct result.

195. From the foregoing explanation we deduce the following

RULE. *Multiply the multiplicand by the numerator of the multiplier, and divide the result by the denominator.*

196. Since a number and its reciprocal have opposite relations to unity, it is evident that the multiplication of a quantity by a number and a division of the same quantity by the reciprocal of the number will give equal results; and that the division of a quantity by a number, and a multiplication of the same by the reciprocal of the number will also give results that are identical; therefore a product and also a quotient may be found by either multiplication or division.

2. How much is $\frac{2}{3}$ of 5.

Operation.

$5 \div 3 = \frac{5}{3}$

$\frac{5}{3} \times 2 = \frac{10}{3} = 3\frac{1}{3}$.

EXPLANATION. We first divide by 3, which gives $\frac{1}{3}$ of 5; we then multiply this quotient by 2 and obtain $3\frac{1}{3}$, which is $\frac{2}{3}$ of 5.

NOTE 1. To the student who has been instructed that the word "of" between fractional expressions implies multiplication, it may seem that this example is identical with the former one. The results are truly the same, but they are reached by different analytical methods. The word "of," whenever employed as in the last example, implies a division of the number after it by the reciprocal of the number preceding it.

3. Multiply $\frac{3}{5}$ by $\frac{4}{7}$.

Operation.

$\frac{3}{5} \times 4 = \frac{12}{5}$

$\frac{12}{5} \div 7 = \frac{12}{35}$

EXPLANATION. This example is the same in principle as the first one given under this case, and calls for the same explanation. We multiply the multiplicand by 4, the number of units in the multiplier, and since these units are regarded in the operation as integral, they are 7 times their true value, thus making it necessary to divide the result by 7.

NOTE 2. The rule given above conforms strictly to the analytical operation of fractional multiplication, but when we compare the product of the last example with the factors, we discover that the numerator of this product is the product of the numerators of the factors, and that the denominator is the product of the denominators of the factors. Hence the following

GENERAL RULE I. *Reduce each factor to a simple fractional form.*

II. *Cancel all factors common to the numerators and denominators.*

III. *Multiply together the remaining factors of the numerator for the numerator of the product, and the remaining factors of the denominator for the denominator of the product.*

4. What is the product of $\frac{1}{7} \times 3\frac{3}{5} \times 14 \times \frac{5}{6}$?

Operation.

$$\frac{1}{\cancel{7}} \times \frac{\overset{3}{\cancel{18}}}{\cancel{5}} \times \frac{\overset{2}{\cancel{14}}}{1} \times \frac{\cancel{5}}{\cancel{6}} = 6$$

EXPLANATION. Expressing each factor in simple fractional form, we have $\frac{1}{7}$, $\frac{18}{5}$, $\frac{14}{1}$, and $\frac{5}{6}$. Cancelling equal factors as previously explained, and multiplying together the uncancelled factors, we have 6, the correct product.

What is the product of

5. $\frac{5}{7} \times \frac{9}{10}$?
6. $\frac{3}{4} \times \frac{2}{5} \times \frac{1}{3} \times 7\frac{1}{2}$?
7. $13\frac{1}{8} \times \frac{17}{80} \times \frac{5}{6}$?
8. $17\frac{1}{5} \times \frac{3}{43} \times 15$?
9. $\frac{3}{5} \times 2\frac{1}{4} \times \frac{5}{7} \times 2\frac{1}{3} \times \frac{4}{9}$?
10. $13\frac{5}{6} \times 21\frac{1}{2} \times \frac{17}{43}$?
11. $12\frac{2}{3} \times \frac{4}{5} \times 5 \times 3\frac{4}{5}$?
12. $117\frac{1}{2} \times 15 \times \frac{18}{30}$?
13. $17\frac{2}{3}$ lbs. $\times 4\frac{1}{2}$?
14. $123\frac{3}{5}$ days $\times 10\frac{3}{5}$?
15. $18\frac{6}{7}$ miles $\times 7\frac{5}{12}$?
16. 125 rods $\times \frac{7}{22}$?

17. At $7\frac{3}{4}$ dollars a ton, what is the value of $12\frac{4}{5}$ tons?

18. A drover bought 23 cows at $31\frac{4}{5}$ dollars each; what was the entire cost?

19. If one man can chop $2\frac{1}{4}$ cords of wood in 1 day, how many cords can 7 men chop in $12\frac{1}{2}$ days?

20. A man owning $\frac{5}{6}$ of $164\frac{3}{4}$ acres of land, sold $\frac{1}{2}$ of $\frac{4}{5}$ of his share; how many acres did he sell?

MULTIPLICATION OF MIXED NUMBERS.

197. It is frequently more convenient to multiply mixed numbers together, especially when large, without reducing them to improper fractions, as shown by the following illustrative example:

1. Multiply $124\frac{2}{3}$ by $8\frac{3}{4}$.

Operation.

$124\frac{2}{3}$
$8\frac{3}{4}$
$\frac{1}{2}$
93
$5\frac{1}{3}$
992
$1090\frac{5}{6}$

EXPLANATION. We write the multiplier under the multiplicand the same as in the multiplication of integral numbers. Commencing at the right, we multiply, separately, first the fractional and then the integral part of the multiplicand by $\frac{1}{4}$. $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$, and $124 \times \frac{3}{4} = 93$. We next multiply each part of the multiplicand by the integral part of the multiplier, and obtain respectively $5\frac{1}{3}$ and 992. Adding these partial products gives $1090\frac{5}{6}$, the complete product. Hence the following

RULE. *Multiply separately, the fractional and the integral parts of the multiplicand by the fractional and the integral parts of the multiplier, respectively, and add the partial products.*

EXAMPLES FOR PRACTICE.

2. Multiply $18\frac{2}{3}$ by $12\frac{1}{2}$.

3. Multiply $35\frac{1}{8}$ by $6\frac{2}{5}$.

4. Multiply $124\frac{3}{7}$ by $21\frac{1}{4}$.

5. What is the value of $15\frac{3}{8}$ tons of coal at $6\frac{3}{4}$ dollars a ton?

6. At $4\frac{1}{2}$ dollars a cord, what will be the cost of $24\frac{3}{8}$ cords of wood?

SPECIAL CASES.

CASE I.

198. When the numbers are small in unit value and the integral unit of the multiplicand is of inferior financial value.

1. What is the value of $9\frac{2}{3}$ pounds of bacon at $13\frac{1}{2}$ cents a pound?

Operation.

$\$.13\frac{1}{2}$
$9\frac{2}{3}$
14
117
$\$1.31$

EXPLANATION. The principle upon which the operation is based is the same as in the former case; but, in business practice, a fractional part of a cent is counted as 1 cent if it is one-half or more, and nothing if less than one-half; therefore, the multiplication of $\frac{1}{2}$ by $\frac{2}{3}$ is disregarded. We take $\frac{2}{3}$ of 13 to the nearest unit, thus obtaining 9, then $\frac{1}{2}$ of 9 in business practice is 5, which, added to 9, makes 14, which we write. To this result we add the product of 13 by 9 and obtain 131 cents. We divide this amount by 100, which is simply done by cutting off 2 places by means of a point, and we have $1.31, the business result. Hence the

RULE I. *Disregard the product of the fractions except when it equals or exceeds* $\frac{1}{2}$, *in which case regard it as 1.*

II. *Multiply the integral part of the multiplicand by the fractional part of the multiplier, regarding the result as the nearest integral number to the exact product; to this number add the product of the integral part of the multiplier by the fractional part of the multiplicand (regarding this product also as the nearest integral number in value), and write the sum below.*

III. Multiply the integral parts of the factors together, and add the partial products.

EXAMPLES FOR PRACTICE.

2. Find the value of $24\frac{2}{3}$ pounds of sugar at $6\frac{1}{4}$ cents a pound.

3. What is the value of $19\frac{2}{3}$ pounds of rice at $5\frac{3}{4}$ cents a pound?

4. What will be the cost of $20\frac{7}{8}$ pounds of raisins at $12\frac{1}{2}$ cents a pound?

5. At $12\frac{1}{2}$ cents a dozen, what will be the cost of 22 dozen and 5 eggs?

6. What is the business result of $32\frac{3}{4}$ cents multiplied by $14\frac{3}{8}$?

CASE II.

199. When the integral parts of the factors are equal, and the sum of the fractions is 1.

1. Multiply $9\frac{3}{4}$ by $9\frac{1}{4}$.

Operation.

$$\begin{array}{r} 9\frac{3}{4} \\ 9\frac{1}{4} \\ \hline 90\frac{3}{16} \end{array}$$

EXPLANATION. The product of the fractions is $\frac{3}{16}$, which we write. Then $\frac{3}{4}$ of $9+\frac{1}{4}$ of 9=once 9; hence the integral part of the product is ten times 9. The explanation suggests the following

RULE. *Increase the integral part of one of the factors by unity and multiply the result by the integral part of the other factor, and to the product annex the product of the fractions.*

NOTE. In commercial examples the fractional product should be omitted if it represents cents, as it can never exceed $\frac{1}{4}$.

EXAMPLES FOR PRACTICE.

What is the product of

2. $8\frac{2}{3}\times 8\frac{1}{3}$?
3. $10\frac{7}{8}\times 10\frac{1}{8}$?
4. $12\frac{1}{2}\times 12\frac{1}{2}$?
5. $11\frac{4}{6}\times 11\frac{1}{6}$?
6. $99\frac{5}{6}\times 99\frac{5}{6}$?
7. $7\frac{6}{7}$ cts. $\times 7\frac{1}{7}$?
8. $8\frac{1}{2}$ cts. $\times 8\frac{1}{2}$?
9. $109\frac{2}{3}\times 109\frac{1}{3}$?
10. $14\frac{5}{6}\times 14\frac{1}{6}$?
11. $999\frac{1}{4}\times 999\frac{3}{4}$?

CASE III.

200. When the fractional parts of the factors are equal.

1. What is the product of $40\frac{3}{4}$ by $12\frac{3}{4}$?

Operation.

$40\frac{3}{4}$
$12\frac{3}{4}$
39
480
$519\frac{9}{16}$

EXPLANATION. Since $\frac{3}{4}$ of 40 plus $\frac{3}{4}$ of 12 equals $\frac{3}{4}$ of (40+12), we write $\frac{3}{4}$ of the sum of the integers, which is 39. To this number we add the product of the integers, and to the result we annex the product of the fractions. Hence the following

RULE. *Multiply the sum of the integers by one of the fractions, and to the result add the product of the integers and the product of the fractions.*

EXAMPLES FOR PRACTICE.

2. Multiply $112\frac{1}{8}$ by $23\frac{1}{8}$.

3. What is the product of $48\frac{2}{3}$ by $13\frac{2}{3}$.

4. What is the value of $21\frac{1}{8}$ pounds of coffee at $18\frac{1}{8}$ cents a pound?

5. At $62\frac{1}{2}$ cents a pound, what will $9\frac{1}{2}$ pounds of tea cost?

6. What is the product of $720\frac{3}{5}$ by $24\frac{2}{5}$?

DIVISION.

201. Since the divisor may be either integral or fractional, the subject presents two cases.

CASE I.

202. To divide a fraction by an integer.

1. Divide $\frac{3}{7}$ by 5.

Operation. $\frac{3}{7} \div 5 = \frac{3}{3\,5}$

EXPLANATION. We make no change of the numerator, but make the denominator 5 times as great in unit value. This operation divides by 5, since we have the same number of fractional units of $\frac{1}{5}$ the value (173, Prin. II).

NOTE. When the numerator is a multiple of the divisor, it is preferable to divide it by the divisor and retain the denominator (173, Prin. I).

203. From the foregoing explanation we deduce the following

RULE. *Divide the numerator by the divisor when a multiple of it, and write the result over the denominator. When the numerator is not a multiple of the divisor, multiply the denominator by the divisor, and over the result write the numerator.*

EXAMPLES FOR PRACTICE.

What is the quotient of

2. $\frac{3}{8} \div 7$?	*5.* $\frac{91}{100} \div 13$?	*8.* $15\frac{2}{3} \div 8$?
3. $\frac{12}{13} \div 4$?	*6.* $\frac{144}{161} \div 16$?	*9.* $26\frac{1}{2} \div 14$?
4. $\frac{112}{243} \div 7$?	*7.* $\frac{31}{140} \div 12$?	*10.* $134\frac{12}{17} \div 5$?

11. At 4 dollars a cord, how many cords of wood can be bought for $17\frac{3}{4}$ dollars?

12. How many acres of land will $284\frac{5}{8}$ dollars buy at 30 dollars an acre?

CASE II.

204. To divide a number by a fraction.

1. Divide 8 by $\frac{3}{5}$.

First Operation.

$8 \div 3 = \frac{8}{3}$

$\frac{8}{3} \times 5 = \frac{40}{3} = 13\frac{1}{3}$

EXPLANATION. According to the true principles of division, we must ascertain how many times the units of the divisor are contained in the dividend; this division gives $\frac{8}{3}$, but since the units employed in the operation are integral, our quotient is only $\frac{1}{5}$ of its true value, and must therefore be multiplied by 5.

205. It has already been shown (196), that the multiplication of a quantity by the reciprocal of a number will give the same result as the division of the same quantity by the number; hence the quotient may be obtained as in the operation below.

Second Operation.

$8 \times \frac{5}{3} = \frac{40}{3} = 13\frac{1}{3}$

EXPLANATION. The reciprocal of the divisor is the quotient of 1 divided by the divisor (161); and since dividing 1 by $\frac{3}{5}$ gives $\frac{5}{3}$, it is evident that dividing 8 by $\frac{3}{5}$ will give 8 times $\frac{5}{3}$. Where this analysis is observed, the reciprocal of *the divisor is the true multiplicand.*

206. From the foregoing investigation of principles, we deduce the following

RULE. *Divide the dividend by the numerator of the divisor and multiply the quotient by the denominator*; or, *multiply the dividend by the reciprocal of the divisor.*

NOTE. The second method is preferable, since it furnishes a means of employing cancellation.

EXAMPLES FOR PRACTICE.

What is the quotient of

2. $17 \div \frac{3}{4}$?
3. $5\frac{3}{4} \div \frac{3}{7}$?
4. $\frac{7}{9} \div \frac{6}{15}$?
5. $\frac{5}{6} \div \frac{4}{9}$?
6. $\frac{121}{140} \div \frac{11}{20}$?
7. $13\frac{5}{6} \div \frac{7}{12}$?
8. 24 hours $\div 2\frac{3}{4}$?
9. $13\frac{2}{5}$ pounds $\div 5\frac{1}{2}$?
10. $5\frac{7}{8}$ dollars $\div 6\frac{1}{4}$ dollars?

11. What quantity of hay at $9\frac{3}{4}$ dollars a ton can be bought for $\frac{5}{8}$ of a dollar?

DIVISION OF MIXED NUMBERS.

CASE I.

207. To divide a mixed number by an integer.

1. Divide $128\frac{2}{3}$ by 15.

Operation.

$$15)128\tfrac{2}{3}(8\tfrac{26}{45}$$
$$\underline{120}$$
$$8\tfrac{2}{3} \times 3 = 26$$
$$\overline{15} \times 3 = \overline{45}$$

EXPLANATION. We write the numbers and perform the work the same as in integral division, and have a remainder of $8\frac{2}{3}$. Placing the divisor under it as instructed in division of integral numbers, we have the complex fraction $\frac{8\frac{2}{3}}{15}$. Since a fraction multiplied by any multiple of its denominator gives an integral number for a product, we multiply both terms of our complex fraction by 3 and obtain the simple fraction $\frac{26}{45}$, which we place to the right of the integral part of the quotient, and we have $8\frac{26}{45}$, the correct result.

EXAMPLES FOR PRACTICE.

Divide

2. $187\frac{1}{2}$ by 23.
3. $4217\frac{1}{4}$ by 127.
4. $2381\frac{1}{4}$ by 95.

Divide

5. $3425\frac{1}{2}$ by 183.
6. $2870\frac{4}{6}$ by 1264.
7. $1382\frac{1}{5}$ by 450.

Divide

8. $285\frac{5}{6}$ by 22.
9. $423\frac{1}{3}$ by 75.
10. $387\frac{1}{8}$ by 103.

CASE II.

208. To divide an integer by a mixed number.

1. Divide 542 by $43\frac{1}{5}$.

Operation.

$43\frac{1}{5})542$

5

$216)2710(12\frac{59}{108}$

216

550

432

118

$\frac{118}{216} = \frac{59}{108}$

EXPLANATION. Since multiplying both dividend and divisor by the same number does not change the value of the quotient, (109, Prin. 5) we multiply the dividend and the divisor by 5, the denominator of the fraction, and obtain integral results; we then proceed as in integral division.

RULE. *Multiply the dividend and the divisor by the denominator of the fractional part of the divisor, and then proceed as in the division of integral numbers.*

EXAMPLES FOR PRACTICE.

Divide	Divide	Divide
2. 35 by $11\frac{2}{3}$.	*5.* 1563 by $93\frac{3}{6}$.	*8.* \$483 by $128\frac{2}{3}$.
3. 285 by $32\frac{1}{6}$.	*6.* 756 by $108\frac{1}{2}$.	*9.* \$563 by \$$91\frac{3}{5}$.
4. 1761 by $113\frac{3}{4}$.	*7.* 2540 by $235\frac{1}{7}$.	*10.* 250 lbs. by $18\frac{7}{8}$.

CASE III.

209. To divide a mixed number by a mixed number.

1. Divide $1284\frac{3}{5}$ by $73\frac{1}{4}$.

Operation.

$73\frac{1}{4})1284\frac{3}{5}$

$12 \quad 12$

$879)15416(17\frac{473}{879}$

879

6626

6153

473

EXPLANATION. We multiply both dividend and divisor by the least common multiple of the denominators of the fractions, and obtain integral numbers as results. We then perform the division as in any other case of division of one integer by another.

RULE. *Multiply the dividend and the divisor by the least common denominator of the given fractions, and then proceed as in any other case of division of an integer by an integer.*

EXAMPLES FOR PRACTICE.

2. What is the quotient of $345\frac{1}{3}$ divided by $25\frac{5}{8}$?

3. How many barrels of flour at $\$5\frac{5}{8}$ a barrel can be bought with $\$202\frac{1}{2}$?

4. A dealer paid $\$787\frac{1}{4}$ for $83\frac{3}{4}$ tons of hay. What was the price per ton?

5. A dealer invested $\$88\frac{3}{8}$ in beans at $\$1\frac{3}{4}$ a bushel. How many bushels did he buy?

210. Reduction of Complex Fractions.

1. Reduce $\frac{\frac{5}{8}+\frac{1}{6}}{7\frac{2}{3}-\frac{1}{4}}$ to a simple fraction.

Operation.

$\frac{5}{8}+\frac{1}{6}=\frac{19}{24}$

$7\frac{2}{3}-\frac{1}{4}=\frac{89}{12}$

$\frac{19}{24}\div\frac{89}{12}=\frac{19}{178}$

EXPLANATION. We first perform the addition indicated in the numerator and obtain for a simplified numerator $\frac{19}{24}$. We next perform the indicated subtraction in the denominator and obtain $\frac{89}{12}$. Then dividing the numerator by the denominator, we obtain the simple fraction $\frac{19}{178}$.

RULE. *Reduce each term to its simplest form by performing the operation indicated, and then divide the numerator by the denominator.*

EXAMPLES FOR PRACTICE.

Reduce each of the following expressions to its most simple form:

2. $\frac{5\frac{3}{8}+3\frac{1}{2}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$.

3. $\frac{7+3\frac{5}{8}}{1\frac{15}{12}}$.

4. $\frac{\frac{2}{3}\times\frac{11}{12}}{\frac{1}{9}\times5\frac{1}{2}}$.

5. $\frac{\frac{3}{5}\frac{7}{0}\frac{5}{0}}{\frac{2}{2}\frac{1}{4}}$.

6. $\frac{\frac{5}{9}\text{ of }\frac{3}{14}}{12\frac{1}{5}-5\frac{4}{15}}$.

7. $\frac{12\frac{3}{5}\times4\frac{4}{9}}{\frac{3}{8}+30\frac{1}{2}+\frac{4}{7}+6\frac{39}{56}+17\frac{1}{7}}$.

8. What is the value of $\frac{13\frac{1}{5}\times(\frac{2}{3}+\frac{7}{8})}{18\frac{2}{5}-(4\frac{1}{2}+\frac{5}{8}-2\frac{1}{3})}$?

NOTE. A parenthesis enclosing numbers is a sign of aggregation, and indicates that the numbers included within it are to be regarded as one whole, thus $(5+3)\times(7-4)$, indicates that the sum of 5 and 3 is to be multiplied by the difference between 7 and 4. A vinculum or bar placed over numbers has the same signification.

GREATEST COMMON DIVISOR OF FRACTIONS.

211. The **Greatest Common Divisor** of two or more fractions is the greatest number that will divide each of them and give an integral quotient.

From the foregoing definition it is evident that the exact divisors of any given fraction, taken in their regular order, are the fraction itself, which will give 1 for a quotient, $\frac{1}{2}$ of the fraction, which will give 2 for a quotient, $\frac{1}{3}$ of the fraction, which will give 3 for a quotient, etc. To illustrate this fact, let us consider a few of the exact divisors of $\frac{7}{12}$ and investigate the principle by which the quotient in any case is obtained. These divisors are $\frac{7}{12}$, $\frac{7}{24}$, $\frac{7}{36}$, $\frac{7}{48}$, $\frac{7}{60}$, etc., giving respectively the quotients 1, 2, 3, 4, and so on. The division of $\frac{7}{12}$ by $\frac{7}{36}$ is effected in the following manner: $\frac{\cancel{7}}{\cancel{12}_{1}} \times \frac{\cancel{36}^{3}}{\cancel{7}_{1}} = 3$. Notice, in the operation, that the numerator of the divisor is exactly contained in the numerator of the dividend, and that the denominator of the dividend is exactly contained in the denominator of the divisor: if such were not the case, the factors of the numbers below the line would not all cancel, and we should have a fractional quotient.

212. From the foregoing investigation we deduce the following facts:

1. A fraction is an exact divisor of a given fraction when its numerator is an exact divisor of the given numerator, and its denominator is a multiple of the given denominator.

2. A fraction is a common divisor of two or more fractions when its numerator is a common divisor of the given numerators, and its denominator is a common multiple of the given denominators.

3. The greatest common divisor of two or more given fractions is a fraction whose numerator is the greatest common divisor of the given numerators, and its denominator is the least common multiple of the given denominators.

1. What is the greatest common divisor of $\frac{3}{8}$, $\frac{3}{10}$, and $\frac{9}{16}$?

EXPLANATION. We first find the greatest common divisor of the numerators 3, 3, and 9, which is 3. We next find the least common multiple

of the denominators 8, 10, and 16, which is 80. Dividing this divisor 3 by the multiple 80 gives $\frac{3}{80}$, which is the greatest common divisor sought.

PROOF.

$\frac{3}{8} \div \frac{3}{80} = 10.$
$\frac{3}{10} \div \frac{3}{80} = 8.$
$\frac{9}{16} \div \frac{3}{80} = 15.$

Since the three quotients have no divisor common to all of them, $\frac{3}{80}$ is the *greatest* common divisor.

RULE. *Divide the greatest common divisor of the numerators by the least common multiple of the denominators.*

NOTE. Before applying the rule, integral numbers and mixed numbers must be reduced to improper fractions, and all fractions to their lowest terms.

EXAMPLES FOR PRACTICE.

Find the greatest common divisor of

2. $\frac{7}{8}$, $\frac{5}{12}$, $\frac{2}{3}$, and $\frac{3}{4}$.
3. $\frac{11}{16}$, $\frac{11}{24}$, and $\frac{22}{27}$.
4. $7\frac{2}{3}$, $18\frac{1}{4}$, and 12.
5. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.
6. $2\frac{3}{5}$, $7\frac{3}{5}$, and $\frac{19}{40}$.
7. $31\frac{1}{2}$, $94\frac{1}{2}$, and $7\frac{7}{8}$.

8. What is the length of the longest measure that can be contained an exact number of times in each of the following distances: $13\frac{3}{4}$ ft., $5\frac{1}{2}$ ft., and $4\frac{1}{8}$ ft.?

LEAST COMMON MULTIPLE OF FRACTIONS.

213. The **Least Common Multiple** of two or more fractions is the least number that will contain each as a divisor an exact number of times.

The multiples of any given fraction are evidently the fraction, 2 times the fraction, 3 times the fraction, etc. Take for example the fraction $\frac{2}{3}$; its multiples are $\frac{2}{3}$, $\frac{4}{3}$, $\frac{6}{3}$, etc. In the division of the multiple $\frac{4}{3}$ by the given fraction $\frac{2}{3}$, we have the following operation $\frac{4}{3} \times \frac{3}{2} = 2$. Notice that the *numerator* of the *multiple* is a *multiple* of the *numerator* of the given fraction, and that the *denominator* of the *multiple* is a *divisor* of the *denominator* of the given fraction. This must be true in every case, otherwise a fractional quotient would be the result of the division.

214. From the foregoing investigation we derive the following facts:

1. A fraction is a multiple of a given fraction when its *numerator is a* multiple of the given numerator, and its *denominator is a* divisor of the given denominator.

2. A fraction is a common multiple of 2 or more given fractions when its numerator is a common multiple of the given numerators, and its denominator is a common divisor of the given denominators.

3. The least common multiple of two or more fractions is a fraction whose numerator is the least common multiple of the given numerators, and its denominator is the greatest common divisor of the given denominators.

1. Find the least common multiple of $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{15}{16}$.

EXPLANATION. We first find the least common multiple of the numerators, 5, 7, and 15, which is 105, and divide this result by the greatest common divisor of the denominators 8, 12, and 16, and obtain $\frac{105}{4}=26\frac{1}{4}$, which is the least common multiple of the given fractions.

RULE. *Divide the least common multiple of the numerators by the greatest common divisor of the denominators.*

NOTE. All numbers should be represented in their most simple fractional forms, before applying the rule.

EXAMPLES FOR PRACTICE.

Find the least common multiple of

2. $\frac{2}{5}$, $\frac{7}{10}$, $\frac{8}{15}$.

3. $4\frac{1}{2}$, 12, and $1\frac{1}{4}$.

4. $1\frac{7}{15}$, $1\frac{1}{21}$, and $\frac{2}{3}$.

5. $\frac{1}{2}$, $\frac{9}{22}$, and $\frac{9}{26}$.

6. $2\frac{15}{16}$, $3\frac{7}{40}$, $5\frac{7}{24}$, and $\frac{189}{198}$.

7. $\frac{1}{7}$, $\frac{3}{7}$, $\frac{5}{7}$, and $\frac{15}{28}$.

8. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, and $\frac{9}{10}$.

9. If the forward wheels of a carriage are each $10\frac{5}{12}$ feet in circumference, and the rear wheels are each $12\frac{2}{3}$ feet, how far will the carriage move before the wheels will be in the same relative position as at starting?

REVIEW OF COMMON FRACTIONS.

1. What is the sum of $132\frac{5}{12}$, $19\frac{5}{6}$, $213\frac{3}{4}$, and $85\frac{1}{3}$?

2. From $\frac{2}{5}$ of $\frac{3}{5}$ of 15 subtract $\frac{1}{2}$ of $\frac{6}{7}$ of $4\frac{2}{3}$.

3. A farmer has in his granary $635\frac{7}{16}$ bushels of wheat, $213\frac{5}{12}$ bushels of barley, $\frac{3}{4}$ as many bushels of oats as of wheat, and $\frac{1}{3}$ as many bushels of corn as of oats. How many bushels of grain are in his granary?

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4. A merchant bought 5 pieces of muslin, containing respectively 43^2, 38^1, 46, 39^3, and 42^1 yards at $5\frac{1}{2}$ cents a yard. If he sell the entire invoice at a gain of $\frac{3}{4}$ of a cent a yard, how much will he receive?

5. A grocer sold to a farmer $45\frac{3}{4}$ pounds of sugar at $6\frac{1}{2}$ cents a pound, $7\frac{1}{2}$ pounds of coffee at $24\frac{3}{4}$ cents a pound, 3 brooms at 25 cents each, 4 sacks of cornmeal at $37\frac{1}{2}$ cents a sack, and 8 pounds of raisins at $12\frac{1}{2}$ cents a pound. He received in payment $27\frac{3}{4}$ pounds of butter at $16\frac{2}{3}$ cents a pound and the remainder in cash. How much money did the merchant receive?

6. A boy lost 15 marbles and had 25 remaining. What fractional part of the whole did he lose?

7. A farmer harvested $605\frac{7}{8}$ bushels of wheat from a field containing $32\frac{3}{4}$ acres. What was the yield per acre?

8. A merchant bought 72 yards of flannel at $18\frac{3}{4}$ cents a yard. If he sells $\frac{7}{16}$ of it for \$6.30, and the remainder at a profit of $2\frac{1}{4}$ cents a yard, what is his total gain?

9. A boy bought $\frac{3}{4}$ of a melon and gave his brother $\frac{2}{3}$ of his purchase. What part of the melon did he retain?

10. A merchant sold $21\frac{3}{4}$ yards of muslin from a piece containing $45\frac{1}{4}$ yards. What fractional part of the piece did he sell?

11. A boy lost $\frac{1}{4}$ of his kite line and then added 80 feet, which increased the original length $\frac{1}{12}$. What was the length of the line before the loss?

12. A firm consists of three partners; A owns $\frac{3}{8}$ of the capital, which is \$2340. How many dollars does each of the other partners own, if B's share is $\frac{7}{16}$ of the whole and C possesses the remainder?

13. How many building lots containing $\frac{5}{16}$ of an acre each can be sold from a tract of land containing $7\frac{1}{2}$ acres?

14. *A farmer* exchanged $1\frac{3}{4}$ bushels of wheat at $76\frac{1}{2}$

cents a bushel for lard at $11\frac{1}{4}$ cents a pound. How many pounds of lard did he receive?

15. A grocer bought 4 bags of coffee, the gross weight of each being respectively $128\frac{7}{8}$, $131\frac{1}{4}$, $132\frac{1}{2}$ and $129\frac{3}{4}$ pounds, paying therefor 24 cents a pound, net. Allowing $1\frac{1}{8}$ pounds for the weight of each bag, what did the grocer pay for the coffee?

16. A can do a piece of work in 5 days and B in 6 days. In what time can it be done if they both work together?

17. A job of work can be done by A in 8 days, by B in 10 days, and by C in 12 days. What time will be required to perform the work if they all work together?

18. A can perform a piece of work alone in 12 days, but when assisted by B the time required is but 7 days. In what time can B perform the same work?

19. If A can mow an acre of grass in $\frac{1}{3}$ of a day, B in $\frac{3}{4}$ of a day, C in $1\frac{1}{4}$ days, and D in $1\frac{1}{3}$ days, in what time can they mow a field containing $4\frac{1}{2}$ acres by working together?

20. A man paid \$195 for a horse and carriage, paying $\frac{6}{7}$ as much for the carriage as for the horse. What was the cost of each?

SUGGESTION. Regard the value of the horse as the unit; then the number of units is $1\frac{6}{7}$; hence we have aggregate value \$195, and number of units $1\frac{6}{7}$ to find unit value. See (120, 5).

21. What number, increased by $\frac{2}{3}$ of itself and by 72, will be made 3 times as great in unit value?

22. If 64 is $\frac{5}{9}$ of a certain number, what is $\frac{4}{7}$ of the same number?

23. If coffee shrink $\frac{1}{14}$ of its weight by roasting, what quantity of green coffee will be required to make 377 pounds of roasted coffee?

24. A merchant's losses, during the first year of business, amounted to $\frac{3}{5}$ of his capital, after which he gained \$960, which made his capital \$7200; how much did he lose?

25. A merchant paid for an invoice of coffee, tea and

sugar $1093\frac{1}{2}$; the tea cost $\frac{1}{4}$ more than the coffee, and the sugar twice as much as both tea and coffee. What was the cost of each?

26. B's age is $\frac{1}{3}$ more than A's and $\frac{1}{5}$ of C's, and the sum of their ages is 96 years; how old is A?

27. A crew consisting of 6 men and 8 boys engage to perform a job of work which they can complete in 12 days; but at the expiration of the third day 5 of the boys are discharged and 2 men are added to the force. Estimating the work of 1 boy $\frac{2}{3}$ that of 1 man, how much time will be required for the performance of the work?

28. Henry was born in 1880, and his age at John's birth was $2\frac{2}{5}$ times John's age in 1898. How old was John in 1905?

SUGGESTION. Let John's age in 1898 be the unit; then, since Henry's age at John's birth was $2\frac{2}{5}$ units, his age in 1898 must have been $3\frac{2}{5}$ units, John's age (1 unit) being added.

29. A's money is what part of B's if B has $\frac{1}{4}$ more than A?

30. How many acres does a farm contain if $\frac{3}{8}$ of it is meadow, $\frac{2}{5}$ plow land, $\frac{3}{20}$ pasture, and the remainder, 18 acres, is timber land?

31. A and B started from different places at the same time, and traveled in a direct line toward each other until they met, A's rate of speed being $\frac{3}{4}$ that of B. Had each traveled during the same time at a rate of speed $\frac{1}{7}$ greater, they would have passed each other and been 14 miles apart. How far apart are the places from which they started?

32. A man bought a horse, sold it at a gain of $\frac{1}{5}$ of the cost, and received $108. How much was the gain?

33. A grocer's sales for February were $\frac{1}{4}$ greater in amount than his January sales; his sales for March exceeded his February sales by $\frac{1}{7}$, and his April sales exceeded his March sales by $\frac{1}{8}$. If his April sales exceeded his January sales by $220, what was the amount of his

34. What is the least number of quarts of wine, expressed by an integral number, that will exactly fill bottles containing either $\frac{3}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, or $\frac{7}{10}$ of a quart?

35. If it takes $\frac{7}{8}$ of a yard of cloth to make a vest, how many vests can be made from a piece containing 22 yards, and what remnant of cloth will be left?

36. The sum of two numbers is $2\frac{5}{12}$, and the difference of their squares is $2\frac{31}{144}$; what are the numbers? (95, Theorem.)

37. A clerk paid $\frac{1}{4}$ of his monthly wages for board, deposited $\frac{4}{5}$ of the remainder in the bank, paid \$5 for shoes, and retained the remainder, \$7, to meet incidental expenses. What amount did he receive a month for his services?

38. In the construction of a house which cost \$4870, $\frac{2}{7}$ less was paid for labor than for material. What did the material cost?

39. The sum of 4 numbers is 372; the second is $\frac{4}{5}$ of the first, the third is $\frac{7}{8}$ of the second, and the fourth is $\frac{6}{7}$ of the third. What is the second number?

40. If a family of 5 persons consume $\frac{5}{9}$ of a barrel of flour in a week when they have 13 boarders, how much flour per week will be required if 12 additional boarders are received?

41. A boy having a certain number of marbles bought $\frac{5}{8}$ as many more; he sold $\frac{3}{5}$ of his purchase to a companion and added the remainder to his previous stock, which increased his number to 20. How many had he at first?

42. The product of 4 factors is $50\frac{5}{9}$. If 3 of the factors are respectively $3\frac{1}{4}$, $2\frac{1}{2}$, and $1\frac{1}{6}$, what is the other factor?

43. What number is that to which, if $7\frac{1}{2}$ be added, $4\frac{3}{4}$ subtracted from the sum, the remainder multiplied by $8\frac{1}{3}$, and the product divided by 25, the quotient will be $5\frac{11}{12}$?

44. What is *the product* of $\frac{\frac{1}{4} \text{ of } \frac{7}{10}}{7}$ multiplied by $\frac{6}{3\frac{6}{11}}$?

DECIMAL FRACTIONS.

215. A **Decimal Fraction** is a fraction whose units represent decimal parts of an integral unit.

NOTE. The term *decimal* is derived from *decem*, which is a Latin word signifying *ten*.

216. In the formation of decimals, an integral unit is divided into ten equal parts, thus forming decimal units of the first order, or *tenths;* one of these divisions may be divided into ten equal parts, thus forming decimal units of the second order, or *hundredths;* and so on, as shown by the following

TABLE OF DECIMAL UNITS.

1 integral unit	equals	10 tenths;
1 tenth	"	10 hundredths;
1 hundredth	"	10 thousandths;
1 thousandth	"	10 ten-thousandths;
etc.		etc.

217. The **Decimal Sign** is a period (.), and indicates that the figures placed at the right of it represent decimal orders; thus, .7 (seven tenths), .32 (thirty-two hundredths).

218. The name of the right hand order is the name of the decimal unit of the fraction. The decimal unit of .1375 is one ten-thousandth.

219. From the foregoing explanations, it will be seen that the decimal notation is the same as the notation of integral numbers; it is therefore unnecessary to write the denominators of decimal fractions, since the local value of any figure is known by its position.

220. The law of local value, exhibited in the table of

decimal units shown above assigns the first place at the right of the decimal point to tenths; the second to hundredths; the third, to thousandths; and so on, as shown by the following

NOTATION AND NUMERATION TABLE.

Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	*Decimal Point.*	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
4	2	1	9	8	6	7	.	3	2	0	9	4	5
Integral Orders.								Decimal Orders.					

221. An examination of the above table will reveal the following facts:

1. The denominator of a decimal fraction is some power of 10.

2. The number of places in any decimal is equal to the number of ciphers necessary to express its denominator. Thus tenths occupy the first place at the right of the point, and the denominator of any number of tenths is 10; thousandths occupy the third place, and the denominator is 1000, etc.

222. A Simple Decimal is one whose value is expressed without the aid of a common fraction; as .15, .014, $.75.

223. A Complex Decimal is one which has a common fraction at the right of its lowest order; as $.27\frac{2}{3}$, $\$.008\frac{1}{7}$.

REMARK. *Terminate* complex decimals are those that can be reduced to simple decimals; *those that* cannot be so reduced are called *interminate*.

224. A Mixed Decimal is one composed of an integer and a decimal fraction; as 12.25, 156.016 pounds.

225. A Decimal Unit is one of the equal decimal parts into which the integral unit is divided. The unit of the right hand order is the unit of the fraction.

226. From the foregoing explanations and illustrations we derive the following

GENERAL PRINCIPLES.

1. *The different orders of decimal units increase from right to left, and decrease from left to right in a tenfold ratio.*

2. *The local value of any decimal figure depends upon its relation, in position, to the decimal point.*

3. *Prefixing a cipher to a decimal, and moving the point to the left of the cipher prefixed, divides the decimal by 10, since it removes every decimal figure one place to the right.*

4. *Annexing ciphers to, or omitting them from, the right of a decimal, produces no change in value, since each order will occupy the same position in relation to the decimal point.*

NOTATION OF DECIMALS.

227. *1.* Write decimally, one hundred five hundred-thousandths.

EXPLANATION. First write the numerator, 105. (The last word, *hundred-thousandths*, expresses the denominator, and that part of the expression preceding it is the numerator.) Since the denominator, 100000, contains 5 ciphers, the decimal must contain 5 decimal places (221, 2), hence we prefix two ciphers to the numerator already written, and we have 00105; but since each order of units is a decimal place, we place the decimal point to the left and we have .00105, the correct result.

228. From the foregoing explanation we deduce the following

RULE I. *Write the decimal expression as if it were an integral number and, when necessary, prefix ciphers to give each figure its true local value.*

II. Place the decimal point before the left-hand figure.

EXAMPLES FOR PRACTICE.

Write decimally the following:

2. Seven tenths.
3. Forty-five thousandths.
4. Thirty-seven tenths.
5. Fifteen millionths.
6. Nine ten-thousandths.
7. Seventy-six thousandths.
8. Three thousand six millionths.
9. Eighty-four ten-millionths.
10. Two hundred thousandths.
11. Ten thousand ten ten-billionths.
12. Twenty-four and seven tenths.
13. Five and twelve thousandths.
14. Two hundred five thousandths.
15. Two hundred and five thousandths.
16. Four hundred ten thousandths.
17. Four hundred ten-thousandths.
18. Six hundred nine and six hundred nine hundred-thousandths.

Write the following expressions decimally:

19. $\frac{15}{1000}$.
20. $\frac{38}{100000}$.
21. $\frac{3214}{10000000}$.
22. $32\frac{51}{10000}$.
23. $325\frac{8}{10000000}$.
24. $308\frac{4}{10000000000}$.

Express the following numbers in words:

25. .209.
26. .500.
27. .00005.
28. 700.007.
29. .707.
30. 2540.000004.
31. 20006.000120$\frac{2}{3}$.
32. 700.00000000036$\frac{3}{8}$.

NUMERATION OF DECIMALS.

229. *1.* Express orally, the decimal .00123.

EXPLANATION. First read the numerator as if it were an integral number *(one hundred twenty-three)*; then numerate from the point, calling

the first order tenths, the next hundreths, etc., until the right-hand figure is named (hundred-thousandths); or, determine the denominator by counting the decimal places according to (221, 2).

RULE. *Read the decimal as if it were an integer, giving it the name of the right-hand order.*

Express the following orally:

2. .129.	*6.* $5.123.	*10.* .0015 bushels.
3. .0045.	*7.* $65.00042.	*11.* 8.075 tons.
4. .37008.	*8.* $1204.0008.	*12.* 111.25 yards.
5. .000016.	*9.* 12.342 pounds.	*13.* 32.01$24\frac{3}{8}$ miles.

REDUCTION.

CASE I.

230. To reduce a decimal to a common fraction.

1. Reduce .625 to an equivalent common fraction.

Operation.

$.625=\frac{625}{1000}=\frac{5}{8}$.

EXPLANATION. We express the decimal value by omitting the decimal point and writing the denominator; thus, $\frac{625}{1000}$; reducing this expression to its lowest terms, gives $\frac{5}{8}$, the correct result.

RULE. *Omit the decimal point, write the denominator, and reduce the fraction to its lowest terms.*

EXAMPLES FOR PRACTICE.

Reduce the following decimals to common fractions:

2. .25.	*7.* .575.	*12.* 5.25 gallons.
3. .125.	*8.* .00015.	*13.* 13.575 acres.
4. .475.	*9.* .96.	*14.* $8.125.
5. .0075.	*10.* .0048.	*15.* .00005 of a mile
6. .00875.	*11.* .0725.	*16.* .03625 of a year

17. Reduce $.16\frac{2}{3}$ to a common fraction.

Operation.

$.16\frac{2}{3}=\frac{16\frac{2}{3}}{100}=\frac{50}{300}=\frac{1}{6}$

EXPLANATION. We omit the point and write the denominator, as in the former illustrative example; the fraction thus formed, being complex, we simplify by multiplying both terms by the denominator of the terminal fraction belonging to the numerator, *we then reduce the* resulting fraction to its lowest terms.

18. Reduce .06¼ to a common fraction.

19. Reduce .000⅛ to a common fraction.

20. Express the most simple value of 28.83⅓ rods.

CASE II.

231. To reduce a common fraction to a decimal.

1. Reduce $\frac{13}{16}$ to a decimal.

Operation.

16)130(.8125
128
20
16
40
32
80
80

EXPLANATION. The value of a fraction is the quotient obtained by dividing the numerator by the denominator, but since the numerator will not contain 16, we multiply it by the radix of the decimal system, 10, and commence the division; we multiply each remainder by 10, and continue the work until the division terminates. An inspection of the work shows that we have annexed four ciphers, which is equivalent to a multiplication by 10000, hence our quotient, 8125, is 10000 times the correct quotient (173, 1), we therefore point off 4 figures, which divides by 10000 and gives the correct result, .8125.

232. From the foregoing explanation we derive the following

RULE I. *Annex ciphers to the numerator and divide by the denominator.*

II. *Point off as many places in the result as are equal to the number of ciphers annexed.*

EXAMPLES FOR PRACTICE.

Reduce the following common fractions to decimals:

2. $\frac{3}{8}$.
3. $\frac{5}{32}$.
4. $\frac{7}{128}$.
5. $\frac{137}{256}$.
6. $\frac{1}{512}$.
7. $\frac{17}{125}$.
8. $\frac{55}{64}$.
9. $\frac{19}{1024}$.
10. $\frac{176}{2500}$.
11. $\frac{7}{50}$ of a mile.
12. $\frac{18}{500}$ of a day.
13. $\frac{317}{6400}$ of an acre.

233. In the reduction of common fractions to decimals only the prime factors 2 and 5 are introduced by the multiplications; therefore, if the denominator of any common fraction in its lowest terms contains other prime factors, it is evident that the division will never terminate, and that the reduction to a perfect decimal will be impos-

sible. An approximate decimal value, however, may be obtained sufficiently accurate for practical purposes.

Reduce the following to approximate decimal values, continuing the work to five decimal places:

NOTE. Whenever the final remainder is less than one-half the divisor, it should be disregarded and the sign + annexed to the last decimal figure. If the final remainder equals or exceeds one-half the divisor in unit value, the last decimal figure should be increased by 1 and the sign — annexed.

1. $\frac{2}{3}$.	*4.* $\frac{23}{36}$.	*7.* $.15\frac{2}{3}$.	*10.* $178.2\frac{4}{9}$.
2. $\frac{5}{7}$.	*5.* $\frac{53}{140}$.	*8.* $.121\frac{1}{7}$.	*11.* $367.000\frac{11}{13}$.
3. $\frac{7}{12}$.	*6.* $\frac{111}{700}$.	*9.* $13.0\frac{8}{11}$.	*12.* $15.400\frac{1}{6}$.

ADDITION OF DECIMALS.

234. Since decimals and integers increase and decrease uniformly by the scale of 10, and are in all respects subject to the same general laws of relation, it is evident that computations of decimals are subject to the same principles and rules as govern the computations of integral numbers.

1. What is the sum of 15.32, 272.7, 48.0035, and .418?

Operation.

15.32
272.7
48.0035
.418
336.4415

EXPLANATION. We write the addends so that units of like orders, whether integral or decimal, shall stand in the same column. We begin at the right and perform the work according to principles explained in addition of integral numbers.

235. For rule governing the addition of decimals, see rule for addition of integral numbers (54).

NOTE. If decimals for addition be so written that the decimal points form a column, it is evident that units of the same order will stand in the same column.

EXAMPLES FOR PRACTICE.

2. Add 187.35, 4.072, .3756, and 38.3.

3. Add 36.4, 8.342, 1564.1, and .3752.

4 What is the sum of 57.1875, 3854.2, 41.63, and 14821.75?

5. What is the sum of $3.8\frac{1}{2}$, $6.344\frac{3}{4}$, $18.7\frac{4}{5}$, and $132.14\frac{5}{8}$?

6. What is the approximate sum, expressed with three decimal places, of $24.5\frac{3}{5}$, $8.1574\frac{1}{8}$, 72.45, $256.12\frac{3}{4}$, $748.0\frac{1}{3}$, and $57.61572\frac{2}{3}$?

EXPLANATION. The first number requires 2 additional decimal places, hence the common fraction $\frac{3}{5}$ must be reduced to a decimal of 2 places, which is 67— (233, Note): The second number contains more than the required number of decimals, hence all except the first three must be dropped and the sign + annexed; the fourth requires an additional decimal place, which a reduction of $\frac{3}{4}$ will furnish. (The reduction gives 7 for the first place, but as the next decimal figure would be 5, it must be regarded as 8—). The last addend contains more than the required number, therefore the last two places must be dropped, but as these places represent more than $\frac{1}{2}$ of a unit of the preceding order, the decimal part of the number will be .616—. The addends, when prepared for addition will be 24.567—, 8.157+, 72.45, 256.128—, 748.011+, and 57.616—.

7. What is the approximate sum, expressed with four decimal places, of $5.3\frac{5}{8}$, $17.4\frac{7}{9}$, $87.4\frac{1}{7}$, $5.784632\frac{1}{8}$, and $28.00\frac{3}{4}$?

8. What is the sum of $218.72\frac{1}{4}$ tons, $54.36\frac{3}{7}$ tons, $225.125342\frac{3}{8}$ tons, and $42.18\frac{5}{12}$ tons, to 5 decimal places?

9. What is the decimal sum of $\frac{17}{128}$, $\frac{13}{16}$, $\frac{5}{7}$, $45\frac{3}{8}$, $12\frac{2}{9}$ and $\frac{15}{64}$, approximately correct to 5 places of decimals?

10. What is the sum of three hundred thousandths, five hundred seven and seven thousandths, three hundred-thousandths, sixteen hundred ten and five ten-thousandths, and two hundred and eight tenths?

11. A man bought $26.5\frac{3}{4}$ cords of wood from A, $15.28\frac{1}{6}$ cords from B, and $72\frac{7}{16}$ cords from C. What was the decimal sum of his purchases?

12. What is the sum of two hundred five and fifteen thousandths miles, one hundred and forty-five thousandths miles, and one thousand one hundred and one hundredth

SUBTRACTION OF DECIMALS.

236. *1.* From 18.3 subtract 5.247.

Operation.

18.300
5.247
13.053

EXPLANATION. We write the subtrahend under the minuend with units of the same order in the same column; we reduce the minuend to the same denomination as the subtrahend by the annexation of two ciphers, and perform the subtraction as explained in the subtraction of integral numbers.

NOTE. The reduction to a common denominator by the annexation of ciphers is not a necessity, and may be dispensed with in practice if desired.

237. For rule governing decimal subtraction, see 63.

EXAMPLES FOR PRACTICE.

Find the difference between

2. 183.542 and 96.08.
3. 13.75 and 12.845.
4. 8.007 and 3.2874.
5. 56.3 and .563.
6. 348.3 and 34.83.
7. 59.32 yards and 36.128 yards.
8. 8.743 lbs. and 1.5446 lbs.
9. 284. and .0284.
10. 76.4 and 58.374.
11. 23.45 miles and 13.460 miles.

12. $62.754\frac{3}{4}$ ounces and $38.33\frac{1}{8}$ ounces.

13. $568.7\frac{1}{7}$ yards and 135.245 yards.

14. $87.6\frac{2}{3}$ and $15.245\frac{1}{2}$ approximately correct to three decimal places.

15. $342.17\frac{5}{8}$ rods and $196.42\frac{1}{7}$ rods, to five decimal places.

16. From three thousandths subtract three hundred-thousandths.

17. From nine thousand subtract nine thousandths.

18. The sum of two numbers is one hundred and twelve thousandths, and one of the numbers is one hundred twelve thousandths. What is the other number?

MULTIPLICATION OF DECIMALS.

238. *1.* Multiply 4.562 by .23.

Operation.

4.562
.23
13686
9124
1.04926

EXPLANATION. We first multiply the numerators of the factors together according to the law of fractional multiplication, and obtain 104926 as the numerator of the product. Since the denominator of the first fraction is 1 followed by *three* ciphers, and the denominator of the other is 1 followed by *two* ciphers, the product of these denominators, which is the denominator of the product, must be 1 followed by 3+2, or 5 ciphers; hence, there must be 5 decimal places in the product. (221, 2).

239. The law of index relations (74, 1), which is general and applies to the multiplication of decimals the same as to integers, furnishes a demonstration of the fact that the product of decimals must contain the same number of decimal places as the factors multiplied together to produce it. Thus, units whose index is —2, (hundredths), multiplied by units whose index is —1, (tenths), will furnish an order whose index is —3, (thousandths). Units whose index is +2, (thousands), multiplied by units whose index is —2 (hundredths), will furnish an order whose index is the sum of +2 and —2, which is zero, (units), etc.

NOTE. Integral indices are positive, and decimal indices, being numbered in the opposite direction, are negative.

RULE. *Multiply as in the multiplication of integral numbers, and point off as many decimal places as are in both factors.*

REMARK 1. If the product does not contain the required number of figures to be pointed off, supply the deficiency by prefixing ciphers.

REM. 2. Decimals may be multiplied by 10, 100, 1000, etc., by moving the decimal point of the multiplicand as many places to the right as there are ciphers in the multiplier.

EXAMPLES FOR PRACTICE.

Multiply

2. 32.35 by 24.
3. 5.037 by .7.
4. 150.14 by 3.15.
5. .02156 by .0004.
6. 3.0024 by 1.322.
7. 120.40 by .05.
8. 3906.25 by .00128.

Multiply

9. .00137 bushels by .032.
10. 5.126 pounds by 3.5.
11. 712.4 yards by 7.15.
12. 34.18$\frac{3}{4}$ miles by 5.21.
13. 61.35$\frac{4}{9}$ acres by .12$\frac{1}{7}$.
14. 142.176 by 100 (Rem. 2).
15. 15.26784 by 100000.

DIVISION OF DECIMALS.

240. *1.* Divide 54.696 by 12.72.

Operation.

12.72) 54.696 (4.3
50 88
3 816
3 816

EXPLANATION. Dividing in the same manner as in the division of one integral number by another, we obtain 4 units of some order and 3 units of the next lower order, the local value being determined by the location of the decimal point. Since the divisor and quotient, taken together, must contain 3 decimal places, the same as the dividend, which is a product of divisor and quotient, it is evident that what the divisor lacks of the requisite number of places must appear in the quotient. The divisor having but two places, the quotient must contain one, as shown in the operation.

241. From the foregoing explanation we derive the following principles:

1. *The dividend must contain as many places of decimals as both divisor and quotient; therefore, it cannot have a less number than the divisor.*

2. *The number of decimal places in the quotient equals the number of decimal places in the dividend minus the number of decimal places in the divisor.*

RULE. *Divide as in the division of one integral number by another, and point off as many decimal places in the quotient as the number of decimal places in the* dividend exceeds the *number of decimal places in the divisor.*

REMARK 1. If a given dividend has fewer decimal places than its divisor, supply the deficiency by annexing ciphers.

REM. 2. If the figures of the quotient are fewer in number than the decimal places required, supply the deficiency by prefixing ciphers.

REM. 3. Decimals may be divided by 10, 100, 1000, etc., by moving the decimal point of the dividend as many places to the left as there are ciphers in the divisor.

EXAMPLES FOR PRACTICE.

Divide

2. 8.75 by 2.5.
3. 11.242 by .73.
4. 4.96 by .0008.
5. 420 by .084.
6. 75.68 by 2.151.
7. 37.256 by 2.1352.

Divide

8. 43.8 acres by .006.
9. 75.075 of a year by .025 of a yr.
10. 456.36 rods by .042 of a rod.
11. .5280 of a mile by 16.5.
12. 61.04$\frac{2}{3}\frac{7}{3}$ of a foot by 37.5$\frac{3}{8}$.
13. $\frac{3}{8}$ by .1875.

Divide (three decimal places if interminal)

14. 52.13$\frac{5}{7}$ by 148.3$\frac{1}{3}$.
15. 5.12 pounds by .7$\frac{3}{8}$.
16. 156.0$\frac{1}{1\,1}$ by 24.002$\frac{1}{9}$.
17. 250.24$\frac{2}{5}$ by 1856.2$\frac{5}{1\,3}$.
18. 9.000$\frac{3}{8}$ bushels by 3$\frac{5}{7}$ bushels.
19. 456.2$\frac{1}{4}$ by .0008$\frac{1}{3}$.
20. 72.46$\frac{4}{5}$ by 18.4$\frac{7}{9}$.

Divide (five decimal places if interminal)

21. 874.12 miles by 4.22$\frac{3}{9}$ miles.
22. 4000.007$\frac{1}{7}$ by .004.
23. 32.63$\frac{1}{4}$ by 78.3$\frac{1}{3}$.
24. 96.117 by 45.44$\frac{4}{9}$.
25. 82.56 by 13.74$\frac{4}{5}$.
26. 308.45 by 17.2164$\frac{2}{3}$.
27. 500 by .025$\frac{1}{4}$.

28. How many tons of coal at 7$\frac{2}{3}$ dollars a ton can be purchased with $41.12?

UNITED STATES MONEY.

242. The denominations of United States money are shown by the following table:

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

243. An inspection of the above table shows that the denominations of United States money increase and decrease in a tenfold ratio according to the law of decimal fractions, hence we conclude that

1. The decimal system of notation should be employed to express United States money.

2. That all computations upon United States money are decimal in character.

NOTE 1. The dollar is the standard or integral unit of United States money.

NOTE 2. The character $ indicates that the number written after it expresses United States money.

244. Since the dollar is the integral unit, and is a tenth part of an eagle, it is evident that eagles are expressed by the second order of integral units; that dimes, (tenths of a dollar), are expressed by the first order of decimal units; that cents, (tenths of a dime), are expressed by the second order of decimal units, and that mills, (tenths of a cent), are expressed by the third order of decimal units. Hence the expression, 2 eagles, 4 dollars, 7 dimes, 6 cents and 5 mills is written decimally thus: $24.765.

245. In commercial practice, the entire integral part

f a number expressing United States money is considered s dollars; dimes are also disregarded, and the first two laces at the right of the point are read as cents. Thus 35.28 instead of being read 3 eagles, 5 dollars, 2 dimes, cents, is read 35 dollars and 28 cents. Hence, the table f United States money in business practice is as follows:

10 mills make 1 cent
100 cents make 1 dollar

246. Since cents represent hundredths of a dollar, it is vident that a single figure expressing cents must occupy he second decimal place, and be preceded by a cipher. Thus, 8 cents should be written $.08.

NOTE 1. Five mills are frequently written ½ of a cent. Thus $.625= $.62½.

NOTE 2. It is customary to express cents on negotiable paper (notes, checks, etc.) as hundredths of a dollar. Thus, 125\frac{50}{100}$.

NOTE 3. In business transactions, a fraction of a cent does not appear in a final result; five or more mills are regarded as a cent, and a less number is rejected. Thus, $1.248 would be called $1.25, and $5.182 would be called $5.18.

REDUCTION OF UNITED STATES MONEY.

247. Since $1=100 cents=1000 mills, it is evident

1. That dollars may be reduced to cents by annexing *two* ciphers; and to mills by annexing *three* ciphers.

2. That cents may be reduced to dollars by pointing off *two* places; and that mills may be reduced to dollars by pointing off *three* places.

3. That, generally, any denomination may be reduced o a lower by annexing the number of ciphers expressing he relation; and that any denomination may be reduced o a higher by pointing off the number of places express- ng the relation.

248. Notwithstanding the fact that the dollar is the standard or integral unit in commercial computations, any

other denomination can be made the integral unit by reduction, and whenever any such change of unit occurs the sign indicating United States money should not be employed. The expression $25 cents is conflicting.

REVIEW OF DECIMALS.

1. Express decimally the sum of $15.5\frac{3}{4}$, $112.23\frac{1}{2}$, $6\frac{7}{8}$, and $245\frac{1}{16}$.

2. Find the difference between $18.52\frac{3}{8}$ bu. and $11.074\frac{1}{7}$ bu.; correct to five decimal places.

3. Reduce the following numbers to hundredths: $18\frac{1}{8}$, 12, 6.3, and 17.5275.

4. If I pay $5.775 for .7 of a ton of coal, what is the price per ton?

5. If $3\frac{1}{2}$ pounds of tea cost $2.275, how much should be paid for 12.35 pounds of the same grade?

6. An invoice of 3640 pounds of merchandise was shipped over two lines of road; the company to whom the same was consigned controlled $202\frac{1}{2}$ miles, and the other company $121\frac{1}{2}$ miles of the distance. If the entire freight bill was at the rate of 45¢ per cwt., how much should the second company receive?

7. Express decimally the difference between $\frac{5}{6}$ of a tenth and $\frac{3}{4}$ of a thousandth.

8. A drover bought 6 head of horses at $85 each, and 12 head at $80 each; at what price per head must he sell them to gain $132?

9. A man bought equal quantities of tea, coffee and sugar, at the following prices per pound: tea, $.62\frac{1}{2}$; coffee, $.24\frac{1}{4}$; and sugar, $.06\frac{1}{4}$. If the total cost was $41.85, how many pounds of each did the purchase contain?

10. If a man travel 963.75 miles in 30 days, how far can he travel in a week of 7 days, if he increase his rate *of speed* .1?

11. If 6 bushels of wheat are worth \$4.71, how much is .8 of a bushel worth?

12. A dealer bought 55 barrels of flour at \$5.80 per barrel, and 250 bushels of corn at \$.42 per bushel; he afterward sold 30 barrels of the flour at \$6.10 per barrel, and all of the corn at \$.45 per bushel. At what price per barrel must he sell the remainder of the flour to realize a gain of \$23.50 on the whole purchase?

13. If B can do .15 more work in a given time than A, how long a time will be required for both working together to perform a job of work that A can do alone in 8 days?

14. If 57.4 acres of land produce 6446.02 bushels of potatoes, how many acres will be required to produce 786.1 bushels?

15. Divide $.0516\frac{51}{64}$ by $.09\frac{3}{16}$, and obtain a quotient true to four decimal places.

16. B's age exceeds A's by .2, and C's age exceeds B's by .75; if the sum of their ages is 86 years, how old is C?

17. A man owning $\frac{3}{4}$ of a steamboat, sold .125 of the boat to A; he afterward sold .25 of his remaining interest to B for \$7250. Computing upon the same basis of value, what was the boat worth?

18. A farmer sold to a merchant $37\frac{1}{2}$ bushels of wheat at \$.82 per bushel, $46\frac{1}{2}$ pounds of butter at \$.18 per pound, and 28 dozen eggs at \$.22 per dozen. He received in exchange 12 yards of dress goods at \$1.75 per yard, $34\frac{3}{4}$ yards of sheeting at \$.08, and the remainder in money; how much money did he receive?

19. If .375 of an acre of land is worth \$33, what is the value of a tract of the same grade of land containing 1245.75 acres?

20. Divide 40.5 by $13\frac{1}{2}$ hundredths.

21. Multiply 37 tenths by 115 hundredths.

22. One thousand five hundred fifty thousandths multiplied by 120 hundredths, equals how many tenths?

23. A speculator bought 1600 bushels of barley at $\$.46\frac{3}{4}$ per bushel, and sold .375 of his purchase at $\$.49\frac{1}{4}$ per bushel. At what price per bushel must he sell the remainder to gain \$52.50 on the whole?

24. If .325 of a ton of coal is worth \$2.73, how much are $12\frac{3}{4}$ tons worth?

25. A can do a certain piece of work in 5.12 days; A and B, working together, can do the same work in 3.2 days, and A and C, working together, can do the same in 2 days. In what time can B and C, working together, do the same work?

26. A man having contracted to pave a certain street, employed a crew of men who performed one-half of the work in 37.5 days, at the end of which time a strike occurred, and the contractor secured the services of a second force, consisting of $\frac{5}{6}$ as many men as belonged to the former crew. How many days were required for the second crew to complete the work, provided that each man thereof could do .9 as much work, during a given time, as a man belonging to the first crew?

27. A has \$150, which is 25 hundredths more than B possesses. B's money is what decimal part of A's?

28. If A's money is $16\frac{2}{3}$ hundredths less than B's, B's is how many hundredths more than A's?

29. A farmer had 160 acres of land, which he could have sold at one time for \$50 an acre, and thereby gained \$800; he afterward sold it at a loss of \$200. What was the selling price per acre?

30. The sum of two numbers is 1.25, and their difference is .0375; what are the numbers?

31. If 4.25 tons of hay are equal in value to $63\frac{3}{4}$

bushels of potatoes, and 1 bushel of potatoes is worth $.40, how much is a ton of hay worth?

32. A grocer bought an invoice of tea and coffee, the aggregate cost of which was $724.50. If the cost of the coffee was .725 of the cost of the tea, what was the cost of each?

33. A manufacturer, during the first year of business, gained a sum equal to .35 of his capital, which he added thereto; during the second year his losses amounted to .15; he then retired with a capital of $8262. How much was his net gain during the two years?

COUNTING-HOUSE PRACTICE.

249. The subject treated under this head is designed to furnish information concerning the leading contractions employed in business transactions.

NOTE. The term *counting-house* is a corruption of the expression *accounting-house*, hence the literal meaning of the term is *a place where accounts are kept.* The term is now employed to convey the idea of a place of *commercial business.*

250. The *elements of commercial traffic* are the following: *Price, Quantity, and Cost.*

251. **Price** is the value of one unit of any kind of commodity involved in a business transaction. (*Unit value*, 119.)

252. **Quantity** is the total number of units involved in a business transaction. (*Number of units*, 119.)

253. **Cost** is the value of the total number of units involved in a business transaction. (*Aggregate value*, 119.)

254. When the price is an aliquot part of a dollar, it is frequently more convenient to compute by a common fraction than by a decimal.

255. An **Aliquot Part** of a number is such a part as can be expressed by a simple fraction having 1 for a numerator.

Parts expressed by the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., are aliquot parts.

256. The aliquot parts of a dollar frequently employed to express price are shown by the following:

TABLE OF ALIQUOT PARTS OF ONE DOLLAR.

50 cents = $\$\frac{1}{2}$	20 cents = $\$\frac{1}{5}$	10 cents = $\$\frac{1}{10}$
$33\frac{1}{3}$ " = $\$\frac{1}{3}$	$16\frac{2}{3}$ " = $\$\frac{1}{6}$	$8\frac{1}{3}$ " = $\$\frac{1}{12}$
25 " = $\$\frac{1}{4}$	$12\frac{1}{2}$ " = $\$\frac{1}{8}$	$6\frac{1}{4}$ " = $\$\frac{1}{16}$

NOTE. Such fractions as $\frac{3}{8}$, $\frac{5}{8}$, $\frac{3}{5}$, etc., express such parts as are sometimes termed *multiple aliquots*.

CASE I.

257. To find the cost when the price is an aliquot part of $1.

1. What is the cost of 42 pounds of butter at $16\frac{2}{3}$ cents a pound?

Operation.

6) $42
$ 7

EXPLANATION. We first write the cost at $1 a pound, which is a number expressing as many dollars as there are units in the quantity. Since the price is $\frac{1}{6}$ of $1 a pound, it is evident that the cost is $\frac{1}{6}$ of $42, which sum we obtain by dividing by 6.

RULE. *Consider the cost at $1 per unit, which amount will be as many dollars as there are units in the given quantity; then take such a part of said amount as the price is part of a dollar.*

EXAMPLES FOR PRACTICE.

Find the cost of

2. 245 bu. corn at $33\frac{1}{3}$c.
3. 1720 lbs. coffee at 25c.
4. 82 doz. eggs at $12\frac{1}{2}$c.
5. 326 yds. flannel at $16\frac{2}{3}$c.
6. 153 bu. barley at 50c.

Find the cost of

7. 324 bbl. flour at $\$5.66\frac{2}{3}$.
8. 78 bu. corn at $37\frac{1}{2}$c.
9. 520 bbl. apples at $3.25.
10. 65 lbs. rice at $8\frac{1}{3}$c.
11. 158 bu. onions at $\$1.12\frac{1}{2}$.

12. Find the cost of 72 bushels of corn at 40 cents a bushel (40c. = $\$\frac{2}{5}$).

13. Find the cost of 96 bushels of barley at 45 cents a bushel. (45 cents = $\$\frac{1}{4} + \$\frac{1}{5}$).

14. Find the cost of 48 pounds of tea at 58⅓ cents a pound. (58⅓=33⅓+25.)

CASE II.

258. To find the quantity when the price is an aliquot part of $1.

1. How many dozen eggs can be bought for $7 at 16⅔ cents a dozen?

Operation.

6 doz.
7
42 doz.

EXPLANATION. Since the price is ⅙ of a dollar, it is evident that $1 will buy 6 dozen; therefore $7 will buy 7 times 6 dozen.

RULE. *Multiply the quantity that $1 can buy by a number equal to the cost.*

EXAMPLES FOR PRACTICE.

Find the number of pounds.

	Cost.	Price per lb.
2.	$13	16⅔¢.
3.	$121	25¢.
4.	$34	33⅓¢.
5.	$156	12½¢.

Find the number of yards.

	Cost.	Price per yd.
6.	$184	50¢.
7.	$721	37½¢.
8.	$152	83⅓¢.
9.	$1524	$2.33⅓.

10. A farmer sold to a merchant 20 bushels of potatoes at 37½ cents a bushel, and received in payment 15 pounds of coffee at 16⅔ cents a pound, and the remainder in cloth at 20 cents a yard. How many yards did he receive?

CASE III.

259. To find the cost when the price is per hundred or per thousand.

1. Find the cost of 1824 feet of flooring at $3.65 per 100 feet.

Operation.

1824
3.65
9120
10944
5472
$66.5760

EXPLANATION. Multiplying the price, $3.65, regarded as the price per foot, by the quantity, gives $6657.60; but since the price is per 100 feet, this sum must be 100 times the correct product; therefore we divide by 100 by removing the point two places to the left, and we have the correct result.

RULE I. *Multiply the price per 100 by the quantity, and point off two more decimal places than are in both factors.*

II. *Multiply the price per 1000 by the quantity, and point off three more decimal places than are in both factors.*

EXAMPLES FOR PRACTICE.

Find the cost of

	Quantity.	Price per 100.
2.	3120 lb. meal at	$2.60.
3.	1435 lb. lard at	$9.75.
4.	1756 lb. flour at	$3.20.
5.	4180 lb. freight at	$.95.
6.	940 cabbages at	$3.25.
7.	6145 lb. beef at	$8.75.

Find the cost of

	Quantity.	Price per 1000.
8.	1876 ft. boards at	$28.50.
9.	3264 ft. boards at	$38.25.
10.	4500 cigars at	$65.00.
11.	2560 R. R. ties at	$280.00.
12.	7250 shingles at	$4.50.
13.	3712 ft. flooring at	$35.25.

CASE IV.

260. To find the cost when the price is per ton of 2000 pounds.

1. Find the cost of 2940 pounds of coal at $6.40 per ton.

Operation.

2940
3.20
588
882
$9.40800

EXPLANATION. Since the price is $6.40 for 2000 pounds, the price for 1000 pounds is $3.20. Applying the principle explained in Case III, we have $9.41.

RULE. *Multiply one-half the price per ton by the quantity, and point off three more decimal places than are in both factors.*

NOTE. It will sometimes be found more convenient to multiply the price per ton by one-half the number of units, or to multiply the price per ton by the number of units and divide the result by 2.

EXAMPLES FOR PRACTICE.

Find the cost of

Quantity.	Price per ton.
. 4850 lb. coal at	$6.25.
. 3875 lb. iron at	$58.75.
. 2764 lb. hay at	$8.75.
. 1342 lb. ice at	$5.80.
. 2840 lb. coal at	$9.45.

Find the cost of

Quantity.	Price per ton.
7. 5840 lb. freight at	$5.85.
8. 3872 lb. hay at	$9.25.
9. 1876 lb. straw at	$4.75.
10. 4830 lb. provd'r at	$43.20.
11. 2560 lb. coal at	$8.80.

CASE V.

261. When the price is per bushel or per barrel.

Grain, seeds, and other farm produce are generally sold by the bushel of a certain number of pounds. The standard weight of a bushel in pounds according to the established law of most states is as follows:

Barley	48	Corn in the ear	70	Rye	56
Beans	60	Corn-meal	50	Rye-meal	50
Buckwheat	48	Oats	32	Timothy seed	45
Clover seed	60	Peas	60	Wheat	60
Corn, shelled	56	Potatoes	60	Wheat bran	20

1. What is the cost of 2184 pounds of barley at 45 cents a bushel?

Operation.

2184
.45
10920
8736
8)982.80(20.47
96
228
192
360
336
24

EXPLANATION. We first find the cost at 45 cents a pound and obtain $982.80; but this result is evidently 48 times the true cost, since the price is 45 cents for 48 pounds; hence we divide the product by 48 and obtain the correct result, $20.48.

RULE. *Multiply the price per bushel by the weight in pounds, and divide by the number of pounds in a bushel.*

EXAMPLES FOR PRACTICE.

Find the cost

	Quantity.	Price per bu.
2.	2460 lb. wheat at	87¢.
3.	1745 lb. oats at	27¢.
4.	845 lb. corn in ear at	42¢.
5.	5480 lb. beans at	$2.10.
6.	560 lb. timothy seed at	$2.20.
7.	4215 lb. potatoes at	35¢.
8.	3740 lb. rye at	64¢.
9.	2525 lb. corn meal at	80¢.
10.	1490 lb. wheat at	90¢.
11.	5724 lb. shelled corn at	58¢.

12. A farmer sold to a grain dealer a load of cor the ear, weighing 2840 pounds, two loads of barley we ing respectively 2545 and 2712 pounds. If the pric corn was 57 cents, and the price of barley 42½ cents, l much should the farmer receive?

BILLS, INVOICES, AND STATEMENTS.

262. A **Bill** or **Invoice** is an itemized statement of merchandise sold, or of service performed.

REMARK 1. The part of a bill or invoice above the head-line is called the *bill head*, and should contain the following essentials: Place and date of business transaction, name of party to whom sale is made, or for whom service is rendered, name of party making the sale or performing the service, and terms. Bill heads commonly contain more or less of other matter of an advertising nature. The body of the bill should contain an itemized list of the goods sold, the price per unit of each commodity, together with the aggregate amount, numbers or identifying marks pertaining to different packages, and any other details that may be of information to the purchaser.

REM. 2. When payment is made the words "*Received Payment*" are usually stamped or written at the bottom, with the signature of the party or firm making the sale. The bill may be receipted by an authorized agent, who should place his initials under the signature of the firm.

263. A **Statement of Account** is a written list of debits and credits copied from the ledger. A statement does not always contain an itemized list of sales. The aggregate of each separate sale is sufficient, inasmuch as an itemized bill has been previously rendered.

264. Abbreviations and characters are of common occurrence in bills and invoices. Those most frequently employed are exhibited in the following list:

COMMERCIAL ABBREVIATIONS AND CHARACTERS.

@.....At	Apr.....April	Bkts.....Baskets
Acct. or %...Account	Aug.....August	Bls.....Bales
Al.....First Quality	Bal.....Balance	Bot.....Bought
Agt.....Agent	Bbl. or brl.....Barrel	Bro.....Brother
Amt.....Amount	Bdls.....Bundles	Brot..... Brought
Astd.....Assorted	Bds.....Boards	Bu.....Bushels
Arr.....Arrived	Bgs.....Bags	Bxs.....Boxes

C.....Hundred
℔ or cts.....Cents
Chgd.....Charged
Chts.....Chests
Cks.....Casks
Co.....Company
C. O. D..{Collect on Delivery
Com.....Commission
Cr.....Credit
Cs.....Cases
Cwt..Hundredweight
d.....Pence
Dec.....December
Dft.....Draft
Disct.....Discount
n/c.....New Account
o/c.....Old Account
c/o.....Care of
%.....Per cent.
Do., Ditto, or The same
Doz.....Dozen
Dr.....Debtor
Dray.....Drayage
Ds. or da.....Days
Ea.....Each
E.&O.E.{Errors and Omissions Excepted
Exch.....Exchange
Feb.....February
Figd.....Figured
Fir.....Firkin
Fol.....Folio or Page
Frt.....Freight
Ft.....Foot or Feet
F. O. B. Free on Board
Gal.....Gallon
Gr. or Gro.....Gross
Guar.....Guarantee
Hlf.....Half
Hhd.....Hogshead
i. e.....That is
In.....Inches
Ins.....Insurance
Inst...Present Month
Int.....Interest
Jan.....January
Jr. or Jun.....Junior
Lb.....Pounds
Ledg.....Ledger
M.....Thousand
Mar.....March
Mdse....Merchandise
Mem...Memorandum
✓.....Check Mark
×..By, as 5×6 inches
Messrs....Gentlemen
Mo.....Month
Mr.....Mister
Mrs.....Mistress
N. B.....Take Notice
No. or #.....Number
Nov.....November
Oct.....October
Oz.....Ounces
P.....Page
Pp.....Pages
Pay't.....Payment
Pd.....Paid
Per or ℔..By or by the
Pkgs.....Packages
P. O.....Postoffice
Pr.....Pair
Ps.....Pieces
Qr.....Quarter
Rec'd Pay't{Received Payment
Rec't.....Receipt
R. R. or Ry..Railroad
Rs. or rls.....Rolls
Ship't.....Shipment
Sept.....September
Stor.....Storage
Sunds.....Sundries
Super.....Superfine
Ult.....Last Month
Wt.....Weight
Yds.....Yards
Yr.....Year

265. Brief forms of expression are a necessity in writing bills, hence they are of frequent occurrence. A few of the more common are explained in the following:

INTERPRETATIONS OF BRIEF EXPRESSIONS.

Forms of expression.	*Interpretations.*
38^{1} yds., 41^{2} yds., 43^{3} yds.....	$38\frac{1}{4}$ yds., $41\frac{2}{4}$ yds., $43\frac{3}{4}$ yds.
62^{24} *bu.*, 125^{35} bu.....	62 bu. 24 lb., 125 bu. 35 lb.
15 doz. $\frac{6}{3.25}$, $\frac{9}{3.50}$.....	15 doz. in all,—6 doz. at $3.25 each, and the remaining 9 doz. at $3.50 each.

34 bxs. $\frac{20}{A}$ @ 2.75, $\frac{14}{B}$ @ 2.60.....	34 boxes in all,—20 boxes of grade A at \$2.75, and the remaining 14 boxes of grade B at \$2.60.
20 doz. $\frac{15}{2}$ @ 2.60, $\frac{5}{3}$ @ 2.50.....	20 doz. in all—15 doz. of No. 2 at \$2.60, and 5 doz. of No. 3, at \$2.50.
3 doz. ea. B. B. $\frac{2.50}{360}$, $\frac{1.90}{480}$, $\frac{3.25}{1250}$	3 doz. blank books No. 360 at \$2.50 per doz. 3 doz. No. 480 at \$1.90, and 3 doz. No. 1250 at \$3.25.
45–2×8—18 @ 26.50.....	45 pieces of dimension stuff, each 2 inches by 8 inches and 18 feet long, at \$26.50 per thousand feet.
Terms: 60 da.; 2% 10 da.	60 days' credit given, but if the purchaser pays the bill in 10 days, $\frac{2}{100}$ of the amount will be deducted as discount.

266. The pupil should be required to copy, with care and neatness, each of the following bills with pen and ink, make the extensions, and "foot up" the columns.

BILL No. 1.

FOLIO 273. CHICAGO, April 18, 1905.

JOHN D. ARNOLD,
City.

Bought of GAYLORD & PHELPS.

TERMS: Cash.

4	lb.	Gunpowder Tea	75	¢	.....		
24	"	Granulated Sugar	6½	"	.....	....	
8	"	Raisins	12½	"	.....		
16	"	Java Coffee	35	"	.....	....	
1¼	Gal.	Maple Syrup	90	"	.....	..	
¾	"	N. O. Molasses	75	"		....	
5	"	Kerosene Oil	18¾	"		....	
4	Cans	Peaches	15	"		....	
3	"	Tomatoes	13	"		....	
8	lb.	Dairy Butter	22	"	.....	....	
							\$..... ....

BILL No. 2.

MILWAUKEE, May 5, 1905.

BRADFORD, BRIGGS & CO.,
Dealers in
DRY GOODS, NOTIONS, CARPETS, SHADES,
LADIES' FURNISHING GOODS, ETC.
Sold to H. B. ALLEN, 312 Grant St., City.

TERM: 30 days.

48	yds. Brussels Carpeting	$1.30	.....	....		
5	pcs. Merrimac Prints 39, 43^1, 45^3, 38, 37^2, ...yds.	$6\frac{1}{2}¢$	.....	....		
12	pcs. L. L. Sheeting, 44^1, 39^2, 42, 38^3, 45, 36^1, 34, 37^1, 47^3, 34, 41^2, 38, ...yds.	$5\frac{1}{2}¢$	.....	....		
	Cooperage.....					80
					$.....	....

BILL No. 3.

ST. PAUL, MINN., Oct. 10, 1905.

JOHN W. MASON,
Lake City, Minn.
Bought of TWIN CITY PAPER CO.

TERMS: 4 mo; 3% 10 days.

Case No.	Quan.	Description	Wt.	Price per lb.				
4240	12	Reams D. M. note	5	19	.....	....		
	8	" " "	6	19	.....	....		
	14	" " Letter hds.	10	17	.....	....		
	6	" R. R. " "	12	$15\frac{1}{2}$	.....	....		
4241	10	M. Envelopes 6. 1620 1.60			.....	....		
	10	" " $6\frac{1}{2}$. 2420 1.75			.....	....		
	8	" " $6\frac{1}{2}$. 1750 1.50			.....	....		
	150	lb. Flat cap D. M.		14	.....	....	.....	....
		Less $\frac{1}{5}$.....					.....	....
							$.....	....

BILL No. 4.

MINNEAPOLIS, Dec. 18, 1905.

HENRY M. JOHNSON,
St. Paul, Minn.

Bought of CASCADE MILLING CO.

TERMS: Cash.

10	brls. Gold Coin Flour	$5.90	.....	....		
15	" Family Favorite Flour	5.75	....	....		
50	½ brl. sacks Corn Meal	.37½	.....	....		
42	½ " " Buckweat Flour	.39	.....	....		
5	½ " " Rye Flour	1.95	.....	....		
12	cwt. Ground Feed	.62½	.....	....		
75	bu. Corn	.35½	.....	...		
50	" Oats	.28	.....	...		
					$.....	....

Received Payment,
CASCADE MILLING CO.
per C. F. D.

BILL No. 5.

All claims for damages or shortage must be made within five days. In reporting errors, ordering duplicate items, etc., give page No.

Page No. 856. Salesman, D. F. G.

CHICAGO, ILL., June 24, 1905.

MR. H. C. DAYTON,
Davenport, Iowa.

Bought of GREGGS, HARPER & CO.

TERMS: 60 days net. *Subject to sight draft if not paid at maturity.*

Lot	Quan.	Items	Price		Extension			
254	1	Doz. Shirts	.....	....	4	60		
382	2	" Overalls	7	75	.....	....		
176	1½	" Pants	8	50	.....	....		
488	½	" "	9	25	.....	....		
B.B.B.	6½	" Underwear	4	25	.....	...		
K. B.	3½	" "	3	75	.....	....		
2420	2½	" Suits	8	80	.....	....		
2448	1½	" "	10	20	.....	....		
1576	2½	" "	7	60	.....	....		
576	6	" Hose		80	.....	....		
748	5½	" "		72	.....	....		
840	4½	" "		45	.....	....		
856	2½	" "		37½	.....	....	$.....	...

BILL No. 6.

PALMYRA, PA., June 23, 1905.

H. G. DE VOE & Co.,
Dixon, Ill.

Bought of the PENNSYLVANIA SHOE CO.

TERMS: { *3 per cent. 10 days.* / *60 days net.* } No discount allowed on part payment.

Case No.	Pairs	Description	Width	Sizes	Price	Amount
5842	12	Pat. Lea. Plain Lace	D	$1\frac{1}{2}/3$	60	.....
7248	24	" " S. H. Lace	D	$5/8$	83	.....
5843	12	" " Button	E	$5/7\frac{1}{2}$	81	.....
5844	12	French Kid Plain Lace	E	$3/5$	76	.....
5845	24	" " Slippers	EE	$4/7$	47	.....
5847	18	" " Plain Lace	EE	$2/3\frac{1}{2}$	45	.....
5848	12	Women's Kid Lace	EE	$5/7$	1 35	.....
5850	12	" Blu. Cong.	E	$3/5\frac{1}{2}$	1 63	.....
5851	24	" " Lace	D	$2\frac{1}{2}/5$	1 70	.....
		2 Cases				$ .....

No. 7. MONTHLY STATEMENT.

Folio 824.

MINNEAPOLIS, Oct. 1, 1905.

H. E. CARROLL,
Sioux Falls, S. D.

To CURTIS & WILSON, *Dr.*

1905				
Sept.	1	As per statement rendered	42	35
"	10	To Invoice	215	10
"	17	" "	28	40
"	22	" "	54	85
"	29	" "	131	65 ....

Write the following bills on bill cap paper in your own name. Assume the date and name of purchaser. Adopt whichever of the preceding forms, in each case, you deem the best suited, supplying additional ruling when necessary.

1. 12 pcs. Indian head sheeting @ $7\frac{1}{2}¢$. Case 84—37, *39*[1], *43*[2], *38, 37*[3], *36*, 42[3], 39[2], 46[1], 45, 44[2], 38[3]. Case 85—5

cs. L.L. Sheeting @ $5\frac{3}{4}¢$, 47^1, 43^2, 45, 39^2, 46^1. Terms,) days net.

2. 13 brls. Full Value Flour @ $5.85, 7 brls. Crystal [ills Flour @ $5.65, 36 $\frac{1}{8}$-brl. sacks Buckwheat Flour @ $2\frac{1}{2}¢$, 25 $\frac{1}{8}$-brl. sacks Rye Flour @ 44¢. Terms, 30 days, ith a discount of 2 per cent. if paid in 10 days.

3. Lot 183, 1 doz. shirts, $4.35. Lot 215, $3\frac{1}{2}$ doz. shirts) $4.90. Lot 1842, 2 doz. fancy hose @ 1.12\frac{1}{2}$. Lot 2150, $\frac{1}{2}$ doz. hose @ $87\frac{1}{2}¢$. Lot 1938, 3 doz. Boys' suits @ $7.45. ot 1656, $7\frac{1}{2}$ doz. underwear @ $4.10. Lot 1680, $5\frac{1}{4}$ doz. nderwear @ $4.37\frac{1}{2}$. Terms, 60 days net, subject to draft not paid at maturity.

4. Case No. 7243, 12 pairs Boys' shoes, sizes from 3 to , @ $87\frac{1}{2}¢$. Case No. 7245, 24 pr. Misses French kid, width , size $1\frac{1}{2}$ to $3\frac{1}{2}$, @ $1.65. Case No. 7260, 18 pr. Ladies lain lace, width EE, size $3\frac{1}{2}$ to 6, @ $2.30. Case No. 7261, 8 pr. Ladies Pat. Lea., width D, size 3 to 5, @ 2.62\frac{1}{2}$. Terms, 60 days, with a discount of 3 per cent. if paid in 10 lays.

DENOMINATE NUMBERS.

267. A **Denominate Number** is a number whose units are employed in measurement.

268. The leading subjects to which measurement applies are *value*, *gravity*, *extension*, *capacity*, *time*, and *angles*.

269. A **Simple Denominate Number** is one whose units are of the same denomination; as, 15 miles, 12 bushels, 8 tons.

270. A **Compound Denominate Number** is one whose units are of different denominations; as (3 pounds, 6 ounces), (7 gallons, 3 quarts, 1 pint).

REMARK 1. The units of a denominate number have application, hence every denominate number is concrete.

REM. 2. A denominate number cannot be a multiplier because a multiplier is abstract.

REM. 3. Some claim that fractional expressions are denominate numbers and that the denominators express the denomination to which the units apply. The claim is true in a certain sense, but the denominate numbers to which this article refers are of a different character. In the expression 5 bushels, the term *bushels* gives *name* to the things to which the 5 units apply, but in the expression $\frac{5}{8}$, the term eighths only expresses the *size* or *magnitude* of the units. $\frac{5}{8}$ may be an abstract number, but 5 bushels cannot.

271. The different parts of a compound denominate number are *similar*, since they may be rendered *like* by reduction. (17).

MEASURES OF VALUE.

272. **Money** is the standard of commercial value, hence all values, (in a commercial sense), are expressed by some *monetary unit.*

273. The money of most civilized countries consists of government obligations, bank bills, and specie.

274. **Government obligations** and **bank bills** are promises of payment made on paper, and are to be redeemed, according to their terms, by the parties issuing them.

275. **Specie** consists of gold, silver, platinum, nickel, and copper coins, stamped by the government and authorized to circulate as a medium of exchange, the number of monetary units they represent being expressed thereon.

276. **Alloy** is a baser metal compounded with gold and silver in making coin, to render such coins more durable.

277. **Bullion** is uncoined gold or silver. The term is also applied to coins so badly mutilated that they cannot remain in circulation.

278. The term **carat**, used in expressing the fineness of gold coin, is one part in 24. Coin that is 22 carats fine contains 22 parts pure gold and 2 parts alloy. The gold coins of the United States are 21.6 carats fine.

UNITED STATES MONEY.

279. **United States Money** is the legal currency of the United States. The standard unit is the gold dollar, which weighs 25.8 grains. All computations of United States money are based upon the decimal system of notation. (242.).

FRENCH MONEY.

280. **French Money** is the legal currency of France. The standard unit is the franc, which is worth in United States money $.193.

TABLE.

10 millimes (m.) = 1 centime (ct.)
10 centimes = 1 decime (dc.)
10 decimes = 1 franc (fr.)

REMARK. French money is expressed in francs and decimal parts of a

franc, and all computations are based on the decimal system of notation. 24 fr. 7 dc. 5 ct. are written 24.75 fr.

GERMAN MONEY.

281. German Money is the legal currency of Germany. The standard unit is the mark, whose intrinsic value in United States money is $.2385.

REMARK. The only fractional denomination in German money is the penny, which is .01 of a mark. 27 marks and 5 pennies are written 27.05 marks. Computations are based upon the decimal system of notation.

CANADA MONEY.

282. Canada Money is the legal currency of the Dominion of Canada. The standard unit is the dollar.

REMARK. The denominations of Canada money are the same as those of United States money, and all computations are decimal. The standard unit, although one dollar, is worth, in United States money, but $.922.

ENGLISH MONEY.

283. English Money (also called sterling money) is the legal currency of Great Britain. The standard unit is the pound or sovereign, the value of which, in United States money, is $4.8665.

TABLE.

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)

REMARK 1. Other denominations than those in the table are in use.

The florin	= 2 shillings
The crown	= 5 shillings
The guinea	= 21 shillings

REM. 2. The character indicating pounds is written before the number; thus, £124. Characters and abbreviations indicating other denominations are written after the numbers to which they refer; thus, 12s. 8d. For the sake of brevity, 12 shillings 8 pence are commonly written thus, 12/8. Farthings are usually written as fourths of a penny.

MEASURE OF WEIGHT.

284. Weight is the measure of the force of gravity.

REMARK 1. All bodies unsupported will be drawn to the earth by an invisible force called *gravity*, or the *attraction of gravity*. The measure of this force is called *weight*.

REM. 2. The kinds of weight in common use are *Troy Weight*, *Diamond Weight*, *Avoirdupois Weight*, and *Apothecaries' Weight*.

REM. 3. The standard unit of weight is the attractive force of the earth upon 22.794377 cubic inches of distilled water at the temperature of 39.83° Fahrenheit, with the barometer at 30 inches. This force just equals the Troy pound.

TROY WEIGHT.

285. Troy Weight is used in philosophical experiments, and in weighing gold, silver, and jewels.

TABLE.

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

DIAMOND WEIGHT.

286. Diamond Weight is used in weighing diamonds and other precious stones. The standard unit is the carat, whose weight is equal to 3.168 Troy grains.

TABLE.

16 parts = 1 carat grain
4 carat grains = 1 carat

REMARK. The term carat, employed in diamond weight, is an *actual weight*, as stated above, and must not be confounded with the *assay carat* (278). The latter is not a weight, but a term employed in expressing the relative purity of gold coins and jewelry.

APOTHECARIES' WEIGHT.

287. Apothecaries' Weight is used by druggists and physicians in compounding medicines.

TABLE.

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (℔.)

REMARK 1. Physicians express numbers in their prescriptions by the Roman notation, employing lower case or small letters instead of capitals. They also substitute j for i when terminal, and write the symbol of the denomination before the numeral expressing the quantity. Thus, 7 ounces, 6 drams, 2 scruples are written ℥ vij ʒ vj ℈ ij.

REM. 2. Dry medicines are bought and sold commercially by avoirdupois weight.

AVOIRDUPOIS WEIGHT.

288. Avoirdupois Weight is the weight usually employed in ordinary commercial transactions.

TABLE.

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight	= 1 ton (T.)

REMARK 1. The following denominations are in common use: 100 pounds of fish=1 quintal; 100 pounds of produce=1 cental; 100 pounds of nails=1 keg; 200 pounds of pork, beef or fish=1 barrel; 196 pounds of flour=1 barrel; 280 pounds of salt at the N. Y. salt works=1 barrel. For the weight per bushel of grain and other products, see 261.

REM. 2. The *long ton* (2240 lb.) was formerly in common use; but it is now seldom used excepting in estimating the weight of certain goods at the U. S. custom houses, and in freighting and wholesaling coal from the Pennsylvania mines.

REM. 3. The corresponding denominations of Troy weight and apothecaries' weight are the same, but both differ from the corresponding denominations of avoirdupois weight as shown by the following:

COMPARISON OF WEIGHTS.

5760 grains	= 1 lb. Troy	480 grains	= 1 oz. Troy
5760 "	= 1 lb. Apoth.	480 "	= 1 oz. Apoth.
7000 "	= 1 lb. Avoir.	437½ "	= 1 oz. Avoir.

REM. 4. The grain is uniform in weight under all conditions.

MEASURES OF EXTENSION.

289. Extension is the act of reaching out in any direction.

290. Measures of Extension are employed in the measurement of lines, surfaces, and solids.

REMARK. Any substance that occupies space has three dimensions, *length*, *breadth* or *width*, and *thickness*. Other terms are sometimes substituted to suit different conditions. Extension measured upward from a given base is termed *height;* extension measured downward is termed *depth*.

LINEAR MEASURE.

291. Linear Measure is used in measuring lines.

REMARK 1. Since every distance can be traced by a line, it is evident that linear measure is employed in ascertaining the measurement of any single dimension of extension, whether length, breadth, or thickness.

ORDINARY TABLE.		SURVEYORS' TABLE.	
12 inches (in.)	= 1 foot (ft.)	7.92 inches	= 1 link (l.)
3 feet	= 1 yard (yd.)	100 links	= 1 chain (ch.)
5½ yards	= 1 rod (rd.)	80 chains	= 1 mile (mi.)
320 rods	= 1 mile (mi.)		

REM. 2. The following denominations, although not belonging to the table, are in common use: 1 size = ⅛ of an inch (applied to shoes); 1 hand = 4 inches (used in measuring the height of horses); 1 span = 9 inches; 1 pace = 3 feet; 1 fathom = 6 feet.

MEASURES OF SURFACE.

292. The measuring unit of a surface is a square.

REMARK. A surface has two dimensions, *length* and *breadth*. The measurement of a surface consists in determining the number of square units it contains.

293. Surface or Square Measure is used in measuring land, boards, paving, and all other surfaces.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

REMARK. *According to* an old English custom, a square rod is some-

times called a *pole* or *perch*, and 40 square rods, a *rood*. These terms have become nearly obsolete in America.

SURVEYORS' SQUARE MEASURE.

294. Surveyors' Square Measure is used by surveyors in measuring land.

TABLE.

10000 square links (sq. l.)	= 1 square chain (sq. ch.)
10 square chains	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

MEASURES OF VOLUME.

295. The measuring unit of a volume is a *cube*, hence the measurement of volume is frequently called *cubic measure*.

296. Measures of Volume are used in the measurement of space.

REMARK 1. Space, considered in its entirety, is infinite and cannot be measured, but a limited portion of space will admit of measurement, and in measuring a material substance, the mathematical solution determines the number of cubic units of space occupied by it.

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

REM. 2. A cord of wood occupies 128 cubic feet of space. The standard cord pile is 8 feet long, 4 feet high, and 4 feet wide, and one foot of the length of the cord pile is a *cord foot*.

REM. 3. A *perch* of masonry contains the following dimensions: Length, $16\frac{1}{2}$ feet; height, 1 foot; thickness, $1\frac{1}{2}$ feet; the volume, therefore, is $16\frac{1}{2} \times 1 \times 1\frac{1}{2} = 24\frac{3}{4}$ cubic feet.

REM. 4. In measuring timber, $\frac{1}{5}$ of the volume is allowed for waste in squaring. A round log containing 50 cubic feet will make 40 cubic feet when squared. In estimating weight, 40 feet of square timber, or 50 feet of round timber ($\frac{1}{5}$ being deducted) is reckoned as a ton.

REM. 5. One hundred cubic feet of the internal capacity of a vessel is *registered a ton.*

MEASURES OF CAPACITY.

297. Capacity signifies room or space.

298. The **Measuring Unit** is a measure of known capacity, and the measurement determines how many of this measuring unit must be employed to fill a given space.

REMARK. Measures of capacity are divided into three classes; *Liquid Measure*, *Apothecaries' Fluid Measure*, and *Dry Measure*.

LIQUID MEASURE.

299. Liquid Measure is used in the measurement of ordinary liquids, or in determining the capacity of vessels containing them.

TABLE.

4 gills (gi.)=1 pint (pt.)
2 pints =1 quart (qt.)
4 quarts =1 gallon (gal.)

REMARK 1. The standard liquid gallon of the United States contains 231 cubic inches. The imperial gallon of Great Britain contains 277.274 cubic inches.

REM. 2. The following denominations are in common use: 31½ gallons=1 barrel; 63 gallons=1 hogshead.

REM. 3. The *tierce*, (42 gallons), the *puncheon*, (84 gallons), the *pipe* or *butt*, (126 gallons), and the *tun*, (252 gallons), are properly names of casks, the capacity here given being their approximate size. In commercial practice each cask is gauged and its exact capacity in gallons is marked upon it.

APOTHECARIES' FLUID MEASURE.

300. Apothecaries' Fluid Measure is used by physicians and by apothecaries in prescribing and compounding liquid medicines.

TABLE.

60 minims (M) =1 fluid drachm (ƒ℥)
8 fluid drachms=1 fluid ounce (ƒ℥)
16 fluid ounces =1 pint (O.)
8 pints =1 gallon (Cong.)

REMARK. *The gallon and the pint of this measure have the same capacity as those of liquid measure.*

DRY MEASURE.

301. Dry Measure is used in measuring grain, fruit, vegetables, and other dry substances.

TABLE.

2 pints (pt.)=1 quart (qt.)
8 quarts =1 peck (pk.)
4 pecks =1 bushel (bu.)

REMARK 1. The standard bushel of the United States contains 2150.42 cubic inches. The imperial bushel of Great Britain contains 2218.192 cubic inches.

REM. 2. The half-peck is sometimes called a gallon, and contains 268.8 cubic inches.

MEASURE OF TIME.

302. Duration is a synonym of eternity, and cannot be measured as a whole, since it is infinite.

303. Time is a limited portion of duration, and since any definite period of time has a beginning and an ending, it may be measured.

REMARK. Two of the denominations or measures of time, the day and the year, are the results of natural causes—the motions of the earth with relation to the sun.

304. A **Solar Day** is the time required for a point on the earth's surface to pass from the vertical rays of the sun and return to the same position.

REMARK. During the day, the earth makes a little more than one revolution on its axis, owing to the fact that it makes a forward movement in its orbit during the time. If the earth's yearly motion were stopped and the daily motion should continue, the day would be shortened about four minutes.

305. A **Solar Year** is the time required for the earth to make one complete revolution around the sun. The exact time is estimated to be 365 days, 5 hours, 48 minutes, and 48 seconds.

REMARK 1. Pope Gregory, the originator of the calendar now in use, *imated the length of* the solar year at 365 days, 5 hours, 48 minutes, and

45.5 seconds. Recent observations have made the number of seconds 49.7. Observations made at different times are slightly at variance, and it is possible, and even probable, that the time of the earth's circuit may vary slightly in consequence of a modifying effect produced by planetary attraction. Our estimate regarding the average time is 365 days, 5 hours, 48 minutes and 48 seconds, as formerly stated; and, if we are wrong by two seconds, the error will amount to only a single hour in 1800 years.

TABLE.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
365 days	= 1 common year* (yr.)
366 days	= 1 leap year (yr.)

*A common year is also called a calendar year.

OTHER DENOMINATIONS.

1 week	= 7 days.
1 fortnight	= 14 days.
1 century	= 100 years.

REM. 2. The months vary in length from 28 to 31 days. The following table gives the order, name, and length of each:

1st	month,	January	31 days	7th	month	July	31 days
2d	"	February	28 or 29 "	8th	"	August	31 "
3d	"	March	31 "	9th	"	September	30 "
4th	"	April	30 "	10th	"	October	31 "
5th	"	May	31 "	11th	"	November	30 "
6th	"	June	30 "	12th	"	December	31 "

REM. 3. Before the time of the introduction of the Julian calendar, the year began March 25th, and contained but 10 months. January and February were added during the reign of Julius Cæsar.

A BRIEF HISTORY OF THE CALENDAR.

306. The first calendar worthy of note was the one in use by the early Romans, and it is said to have been introduced by Romulus, the founder of Rome, about 738 years before the birth of Christ. This calendar was modified by Numa Pompilius, the second ruler of Rome, about 713 B. C. Estimates regarding the length of the year proving erroneous, other modifications were afterward made. The return of the seasons furnished the first estimates of the length of the year, and as the time from seed time to seed time, or from harvest to harvest, was subject to much variation, it is evident that the calendars of ancient time were very imperfect, that changes were frequent, and that much confusion prevailed.

The first study of the earth's relations to the sun resulted in a general belief that the sun moved in a spiral path resembling the threads of a screw, around the earth, and that during the year it moved across what is now designated as the torrid zone, and back again to its first position. The Romans, during the fourth century of their history, estimated that the time required for this transit of the sun in both directions was 365 days, and this was made the year of the calendar for about three and one-half centuries, or until the reign of Julius Cæsar. During the early part of Cæsar's administration, Sosigenes, an Alexandrian mathematician and astronomer, considering the imperfect condition of the calendar, which marked the winter by the spring months, made calculations to the effect that the year contained 365 days and 6 hours. Cæsar, having implicit faith in Sosigenes' calculations, reformed the calendar by making every fourth year to contain 366 days; this was the origin of bissextile,* or leap year. It was also necessary in the correction of the calendar, to add 90 days to the year in which the reformation occurred; hence the year 46 B. C., the date of the introduction of the Julian calendar, contained 455 days. This was denominated "the last year of confusion," it being supposed that all defects of the calendar had been corrected. Time was reckoned by the Julian calendar during a period of fifteen centuries, or until 1582, at which time Pope Gregory XIII, governed by the calculations made by astronomers of that time, declared the length of the year to be 365 days, 5 hours, 48 minutes and 45½ seconds, and introduced what has since been known as the Gregorian calendar, which is still in use by all civilized countries except Russia. At the time of the reformation by Pope *Gregory* the error of the Julian calendar had grown to ten

* *The lengthened year* was called bissextile because the 6th of the calends of March was repeated. At a later date the additional day was added to February.

days, hence, October 5th, 1582, was made October 15. Roman Catholic countries at once adopted the reform. Protestant states, one after another, followed the example. When the new style was adopted in England by an act of Parliament 1752, it was necessary to strike out eleven days, the 3d of October being made the 14th. During the period when both calendars were in use, dates corresponding to the Julian calendar were marked O. S. (Old Style) and those corresponding to the Gregorian plan were marked N. S. (New Style).

The adoption of the Gregorian arrangement requires an observance of the following

RULES CONCERNING LEAP YEARS.

307. I. *Years exactly divisible by 4, except the closing years of centuries, called centennial years, are leap years.*

II. *Centennial years divisible by 400 are leap years.*

REMARK. Continuing our reckoning of time by the above rules during a period of 4000 years, an addition of 970 days will be made to the common year. The correct amount to be added is 968 days, 21 hours, 20 minutes; hence the addition is too much by 1 day, 2 hours, 40 minutes; the following modification of Rule 2 has therefore been suggested: "*Every millennial year exactly divisible by 4000 is not a leap year.*"

MEASURE OF ANGLES.

308. The **Measure of Angles,** (the circle and its divisions), is used in surveying, navigation, astronomy, for reckoning latitude and longitude, determining location of places and vessels, and comparing differences of time.

309. An **Angle** is the opening between two lines that meet at a common point.

REMARK 1. The entire space encircling a point is called the *angular space.* The magnitude of an angle depends upon the difference in direction of the lines that form it. Angles are measured by arcs of a circle. The entire angular space *about a point* is 360 degrees, and as the number of

degrees belonging to the circle surrounding the point is the same, this measure is commonly called *circular measure*. The expression, 1 degree, means simply $\frac{1}{360}$ part of a circle, regardless of size, or, more nearly correct, the angle measured by $\frac{1}{360}$ of a circle.

TABLE.

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
30 degrees	= 1 sign (S.)
12 signs or 360°	= 1 circle (C.)

REM. 2. In astronomical calculations, seconds are sometimes subdivided into *thirds*, thirds into *fourths*, etc., the relation in each case being 60, the same as between minutes (firsts) and seconds.

REM. 3. A minute of the earth's circumference is called by sailors a knot or *geographic mile*. It is equal to a small fraction more than a common mile.

SPECIAL OR MISCELLANEOUS MEASURES.

310. Counting.

12 units or things	= 1 doz (doz.)
12 dozen	= 1 gross (grs.)
12 gross	= 1 great gross (g. grs.)
20 units or things	= 1 score (scr.)

311. Paper.

24 sheets	= 1 quire (qr.)
20 quires	= 1 ream (rm.)
2 reams	= 1 bundle (bdl.)
5 bundles	= 1 bale (bl.)

TYPE FOUNDERS' AND PRINTERS' MEASURES.

312. Type founders and printers make use of two standard units in their measurements, the *point* and the *pica*.

A point is $\frac{1}{72}$ of an inch.
A pica is $\frac{1}{6}$ of an inch.

REMARK 1. The size of ordinary type used in book work and in common newspaper work is designated by the point. Each size has also a name. Sizes and corresponding names are shown by the following

TABLE.

4½	point is Diamond	10	point is Long Primer
5	" " Pearl	11	" " Small Pica
5½	" " Agate	12	" " Pica
6	" " Nonpareil	14	" " English
7	" " Minion	16	" " Columbian
8	" " Brevier	18	" " Great Primer
9	" " Bourgeois	20	" " Paragon

r. 2. Larger sizes than those named in the above table are desig-

nated by the pica or nonpareil. Thus, 2 *line pica* or 4 *line nonpareil;* 6 *line pica* or 12 *line nonpareil*, etc.

REM. 3. A piece of type, when set in position for printing, stands $\frac{1}{1}\frac{5}{6}$ of an inch high. The size refers to the body of the metal upon which the face is cast, measured in the direction of the column in which it stands.

313. Composition is the act of setting type. The term also applies to the type when set for printing.

REMARK. Composition is estimated by the 1000 ems, an em being a piece of type (of any size) exactly square. A pica em is $\frac{1}{6}$ of an inch square; a brevier em is 8 points or $\frac{1}{12}$ of an inch square. The term does not apply to the letter M, but to a *quad* which is shorter than faced type, and is used at the beginning of paragraphs, to fill out incomplete lines, and for spacing purposes.

314. The most common width of newspaper columns is 13 pica ems, or $2\frac{1}{6}$ inches. To estimate the distance up or down a column of this width necessary to contain 1000 pica ems, we have only to divide 1000 by 13, which gives $76\frac{12}{13}$ picas or $76\frac{12}{13} \div 6 = 12\frac{3}{3}\frac{2}{9}$ inches. The number of ems of different sizes of type necessary for one line of a column 13 pica ems wide is shown by the following

TABLE OF EMS TO THIRTEEN PICA COLUMN.

Solid. No leads between lines.	Agate	Nonp'l	Minion	Brev.	Bourg.	Lg. Pr.	S. Pica
Number of ems in line	$28\frac{3}{4}$	26	$22\frac{1}{2}$	$19\frac{1}{2}$	$17\frac{1}{2}$	$15\frac{1}{2}$	$14\frac{3}{11}$
Number lines to 1000 ems	$35\frac{1}{3}$	$38\frac{1}{2}$	45	$51\frac{1}{3}$	$57\frac{2}{3}$	$64\frac{1}{2}$	$70\frac{1}{11}$
Number inches to 1000 ems	$2\frac{2}{3}$	$3\frac{1}{4}$	$4\frac{1}{3}$	$5\frac{1}{3}$	$7\frac{1}{4}$	9	$10\frac{2}{3}$

REMARK. Whether solid or leaded, the figures would be the same except in number of lines to 1000 ems. The leads are counted in composition the same as the type.

SIZES OF BOOKS.

315. Book pages are printed on full sized sheets of paper. The sheet is then cut into half sheets, as a general rule, after which the half sheets are folded into book leaves. The standard sizes of paper vary, hence books of the same description vary somewhat in size.

Folio. The half sheet folded in two leaves, or four pages, makes a book called a *folio*.

Quarto. The half sheet folded in four leaves, or eight pages, forms a *quarto*.

Octavo. The half sheet folded in eight leaves, or sixteen pages, forms an *octavo*.

Duodecimo. The half sheet folded in twelve leaves, or twenty-four pages, forms a *duodecimo*.

Sixteen leaves to the half-sheet is a 16*mo*; eighteen leaves is an 18*mo*; twenty-four leaves a 24*mo*, etc.

Marks occasionally found at the bottom of book pages, as a, b, c; or 1, 2, 3, are termed signatures, and are used for the direction of the pressman and the binder in printing, folding, and gathering the sheets.

REMARK. The terms folio, quarto, etc., applied to newspapers, have reference to the number of leaves folded from the *full* sheet.

REDUCTION OF DENOMINATE NUMBERS.

316. Reduction of Denominate Numbers is the process of changing them from one denomination to another without producing a change in value.

Reduction is of two kinds, distinguished as *Reduction Descending* and *Reduction Ascending*.

317. Reduction Descending is the process of reducing a denominate number to a *lower* denomination.

318. Reduction Ascending is the process of reducing a denominate number to a *higher* denomination.

CASE I.

319. Reduction of a compound number to a required *lower* denomination.

1. Reduce 14 bu. 3 pk. 5 qt. to pints.

Analytical Operation.

4 pk. × 14 + 3 pk. = 59 pk.
8 qt. × 59 + 5 qt. = 477 qt.
2 pt. × 477 = 954 pt.

EXPLANATION. We first reduce the highest denomination (14 bu.) to pecks, by multiplying the number of pecks in one bushel by 14. To this product we add the 3 pecks and obtain 59 pecks. This number of pecks we reduce to quarts by multiplying the number of quarts in one peck by it, and to the result we add the 5 quarts, making 477 quarts. Then multiplying the number of pints in one quart by 477, gives 954, the number of pints to which 14 bu. 3 pk. 5 qt. are equal.

Practical operation.

14 bu. 3 pk. 5 qt.
4
59
8
477
2
954

EXPLANATION. This operation, although not quite so logical as the former, is the one in general use and will admit of analysis. Knowing from the unit of relation between the bushel and the peck that there are 4 times as many pecks as bushels, we first multiply by 4 and to the result we add the 3 pecks mentally and obtain 59 pecks. There being in this quantity 8 times as many quarts as pecks, we multiply by 8 and add the 5 quarts, obtaining 477 quarts. We then multiply by 2, since there are 2 times as many pints as quarts, and obtain 954 pints, the same as found by the first method.

RULE I. *Multiply the highest denomination by the number expressing the unit of relation between it and the next lower denomination, and to the product add the given number, if any, of such lower denomination.*

II. *Reduce the result thus obtained, in a similar manner, to the next lower denomination, and so continue until the given compound number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

MEASURES OF VALUE.

Reduce

2. $7 and 6 dimes to mills.

3. $12 and 13c to cents.

4. £21 12s. 8d. to pence.

5. £124 10d. to farthings.

Reduce

6. 21 marks 8 pennies to pennies.
7. $35 72c to mills.
8. £73 18s. 9d. to farthings.
9. 54 fr. 8 dc. 5 ct. to millimes.
10. 15 guineas to pence.

Find value in U. S. money of

11. 735 fr. (France.)
12. £132 15s. (England.)

MEASURES OF WEIGHT.

Reduce

13. 17 lb. 10 oz. 14 pwt. to grains.
14. 5 tons 7 cwt. 35 lb. to ounces.
15. 132 tons 15 lb. to pounds.
16. 8 ℔ 11℥ 5℈ 1℈ to grains.
17. 18 lb. 13 pwt. to grains.
18. ℥vj ℥iij ℈ij to scruples.
19. 145 long tons 572 lb. to pounds.
20. 90 bbl. flour to pounds.
21. 375 bu. oats to pounds.
22. 125 bu. wheat to pounds.

MEASURES OF EXTENSION.

Reduce

23. 4 mi. 215 rd. 2 ft. to inches.
24. 18 A. 64 sq. rd. 135 sq. yd. to sq. ft.
25. 5 mi. 54 ch. 72 l. to inches.
26. 85 A. 4 sq. ch. 12 sq. rd. to sq. rd.
27. 12 mi. 4 yd. 7 in. to inches.
28. 21 sq. mi. 135 A. to sq. ch.
29. 16½ hands to inches.
30. 12 fath. 5 ft. 11 in. to inches.
31. 32 cu. yd. 19 cu. ft. to cu. in.
32. 27 cd. 112 cu. ft. to cu. ft.
33. 72 ch. 17 l. to inches.
34. 15 A. 84 sq. rd. to sq. yd.

MEASURES OF CAPACITY.

duce

35. 17 bu. 3 pk. to quarts.
36. 25 gal. 3 qt. 1 pt. to gills.
37. fʒij fʒvj to mɻ.
38. 175 bu. 7 qt. to pints.
39. 35 bbl. 24 gal. to pints.
40. 240 gal. 1 pt. to gills.
41. 7 pk. 3 qt. to pints.
42. 2400 lb. wheat to quarts.

MEASURES OF TIME.

duce

43. 5 yr. 147 da. 10 hr. to seconds.
44. 1 leap yr. 58 da. 4 min. to min.
45. The 19th century to hours.
46. 275 da. 36 min. to seconds.
47. The 1st 3 mo. of 1904 to sec.
48. The mo. of Nov. to min.
49. The mo. of Feb., 1900, to sec.
50. 17 weeks 3 da. to min.

OTHER MEASURES.

duce

51. 32° 42′ 20″ to seconds.
52. 7 S. 25° to minutes.
53. 3 bales of paper to quires.
54. 15 g. gr. 5 gr. 7 doz. to doz.
55. 3 score and 10 yr. to hours.*
56. 12 reams 5 quires to sheets.
57. 1 circle to seconds.
58. 5° 20′ to geographic miles.
59. How many points wide is a column that will contain 18 brevier ems?

e-fourth the *number of years* are leap years.

CASE II.

320. Reduction of denominate numbers to higher denominations.

1. Reduce 4365 pennyweights to higher denominations.

Operation.

20)4365
12)218+5 pwt.
18+2 oz.
Ans. 18 lb. 2 oz. 5 pwt.

EXPLANATION. The analytical statement calling for the first step of the operation is as follows: Since there is one ounce to every 20 pennyweights, there are as many ounces in 4365 pennyweights as 20 pennyweights are contained times in 4365 pwt. This number is contained 218 times with a remainder of 5 pwt.; therefore the number of ounces is 218 (a concrete number of the same unit value as the abstract quotient. The expression in the statement "*as many as*" shows that the quotient is not the number of ounces). The above explanation will render clear the analysis of the division by 12; the pupil should be required to give it.

RULE I. *Divide the given number by the number expressing the unit relation between it and the next higher denomination, and reserve the remainder, if any, as a part of the required answer.*

II. *Proceed in like manner with the quotient, and with each quotient succeeding, until the required denomination is reached.*

III. *The last quotient with the several remainders annexed will express the answer.*

NOTE. A denominate number may be reduced to a required higher denomination in one division, by dividing it by the number expressing the unit relation between it and the required denomination. Had the above illustrative example required a reduction of 4365 pwt. to *pounds*, instead of to *higher denominations*, we would suggest the following operation: $4365 \div 240 = 18\frac{3}{16}$ lbs.

EXAMPLES FOR PRACTICE.

MEASURES OF VALUE.

Reduce

2. 18356 farthings to £'s.
3. 3872 mills to dollars.
4. 17624 pence to £'s.
5. 5837 pennies to marks.
6. *21312* centimes to fr.
7. $87.75 to francs.
8. $135.72 to marks.
9. $128.50 to £'s.
10. $60.467 to English values.
11. $293.36 to French values.

MEASURES OF WEIGHT.

Reduce

2. 12373 gr. to pounds Troy.
3. 132176 oz. Avoir. to tons.
4. 8749 ℈ to pounds Apoth.
5. 32876 oz. Avoir. to cwt.
6. 19743 gr. to higher denom., Troy.
17. 17215 ℈ to higher denom.
18. 840 pwt. to lb. Apoth.
19. 72 lb. Troy to lb. Avoir.
20. 32465 gr. to higher denom. Avoir.
21. 6584 lb. wheat to bu.

MEASURES OF EXTENSION.

Reduce to higher denom.:

12. 98745 inches.
13. 174831 square feet.
14. 38325 square yards.
15. 213145 cubic inches.
16. 21485 links.

Reduce

27. 171072 cu. in. to cu. yd.
28. 26977.5 ft. to miles.
29. 331776 cu. in. to cords.
30. 300000 sq. l. to acres.
31. 74 cd. ft. to cords.

MEASURES OF CAPACITY.

Reduce to higher denom.:

22. 2648 qt. (dry measure).
23. 3861 gills.
24. 18240 pt. (liq. measure).

Reduce

35. 17824 pt. to bushels.
36. 32516 f ʒ to gallons.
37. 924 pt. to barrels.

OTHER MEASURES.

Reduce to higher denom.:

38. 146385″.
39. 3658241 seconds.
40. 6553 sheets of paper.
41. 6437 pence.
42. 14738 hours.

Reduce

43. 138 years to scores.
44. 19428′ to degrees.
45. 3874 wks. to common yrs.
46. 3724 geographic miles to degrees.
47. 22736 oz. flour to brls.

CASE III.

321. Reduction of a denominate fraction to a specified lower denomination.

1. Reduce $\frac{1}{36}$ of a gallon to a fraction of a gill.

Operation.

$$\frac{1}{\cancel{36}_{9}} \times \frac{\cancel{4}}{1} \times \frac{2}{1} \times \frac{4}{1} = \frac{8}{9} \text{ gi.}$$

EXPLANATION. The principles of reduction explained in Case I will apply with equal force to fractional numbers; hence we make the units of relation the several multipliers as in that case. The first multiplier, 4, reduces to a fraction of a quart; the second, 2, reduces to a fraction of a pint; the third, 4, reduces to the specified denomination, and furnishes for a result $\frac{8}{9}$ of a gill.

EXAMPLES FOR PRACTICE.

What fraction of

2. A pint is $\frac{1}{320}$ of a bu.?

3. A grain is $\frac{3}{28800}$ of a lb.?

4. An hour is $\frac{5}{1176}$ of a wk.?

5. A cu. ft. is $\frac{1}{288}$ of a cord?

What decimal of

6. An oz. is .000015 of a T.?

7. A penny is £.0032?

8. A quart is .0175 of a bu.?

9. A second is .00023 of an hr.?

CASE IV.

322. Reduction of a denominate fraction to a specified higher denomination.

1. Reduce $\frac{3}{4}$ of an inch to the fraction of a rod.

Operation.

$$\frac{\cancel{3}}{\cancel{4}_{2}} \times \frac{1}{\cancel{12}_{4}} \times \frac{\cancel{2}}{33} = \frac{1}{264}$$

EXPLANATION. The principles of reduction are the same as explained in Case II. Dividing by 12, or multiplying by its reciprocal, reduces to feet, and again dividing by $16\frac{1}{2}$, the number of feet in a rod, or multiplying by its reciprocal, $\frac{2}{33}$, reduces to the fraction of a rod as required.

EXAMPLES FOR PRACTICE.

What fraction of

2. A day is $\frac{5}{8}$ of a min.?

3. A pound Troy is $\frac{8}{9}$ of a pwt.?

4. A gallon is $\frac{4}{6}$ of a pt.?

What decimal of

5. A mile is .8976 of a foot?

6. A cu. yd. is .1073088 of a cu. in.?

7. An acre is .39204 of a sq. ft.?

CASE V.

323. Reduction of denominate common fractions to integers of lower denominations.

1. Reduce $\frac{7}{16}$ of a bushel to integers of lower denominations.

Operation.

7
4
6)28(1 pk.
16
12
8
6)96(6 qt.
96

EXPLANATION. The explanation in Case I is equally applicable to the reduction of fractions. Hence to reduce $\frac{7}{16}$ of a bushel to the next lower denomination requires a multiplication by 4. This is accomplished, as explained in fractional multiplication, by multiplying the numerator and dividing the result by the denominator. The product is 1 pk. and $\frac{12}{16}$. The fractional part of a peck is reduced to quarts by multiplying by 8; thus, 8 times the numerator divided by the denominator, gives 6 quarts. Hence the complete result is 1 pk. 6 qt.

EXAMPLES FOR PRACTICE.

Reduce to integers of lower denominations

2. $\frac{3}{5}$ of a lb. Troy.
3. $\frac{7}{9}$ of a day.
4. $\frac{7}{22}$ of an A.
5. $\frac{4}{5}$ of a lb Apoth.
6. $\frac{11}{16}$ of a T.
7. $\frac{7}{8}$ of a £.
8. $\frac{5}{9}$ of a year.
9. $\frac{7}{12}$ of a cord.
10. $\frac{1}{7}$ of a mile.

CASE VI.

324. Reduction of denominate decimals to integers of lower denominations.

1. Reduce £.6375 to integers of lower denominations.

Operation.

.6375
20
s. 12.75
12
150
75
d. 9.00

EXPLANATION. Applying the principles of the former case, which are universal in all cases of reduction descending, we first reduce to the next lower denomination, shillings, by multiplying by the number of shillings in a £, and obtain 12.75s. We retain the integral part of the product as the first term of the required answer, and multiply the decimal part by 12, the number of pence in a shilling, and obtain 9d.; hence our complete answer *is 12s. 9d.*

EXAMPLES FOR PRACTICE.

Reduce to integers of lower denominations:

2. .85 of a yr.
3. .9375 of a gal.
4. .4375 of a lb. Troy.
5. .3183125 of a ton.
6. .5275 of a cu. yd.
7. .875 of a bu.
8. .4375 of a ℔ Apoth.
9. .8469 of a degree.
10. .6625 of a mile.

CASE VII.

325. Reduction of a compound number to a common fraction of a specified higher denomination.

1. What part of a day is 13 hours 20 minutes?

Operation.

13 hr. 20 min.=800 min.
1 day=1440 minutes.
800÷1440=$\frac{5}{9}$ of a day.

EXPLANATION. We first reduce the compound number to its lowest denomination; the result is 800 minutes. We next divide the number of minutes by the number of minutes in a day to ascertain the number of days.

EXAMPLES FOR PRACTICE.

What fraction of

2. a pound are 5 oz. 5 pwt.?
3. a yard are 2 ft. 8 in.?
4. a mile are 123 rd. 1 ft. $3\frac{3}{15}$ in.?
5. a gallon are 1 qt. 1 pt. $1\frac{1}{2}$ gills?
6. a degree are 12′ 36″?
7. a ton are 777 lb. $12\frac{4}{5}$ oz.?

8. What is the value of 4 bundles, 6 quires, 16 sheets of paper at $6.60 a bale?

CASE VIII.

326. Reduction of a compound number to a decimal of *a specified higher* denomination.

1. Reduce 9 ʒ 5 ʒ 2 ℈ to the decimal of a pound.

Operation.

3)2.0
8)5.667—
12)9.708+
.809 ℔

EXPLANATION. We begin with the lowest denomination and divide by the number that will reduce it to the next higher and obtain .667—ʒ; to this we prefix (add) the 5 ʒ given in the example. We then reduce to ʒ's by dividing by 8. We then add the 9 ʒ and reduce the result to ℔ according to the former explanations.

EXAMPLES FOR PRACTICE.

What decimal of

2. a mile are 28 rd. 2 yd. 1 ft. 11.04 in.?

3. a pound are 3 ℈ 3 ʒ?

4. a fathom are 3 ft. 9 in.?

5. a sign are 3° 27′ 46.44″?

6. 1½ bu. is $\frac{9}{10}$ of a peck?

7. 3 A. 20 rd. are 110 rd.?

8. a gal. are 3 qt. 1 pt. 3 gi.?

9. 2 bu. are 3 pk. 5 qt.?

10. A common year is what decimal part of a solar year?

11. A Troy pound is what decimal part of an avoirdupois pound?

ADDITION OF COMPOUND NUMBERS.

327. *1.* What is the sum of 13 bu. 1 pk. 5 qt., 24 bu. 3 pk. 7 qt., 19 bu. 2 pk. 4 qt., and 23 bu. 2 pk. 6 qt.?

Operation.

bu.	pk.	qt.
13	1	5
24	3	7
19	2	4
23	2	6
81	2	6

EXPLANATION. We write the addends in such a manner as to have units of the same denomination in the same column. Beginning with the lowest denomination we ascertain the sum of the column of quarts, which is 22 quarts, or 2 pecks 6 quarts. We write the 6 quarts under its respective column and carry the 2 pecks to the column of pecks, obtaining 10 pecks, or 2 bushels 2 pecks. We write the number of pecks under its appropriate column and carry the units of the next higher denomination as before.

REMARK. The computations of compound numbers are governed by

the same principles as apply to simple numbers. In simple numbers we carry 1 for each 10, the scale being unvarying. In compound numbers the scale is usually varying, but in either simple or compound numbers we carry 1 for the number expressing the relation. In view of the similarity we deem special rules unnecessary.

EXAMPLES FOR PRACTICE.

Find the sum of

(*2.*)

gal.	qt.	pt.	gi.
36	3	1	2
15	1	1	3
24	2	1	1
18	3	1	3

(*3.*)

mi.	rd.	yd.	ft.	in.
27	214	5	2	10
34	86	3	1	8
17	113	4	2	7
53	298	2	1	11

(*4.*)

A.	sq.rd.	sq.yd.	sq.ft.
120	82	25	7
136	57	19	5
128	105	27	8
324	92	23	6

5. Add £220 15s. 9d., £143 16s. 8d. 3 far., £87 5s. 10d., and £185 18s. 6d. 1 far.

6. 24 A. 8 sq. ch. 12 sq. rd. of a man's farm is timber land; 52 A. 6 sq. ch. 10 sq. rd. is in meadow; 37 A. 12 sq. ch. 8 sq. rd. is plow land; and the remainder, 16 A. 155 sq. rd., is pasture. How much land comprises his farm?

7. What is the sum of $\frac{5}{8}$ of a day, $5\frac{5}{6}$ hours, $\frac{7}{12}$ of a minute, and 8 da. 13 hr. 18 min.?

8. Add 12.725 bu. 3.875 pk. and 5.625 qt.

SUBTRACTION OF COMPOUND NUMBERS.

328. *1.* Subtract 7 lb. 10 oz. 12 pwt. from 12 lb. 8 oz. 10 pwt.

Operation.

lb.	oz.	pwt.
12	8	10
7	10	12
4	9	18

EXPLANATION. We follow the instructions given in subtraction of simple numbers by writing the subtrahend under the minuend with units of like denominations in the same column. We begin with the lowest denomination, and not being able to subtract 12 pwt. from 10 pwt., we take 1 oz. (20 pwt.) from the 8 oz., which, combined with the 10 pwt. make 30 pwt., and 12 pwt. subtracted from 30 pwt. leave 18 pwt., which we write below. Not being able to take 10 oz. from 7 oz., we take 1 from the next higher denom-

ination as before; thus 7 oz. + 12 oz. — 10 oz. = 9 oz., and 7 lb. from 11 lb. = 4 lb.

EXAMPLES FOR PRACTICE.

2. From 14 mi. 85 rd. 3 yd. 1 ft. subtract 11 mi. 238 rd. 4 yd. 2 ft.

3. From 58 bu. 1 pk. 2 qt. 1 pt. take 35 bu. 3 pk. 1 qt. 1 pt.

4. From 8 rd. 5 yd. 2 ft. 11 in. take 9 rd. 1 ft. 4 in.

5. Subtract 17 A. 113 sq. rd. 4 sq. yd. 5 sq. ft. from 32 A. 96 sq. rd. 3 sq. yd. 8 sq. ft.

6. From a 20-acre field a man sold a piece of land containing 4 acres 7 sq. ch. 13 sq. rd. What was the remainder worth at $40 an acre?

7. London is 51° 32′ N. latitude, and Boston is 42° 23′; what is the difference of latitude between the two cities?

8. A ship loaded with 824 T. 1150 lb. of coal encountered a tempest, making it necessary to throw a portion overboard; on arriving in port she had on board 328 T. 1780 lb. How much was lost?

9. From .1375 of a mile subtract $\frac{7}{8}$ of a rod.

10. Subtract $\frac{7}{8}$ of a sq. yd. from .125 of a sq. rd.

329. To find the difference in time between two dates.

NOTE. The ordinary method employed in finding the difference in time between dates is to subtract the first date from the last; but this plan usually furnishes only an approximate result, owing to the fact that 30 days are reckoned for each month. The results obtained by this method are sufficiently accurate for most purposes.

1. Find the interval of time, by compound subtraction, between June 13, 1898, and March 25, 1904.

REMARK. The mathematical expression for a date requires that the *numbers* of the months be written instead of their *names*. The dates in the above example should be written as shown below:

1904	yr.	3	mo.	25 da.
1898	"	6	"	13 "
5	*yr.*	*9*	*mo.*	*12 da.*

No explanation is considered necessary, as the principles of compound subtraction have already been explained.

EXAMPLES FOR PRACTICE.

By compound subtraction find the interval between

2. Nov. 17, 1901, and May 5, 1903.
3. Aug. 29, 1899, and Feb. 12, 1901.
4. Dec. 31, 1901, and June 5, 1904.
5. May 31, 1900, and Sept. 1, 1902.
6. July 19, 1742, and Apr. 3, 1899.
7. Jan. 5, 1812, and Nov. 1, 1900.
8. Feb. 29, 1896, and Jan. 18, 1904.
9. July 4, 1776, and Apr. 19, 1861.

330. To find the exact time in days between two dates.

1. Find the exact number of days between Dec. 23, 1903, and March 16, 1904.

Operation.

8 Dec.
31 Jan.
29 Feb.
16 Mar.
84

EXPLANATION. The interval includes the remaining 8 days of December, (31—23), all of January (31 da.), all of February (29 da.) and the first 16 days of March. The sum of these respective numbers will evidently be the required number of days.

NOTE. In finding the interval between two dates, the first day is not included, but the last day is counted.

EXAMPLES FOR PRACTICE.

Find the exact number of days between

2. June 5, 1902, and Sept. 19, 1902.
3. Oct. 17, 1903, and May 30, 1904.
4. Feb. 20, 1901, and Aug. 31, 1901.
5. April 4, 1900, and Sept. 12, 1900.
6. Nov. 30, 1904, and April 3, 1905.
7. July 4, 1901, and Jan. 11, 1902.
8. March 29, 1902, and Aug. 8, 1902.
9. Dec. 1, 1903, and May 24, 1904.
10. Sept. 3, 1903, and Jan. 18, 1904.
11. Oct. 13, 1904, and Feb. 5, 1905.

12. A note, dated Jan. 20, 1904, was due, by its terms, ıpr. 19, 1905. How many days did it run?

13. A note due Aug. 8, 1905, was discounted at a ›ank June 3, 1905. For how many days should the bank harge interest?

14. How many minutes from 20 min. past 3 o'clock p. n. Aug. 19, 1890, to 15 min. past 9 o'clock a. m. May 3, .903, due allowance being made for leap years?

MULTIPLICATION OF COMPOUND NUMBERS.

331. Multiply 12 gal. 3 qt. 1 pt. by 7.

Operation.

gal.	qt.	pt.
12	3	1
		7
90	0	1

EXPLANATION. We place the multiplier, for convenience, under the lowest denomination of the multiplicand, and proceed with the work according to the principles of multiplication of simple numbers. Thus, 1 pt.×7=7 pt.=3 qt. 1 pt. We write the 1 pt. in its order and carry the 3 qt. to its appropriate column. 3 qt.×7=21 qt., to which we add the 3 qt. rom the former multiplication, making 24 qt., or 6 gal. 0 qt. Multiplying he gallons (12 gal.×7) and carrying the six gallons, we have the complete ›roduct, 90 gal. 0 qt. 1 pt.

EXAMPLES FOR PRACTICE.

Multiply

2. 15 mi. 94 rd. 4 yd. 1 ft. by 5.

3. 82 A. 112 sq. rd. 5 sq. ch. by 8.

4. 320 cu. yd. 22 cu. ft. 830 cu. in. by 11.

5. £140 16s. 8d. 3 far. by 25.

6. 7 ℔ 8 ℥ 5 ʒ 1 ℈ by 42.

7. 150 bu. 5 qt. by 12.

8. 165 T. 1345 lb. by 18.

9. 120° 42′ 18″ by 15.

10. What is the weight of 8 barrels of sugar, if each weighs 287 lb. 13 oz.?

11. A dealer bought a piece of English goods containing 38 yds. invoiced at £1 4s. 6d. What was the cost?

12. A farmer purchased 5 wood lots at $60 an acre. If each lot contained 2 A. 7 sq. ch. 4 sq. rd., how much should he pay for the purchase?

13. How many cords of wood can be cut from the above specified tract, estimating that the average yield per square rod is 2 cd. 16 cu. ft.?

14. If the average speed of a vessel is 23 mi. 95 rd. 12 ft. per hour, how far will she sail in a week of 7 days?

15. Multiply $\frac{7}{16}$ da. by 12.

16. Find the product of $\frac{3}{20}$ pwt. by 50.

17. Find the product of .01845 pk. by 250.

DIVISION OF COMPOUND NUMBERS.

CASE I.

332. Division of compound numbers by abstract numbers.

1. Divide 6 lb. 5 oz. 15 pwt. by 4.

Operation.

	lb.	oz.	pwt.	
4)	6	5	15	
	1	7	8	18 gr.

EXPLANATION. The principles are the same as those employed in the division of simple numbers. We begin with the highest denomination, and reduce each remainder to, and combine the result with the units of the next lower denomination. Thus, $\frac{1}{4}$ of 6 lb.=1 lb., and a remainder of 2 lb., which are equal to 24 oz. 24 oz.+5 oz.=29 oz. $\frac{1}{4}$ of 29 oz.=7 oz. and a remainder of 1 oz. or 20 pwt. 20 pwt.+15 pwt.=35 pwt., $\frac{1}{4}$ of which is 8 pwt., with a remainder of 3 pwt. 3 pwt.=72 gr., $\frac{1}{4}$ of which is 18 gr. Hence the quotient is 1 lb. 7 oz. 8 pwt. 18 gr.

EXAMPLES FOR PRACTICE.

Divide

2. 14 bu. 3 pk. 5 qt. by 3.

3. 364 mi. 216 rd. 5 yd. by 8.

4. 85 lb. 8 oz. Avoir. by 7.

5. £25 17s. 9d. by 6.

Divide

6. 215 acres 113 sq. rd. 19 sq. yd. 4 sq. ft. by 5.

7. 74° 36′ 24″ by 9.

8. 31 ℔ 6 ℥ 5 ʒ 1 ℈ by 10.

9. 17 tons 7 cwt. 15 lb. by 15=(5×3).

10. 754 gal. 3 qt. 1 pt. by 37.

Operation.

gal.	qt.	pt.	
37)754	3	1	(20 gal.
74			
14			
4			
37) 59			(1 qt.
37			
22			
2			
37) 45			($1\frac{8}{37}$ pt.
37			
8			

REMARK 1. When the divisor exceeds 12, and is not composite, the work should be performed by long division, as shown by the accompanying operation.

REM. 2. The solution is the same in principle as that in example 1, which contains a full explanation. The reductions are here written out, instead of being performed mentally, as in the former case. We are confident that a clear understanding of the former explanation will render this solution clear.

11. A tract of land containing 5 A. 40 sq. rd. was surveyed into 26 town lots of equal size. What amount of land did each lot contain?

12. If 24 watch cases were made from 6 lb. 7 oz. 13 pwt. of silver, what was the weight of each case?

13. In walking a distance of 120 rd. 3 yd. 2 ft. 4 in., a pedestrian took 824 steps. What was the average length of each step?

14. A syndicate bought 5270 acres of western land and divided it into 37 farms of equal size. What was the area of each farm?

15. Divide $3\frac{1}{4}$ sq. rd. by 4.

16. Divide 15.0645 gal. by .12.

CASE II.

333. Division of one compound number by another.

1. Divide 36 lb. 8 oz. 12 pwt. by 9 oz. 6 pwt.

Operation.

36 lb. 8 oz. 12 pwt. = 8812 pwt.
9 oz. 6 pwt. = 186 pwt.
8812 pwt. ÷ 186 pwt. = $47\frac{10}{93}$.

EXPLANATION. We first reduce both divisor and dividend to simple denominate numbers of the lowest denomination expressed by either term, which in this case is pennyweights. We next divide the dividend, 8812 pwt., by the divisor, 186 pwt., and obtain the quotient $47\frac{10}{93}$.

EXAMPLES FOR PRACTICE.

2. Divide 23 lb. 11 oz. 15 pwt. by 1 lb. 7 oz. 13 pwt.

3. Divide 91 bu. 1 pk. 1 qt. by 3 bu. 3 pk. 7 qt.

4. What is the quotient of 24 mi. 124 rd. 0 yd. 1 ft. divided by 243 rd. 4 yd. 2 ft. 6 in.?

5. Divide 3 A. 130 sq. rd. 5 sq. yd. by 2 sq. yd. 4 sq. ft. 72 sq. in.

6. Divide 1 circle by 6° 6′ 40″.

7. How many yards of cloth at 8s. 4d. per yard can be bought with £13 15s.?

8. How many carats fine are rings manufactured from a compound of 1 oz. 5 pwt. 12 gr. of pure gold and 12 pwt. 18 gr. alloy?

9. How many revolutions will a carriage wheel 13 ft. 4 in. in circumference make while the carriage is moving 5 miles?

10. How many prescriptions can be compounded from a certain kind of medicine weighing $3\frac{1}{2}$ lb. if ℈ ij gr. v are required for each?

REVIEW OF DENOMINATE NUMBERS.

334. *1.* From an invoice of sugar weighing $\frac{7}{8}$ of a T., *1256 lb. 12 oz.* were sold. What was the value of the *remainder at 6¢ a pound*?

2. A grocer bought 3 bu. 3 pk. of blueberries at 8¢ a quart. What did they cost?

3. What should be paid for 35 lb. of flour at $6.40 a barrel?

4. A retail fruit dealer sold 8 bu. 3 pk. 4 qt. of pears daily for 12 days; during the next two weeks his sales were only $\frac{3}{4}$ of that quantity. If the price charged was 60¢ a peck, how much did he receive for the four weeks' sales?

5. What decimal of a rod is $\frac{5}{16}$ of an inch?

6. How many rings weighing 8 pwt. 6 gr. each, and 18 carats fine, can be manufactured from a compound consisting of 9 oz. 18 pwt. of pure gold and the required amount of alloy?

7. A man purchased a tract of land consisting of 529 A. 51 sq. rd. 12 sq. yd., and divided it equally among his 4 sons. What was the share of each?

8. A farmer bought a bill of groceries amounting to $52.44 and gave in payment 4 T. 1280 lb. of hay at $6 a ton, and 5 cd. 1 cd. ft. of wood. What was the price of the wood per cord?

9. If a horse consume 5 quarts of oats at a feed, and is given 3 feeds a day, how many days will 138 bu. 3 pk. sustain 8 horses, provided they are fed at the same rate?

10. A grocer paid $2.50 for a bushel of blueberries and sold from the purchase as many full quarts, liquid measure, as the quantity contained, at 10 cents a quart, and gave the remainder to a child. How much did he gain?

11. How many feet of barbed wire will be required to fence a field 15 ch. 60 l. long and 12 ch. 80 l. wide, if the wire encircle the field 4 times?

12. If 1 bu. 1 pk. 3 qt. are required to sow an acre of

ground, how much seed will be required to sow a field containing 32 A. 120 sq. rd.?

13. A coal dealer buys coal at the mines for $4.20 per long ton, pays 15 cents per cwt. for transportation, and retails it at $8 per short ton. How many long tons must be involved in the deal to make his profit $357?

14. If the average yield of potatoes per acre is 124 bu. 1 pk. 5 qt., what quantity can be grown from a field containing 40 acres.?

15. How many gallons of medicine will be required to fill 42 gross of vials holding $f\,\overline{3}\,ij$ each?

16. If 1 pt. 2 gi. of water be mixed with 2 qt. of milk, what decimal part of the mixture will be milk?

17. For what number of days will the maker of a note be required to pay interest if the note be given March 17 and paid the 10th day of the following October?

18. A milkman adulterated the milk sold to his customers by adding 1 pint of water to each gallon. If his average sales were 24 gallons a day, and he received 6 cents a quart, how much more did he receive than was honestly due him during a period of 12 days?

19. Reduce $\frac{1}{8}$ of $\frac{2}{11}$ of £3 to the fraction of a penny.

20. Counting at the rate of 180 in 1 min. 48 sec., and continuing the work daily, 10 hours each day, how long a time will be required to count a trillion, if each year be counted as 365 days?

21. If a horse travel a mile in 6 min. 8 sec., in what time can he travel 88 miles?

22. A jeweler contracted with a lady customer for a jar of butter, agreeing to pay 24 cents a pound. The net weight, by the jeweler's scales (Troy), was 10 lb. 6 pwt. 6 gr. How much should the lady receive?

23. What is the value of 3270 pounds of wheat at 85½ cents a bushel?

24. What is the cost of 115 lb. 12 oz. of beef at $18.60 a barrel?

25. What decimal part of a mile is 40.04 fathoms?

26. How many times is $\frac{5}{8}$ of a yard contained in $\frac{3}{4}$ of a mile?

27. If gold is worth $20 an ounce, how much greater is the value of the material of an 18 carat gold ring weighing 5 pwt., than that of a 14 carat gold ring of the same weight, the alloy in each being regarded as valueless?

28. A clock, which registers correct time at noon April 5th, loses time at the rate of 1 min. 5 sec. in 18 hours. What will be the correct time May 12th, when the clock registers 1 hr. 30 min. p. m.?

29. Find the difference, by compound subtraction, between the following dates: June 16, 1893, and March 29, 1904.

30. What decimal part of a bushel is 4 quarts, liquid measure?

31. A and B are joint owners of a farm; A's interest is $\frac{7}{12}$ and B's is $\frac{5}{12}$, and the difference between their shares is 14 A. 112 sq. rd. If B sells his interest to C at $50 an acre, how much should he receive?

32. At 45 cents a pound, how much should be paid for 13 chests of tea, each weighing 76 lb. 12 oz.?

33. From $\frac{5}{7}$ of 3 T. 10 cwt. subtract $\frac{4}{13}$ of 7 T. 3 cwt. 26 lb.

34. What decimal of a rod are 3 ft. 3.6 in.?

35. How many yards are equal in distance to 27 ch. 25 l.?

36. What number of hours will the 20th century contain?

37. The forward wheels of a wagon are 10 ft. 8 in. in circumference, and the hind wheels 14 ft. 6 in. How many

the same principles as apply to simple numbers. In simple numbers we carry 1 for each 10, the scale being unvarying. In compound numbers the scale is usually varying, but in either simple or compound numbers we carry 1 for the number expressing the relation. In view of the similarity we deem special rules unnecessary.

EXAMPLES FOR PRACTICE.

Find the sum of

(*2.*)

gal.	qt.	pt.	gi.
36	3	1	2
15	1	1	3
24	2	1	1
18	3	1	3

(*3.*)

mi.	rd.	yd.	ft.	in.
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34	86	3	1	8
17	113	4	2	7
53	298	2	1	11

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A.	sq.rd.	sq.yd.	sq.ft.
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5. Add £220 15s. 9d., £143 16s. 8d. 3 far., £87 5s. 10d., and £185 18s. 6d. 1 far.

6. 24 A. 8 sq. ch. 12 sq. rd. of a man's farm is timber land; 52 A. 6 sq. ch. 10 sq. rd. is in meadow; 37 A. 12 sq. ch. 8 sq. rd. is plow land; and the remainder, 16 A. 155 sq. rd., is pasture. How much land comprises his farm?

7. What is the sum of $\frac{5}{8}$ of a day, $5\frac{5}{6}$ hours, $\frac{7}{12}$ of a minute, and 8 da. 13 hr. 18 min.?

8. Add 12.725 bu. 3.875 pk. and 5.625 qt.

SUBTRACTION OF COMPOUND NUMBERS.

328. *1.* Subtract 7 lb. 10 oz. 12 pwt. from 12 lb. 8 oz. 10 pwt.

Operation.

lb.	oz.	pwt.
12	8	10
7	10	12
4	9	18

EXPLANATION. We follow the instructions given in subtraction of simple numbers by writing the subtrahend under the minuend with units of like denominations in the same column. We begin with the lowest denomination, and not being able to subtract 12 pwt. from 10 pwt., we take 1 oz. (20 pwt.) from the 8 oz., which, combined with the 10 pwt. make 30 pwt., and 12 pwt. subtracted from 30 pwt. leave 18 pwt., which we write below. Not being able to take 10 oz. from 7 oz., we take 1 from the next higher denom-

ation as before; thus 7 oz. + 12 oz. — 10 oz. = 9 oz., and 7 lb. from 11). = 4 lb.

EXAMPLES FOR PRACTICE.

2. From 14 mi. 85 rd. 3 yd. 1 ft. subtract 11 mi. 238 d. 4 yd. 2 ft.

3. From 58 bu. 1 pk. 2 qt. 1 pt. take 35 bu. 3 pk. 1 qt. pt.

4. From 8 rd. 5 yd. 2 ft. 11 in. take 9 rd. 1 ft. 4 in.

5. Subtract 17 A. 113 sq. rd. 4 sq. yd. 5 sq. ft. from 2 A. 96 sq. rd. 3 sq. yd. 8 sq. ft.

6. From a 20-acre field a man sold a piece of land containing 4 acres 7 sq. ch. 13 sq. rd. What was the remainder worth at $40 an acre?

7. London is 51° 32′ N. latitude, and Boston is 42° 23′; what is the difference of latitude between the two cities?

8. A ship loaded with 824 T. 1150 lb. of coal encountered a tempest, making it necessary to throw a portion overboard; on arriving in port she had on board 328 T. 780 lb. How much was lost?

9. From .1375 of a mile subtract $\frac{7}{8}$ of a rod.

10. Subtract $\frac{7}{8}$ of a sq. yd. from .125 of a sq. rd.

329. To find the difference in time between two dates.

NOTE. The ordinary method employed in finding the difference in time between dates is to subtract the first date from the last; but this plan usually furnishes only an approximate result, owing to the fact that 30 days are reckoned for each month. The results obtained by this method are sufficiently accurate for most purposes.

1. Find the interval of time, by compound subtraction, between June 13, 1898, and March 25, 1904.

REMARK. The mathematical expression for a date requires that the *numbers* of the months be written instead of their *names*. The dates in the above example should be written as shown below:

1904 yr. 3 mo. 25 da.
1898 " 6 " 13 "
5 yr. 9 mo. 12 da.

No explanation is considered necessary, as the principles of compound subtraction have already been explained.

EXAMPLES FOR PRACTICE.

By compound subtraction find the interval between

2. Nov. 17, 1901, and May 5, 1903.
3. Aug. 29, 1899, and Feb. 12, 1901.
4. Dec. 31, 1901, and June 5, 1904.
5. May 31, 1900, and Sept. 1, 1902.
6. July 19, 1742, and Apr. 3, 1899.
7. Jan. 5, 1812, and Nov. 1, 1900.
8. Feb. 29, 1896, and Jan. 18, 1904.
9. July 4, 1776, and Apr. 19, 1861.

330. To find the exact time in days between two dates.

1. Find the exact number of days between Dec. 23, 1903, and March 16, 1904.

Operation.

8 Dec.
31 Jan.
29 Feb.
16 Mar.
84

EXPLANATION. The interval includes the remaining 8 days of December, (31—23), all of January (31 da.), all of February (29 da.) and the first 16 days of March. The sum of these respective numbers will evidently be the required number of days.

NOTE. In finding the interval between two dates, the first day is not included, but the last day is counted.

EXAMPLES FOR PRACTICE.

Find the exact number of days between

2. June 5, 1902, and Sept. 19, 1902.
3. Oct. 17, 1903, and May 30, 1904.
4. Feb. 20, 1901, and Aug. 31, 1901.
5. April 4, 1900, and Sept. 12, 1900.
6. Nov. 30, 1904, and April 3, 1905.
7. July 4, 1901, and Jan. 11, 1902.
8. March 29, 1902, and Aug. 8, 1902.
9. Dec. 1, 1903, and May 24, 1904.
10. Sept. 3, 1903, and Jan. 18, 1904.
11. Oct. 13, 1904, and Feb. 5, 1905.

12. A note, dated Jan. 20, 1904, was due, by its terms, Apr. 19, 1905. How many days did it run?

13. A note due Aug. 8, 1905, was discounted at a bank June 3, 1905. For how many days should the bank charge interest?

14. How many minutes from 20 min. past 3 o'clock p. m. Aug. 19, 1890, to 15 min. past 9 o'clock a. m. May 3, 1903, due allowance being made for leap years?

MULTIPLICATION OF COMPOUND NUMBERS.

331. Multiply 12 gal. 3 qt. 1 pt. by 7.

Operation.

gal.	qt.	pt.
12	3	1
		7
90	0	1

EXPLANATION. We place the multiplier, for convenience, under the lowest denomination of the multiplicand, and proceed with the work according to the principles of multiplication of simple numbers. Thus, 1 pt.×7=7 pt.=3 qt. 1 pt. We write the 1 pt. in its order and carry the 3 qt. to its appropriate column. 3 qt.×7=21 qt., to which we add the 3 qt. from the former multiplication, making 24 qt., or 6 gal. 0 qt. Multiplying the gallons (12 gal.×7) and carrying the six gallons, we have the complete product, 90 gal. 0 qt. 1 pt.

EXAMPLES FOR PRACTICE.

Multiply

2. 15 mi. 94 rd. 4 yd. 1 ft. by 5.

3. 82 A. 112 sq. rd. 5 sq. ch. by 8.

4. 320 cu. yd. 22 cu. ft. 830 cu. in. by 11.

5. £140 16s. 8d. 3 far. by 25.

6. 7 ℔ 8 ℥ 5 ʒ 1 ℈ by 42.

7. 150 bu. 5 qt. by 12.

8. 165 T. 1345 lb. by 18.

9. 120° 42′ 18″ by 15.

10. What is the weight of 8 barrels of sugar, if each weighs 287 lb. 13 oz.?

11. A dealer bought a piece of English goods containing 38 yds. invoiced at £1 4s. 6d. What was the cost?

12. A farmer purchased 5 wood lots at $60 an acre. If each lot contained 2 A. 7 sq. ch. 4 sq. rd., how much should he pay for the purchase?

13. How many cords of wood can be cut from the above specified tract, estimating that the average yield per square rod is 2 cd. 16 cu. ft.?

14. If the average speed of a vessel is 23 mi. 95 rd. 12 ft. per hour, how far will she sail in a week of 7 days?

15. Multiply $\frac{7}{16}$ da. by 12.

16. Find the product of $\frac{3}{20}$ pwt. by 50.

17. Find the product of .01845 pk. by 250.

DIVISION OF COMPOUND NUMBERS.

CASE I.

332. Division of compound numbers by abstract numbers.

1. Divide 6 lb. 5 oz. 15 pwt. by 4.

Operation.

	lb.	oz.	pwt.	
4)	6	5	15	
	1	7	8	18 gr.

EXPLANATION. The principles are the same as those employed in the division of simple numbers. We begin with the highest denomination, and reduce each remainder to, and combine the result with the units of the next lower denomination. Thus, $\frac{1}{4}$ of 6 lb.=1 lb., and a remainder of 2 lb., which are equal to 24 oz. 24 oz.+5 oz.=29 oz. $\frac{1}{4}$ of 29 oz.=7 oz. and a remainder of 1 oz. or 20 pwt. 20 pwt.+15 pwt.=35 pwt., $\frac{1}{4}$ of which is 8 pwt., with a remainder of 3 pwt. 3 pwt.=72 gr., $\frac{1}{4}$ of which is 18 gr. Hence the quotient is 1 lb. 7 oz. 8 pwt. 18 gr.

EXAMPLES FOR PRACTICE.

Divide

2. 14 bu. 3 pk. 5 qt. by 3.

3. 364 mi. 216 rd. 5 yd. by 8.

4. 85 lb. 8 oz. Avoir. by 7.

5. £25 17s. 9d. by 6.

Divide

6. 215 acres 113 sq. rd. 19 sq. yd. 4 sq. ft. by 5.

7. 74° 36′ 24″ by 9.

8. 31 ℔ 6 ℥ 5 ʒ 1 ℈ by 10.

9. 17 tons 7 cwt. 15 lb. by 15=(5×3).

10. 754 gal. 3 qt. 1 pt. by 37.

Operation.

gal.	qt.	pt.	
37)754	3	1	(20 gal.
74			
14			
4			
37) 59			(1 qt.
37			
22			
2			
37) 45			($1\frac{8}{37}$ pt.
37			
8			

REMARK 1. When the divisor exceeds 12, and is not composite, the work should be performed by long division, as shown by the accompanying operation.

REM. 2. The solution is the same in principle as that in example 1, which contains a full explanation. The reductions are here written out, instead of being performed mentally, as in the former case. We are confident that a clear understanding of the former explanation will render this solution clear.

11. A tract of land containing 5 A. 40 sq. rd. was surveyed into 26 town lots of equal size. What amount of land did each lot contain?

12. If 24 watch cases were made from 6 lb. 7 oz. 13 pwt. of silver, what was the weight of each case?

13. In walking a distance of 120 rd. 3 yd. 2 ft. 4 in., a pedestrian took 824 steps. What was the average length of each step?

14. A syndicate bought 5270 acres of western land and divided it into 37 farms of equal size. What was the area of each farm?

15. Divide $3\frac{5}{7}$ sq. rd. by 4.

16. Divide 15.0645 gal. by .12.

CASE II.

333. Division of one compound number by another.

1. Divide 36 lb. 8 oz. 12 pwt. by 9 oz. 6 pwt.

Operation.

36 lb. 8 oz. 12 pwt.=8812 pwt.
9 oz. 6 pwt.=186 pwt.
8812 pwt.÷186 pwt.=$47\frac{5}{93}$.

EXPLANATION. We first reduce both divisor and dividend to simple denominate numbers of the lowest denomination expressed by either term, which in this case is pennyweights. We next divide the dividend, 8812 pwt., by the divisor, 186 pwt., and obtain the quotient $47\frac{5}{93}$.

EXAMPLES FOR PRACTICE.

2. Divide 23 lb. 11 oz. 15 pwt. by 1 lb. 7 oz. 13 pwt.

3. Divide 91 bu. 1 pk. 1 qt. by 3 bu. 3 pk. 7 qt.

4. What is the quotient of 24 mi. 124 rd. 0 yd. 1 ft. divided by 243 rd. 4 yd. 2 ft. 6 in.?

5. Divide 3 A. 130 sq. rd. 5 sq. yd. by 2 sq. yd. 4 sq. ft. 72 sq. in.

6. Divide 1 circle by 6° 6′ 40″.

7. How many yards of cloth at 8s. 4d. per yard can be bought with £13 15s.?

8. How many carats fine are rings manufactured from a compound of 1 oz. 5 pwt. 12 gr. of pure gold and 12 pwt. 18 gr. alloy?

9. How many revolutions will a carriage wheel 13 ft. 4 in. in circumference make while the carriage is moving 5 miles?

10. How many prescriptions can be compounded from a certain kind of medicine weighing $3\frac{1}{2}$ lb. if ℈ ij gr. v are required for each?

REVIEW OF DENOMINATE NUMBERS.

334. *1.* From an invoice of sugar weighing $\frac{7}{8}$ of a T., 1256 lb. 12 oz. were sold. What was the value of the *remainder at 6¢* a pound?

2. A grocer bought 3 bu. 3 pk. of blueberries at 8¢ a uart. What did they cost?

3. What should be paid for 35 lb. of flour at $6.40 a arrel?

4. A retail fruit dealer sold 8 bu. 3 pk. 4 qt. of pears aily for 12 days; during the next two weeks his sales ere only $\frac{3}{4}$ of that quantity. If the price charged was 0¢ a peck, how mueh did he receive for the four weeks' ales?

5. What decimal of a rod is $\frac{5}{16}$ of an inch?

6. How many rings weighing 8 pwt. 6 gr. each, and 8 carats fine, can be manufactured from a compound consisting of 9 oz. 18 pwt. of pure gold and the required mount of alloy?

7. A man purchased a tract of land consisting of 29 A. 51 sq. rd. 12 sq. yd., and divided it equally among is 4 sons. What was the share of each?

8. A farmer bought a bill of groceries amounting to 52.44 and gave in payment 4 T. 1280 lb. of hay at $6 a on, and 5 cd. 1 cd. ft. of wood. What was the price of he wood per cord?

9. If a horse consume 5 quarts of oats at a feed, and is iven 3 feeds a day, how many days will 138 bu. 3 pk. ustain 8 horses, provided they are fed at the same rate?

10. A grocer paid $2.50 for a bushel of blueberries nd sold from the purchase as many full quarts, liquid ieasure, as the quantity contained, at 10 cents a quart, nd gave the remainder to a child. How much did he ain?

11. How many feet of barbed wire will be required to ence a field 15 ch. 60 l. long and 12 ch. 80 l. wide, if the ire encircle the field 4 times?

12. If 1 bu. 1 pk. 3 qt. are required to sow an acre of

ground, how much seed will be required to sow a field containing 32 A. 120 sq. rd.?

13. A coal dealer buys coal at the mines for $4.20 per long ton, pays 15 cents per cwt. for transportation, and retails it at $8 per short ton. How many long tons must be involved in the deal to make his profit $357?

14. If the average yield of potatoes per acre is 124 bu. 1 pk. 5 qt., what quantity can be grown from a field containing 40 acres.?

15. How many gallons of medicine will be required to fill 42 gross of vials holding $f\,\bar{3}\,ij$ each?

16. If 1 pt. 2 gi. of water be mixed with 2 qt. of milk, what decimal part of the mixture will be milk?

17. For what number of days will the maker of a note be required to pay interest if the note be given March 17 and paid the 10th day of the following October?

18. A milkman adulterated the milk sold to his customers by adding 1 pint of water to each gallon. If his average sales were 24 gallons a day, and he received 6 cents a quart, how much more did he receive than was honestly due him during a period of 12 days?

19. Reduce $\frac{1}{8}$ of $\frac{2}{11}$ of £3 to the fraction of a penny.

20. Counting at the rate of 180 in 1 min. 48 sec., and continuing the work daily, 10 hours each day, how long a time will be required to count a trillion, if each year be counted as 365 days?

21. If a horse travel a mile in 6 min. 8 sec., in what time can he travel 88 miles?

22. A jeweler contracted with a lady customer for a jar of butter, agreeing to pay 24 cents a pound. The net weight, by the jeweler's scales (Troy), was 10 lb. 6 pwt. 6 gr. How much should the lady receive?

23. What is the value of 3270 pounds of wheat at $85\frac{1}{2}$ *cents a bushel?*

24. What is the cost of 115 lb. 12 oz. of beef at $18.60 a barrel?

25. What decimal part of a mile is 40.04 fathoms?

26. How many times is $\frac{5}{8}$ of a yard contained in $\frac{3}{4}$ of a mile?

27. If gold is worth $20 an ounce, how much greater is the value of the material of an 18 carat gold ring weighing 5 pwt., than that of a 14 carat gold ring of the same weight, the alloy in each being regarded as valueless?

28. A clock, which registers correct time at noon April 5th, loses time at the rate of 1 min. 5 sec. in 18 hours. What will be the correct time May 12th, when the clock registers 1 hr. 30 min. p. m.?

29. Find the difference, by compound subtraction, between the following dates: June 16, 1893, and March 29, 1904.

30. What decimal part of a bushel is 4 quarts, liquid measure?

31. A and B are joint owners of a farm; A's interest is $\frac{7}{12}$ and B's is $\frac{5}{12}$, and the difference between their shares is 14 A. 112 sq. rd. If B sells his interest to C at $50 an acre, how much should he receive?

32. At 45 cents a pound, how much should be paid for 13 chests of tea, each weighing 76 lb. 12 oz.?

33. From $\frac{5}{7}$ of 3 T. 10 cwt. subtract $\frac{4}{13}$ of 7 T. 3 cwt. 26 lb.

34. What decimal of a rod are 3 ft. 3.6 in.?

35. How many yards are equal in distance to 27 ch. 25 l.?

36. What number of hours will the 20th century contain?

37. The forward wheels of a wagon are 10 ft. 8 in. in circumference, and the hind wheels 14 ft. 6 in. How many

more revolutions will the forward wheels make than the hind wheels in going a distance of 90 miles?

38. To $\frac{3}{8}$ of $20\frac{1}{2}$ yards, add $\frac{3}{4}$ of $8\frac{1}{2}$ feet.

39. From $\frac{4}{5}$ of $19\frac{1}{4}$ gallons subtract $\frac{1}{2}$ of $3\frac{4}{5}$ quarts.

40. What part of a degree are 11′ 15″?

41. Reduce $\frac{11}{16}$ of an acre to integers of lower denominations.

42. What decimal of a pound is 9ʒ 5ʒ 2ϑ?

43. At $4.8655 per £, what is the value in U. S. money of £15 8s. 9d.?

44. At $.238 to the mark, how many marks are equal in value to $47.60, U. S. money?

LATITUDE, LONGITUDE, AND TIME.

335. The **Latitude** of a place is the position it occupies (marked by degrees, minutes, and seconds), north or south of the equator.

REMARK. Places north of the equator are in *North Latitude*, and places south of the equator are in *South Latitude*.

336. The **Longitude** of a place is the position it occupies (marked by degrees, minutes, and seconds), east or west from a given prime meridian.

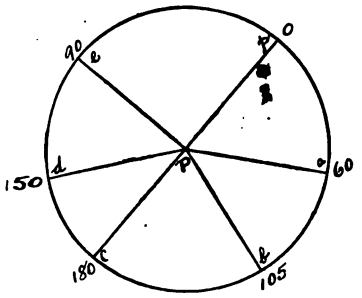
REMARK. Places east of the prime meridian are in *East Longitude*, and places west of the prime meridian are in *West Longitude*.

337. Parallels of Latitude are imaginary lines encircling the earth parallel to the equator.

338. Meridians are imaginary lines crossing the equator at right angles, and extending from pole to pole.

REMARK 1. Two opposite meridians form what is called a meridian circle.

REM. 2. Since the distance from the equator to either pole is $\frac{1}{4}$ a meridian circle, it is evident that the greatest latitude a place can have is *90°. The greatest longitude* a place can have is $\frac{1}{4}$ the distance around the *earth from the prime* meridian, hence 180°.



339. The accompanying figure is designed to illustrate the methods of finding the difference in longitude between any two places. The circumference of the circle represents the earth's equator. The point P, in the center is one of the poles, and the lines pP, aP, etc., are meridians. The line pP represents the prime meridian crossing the equator at p. The arc pa is 60° in length, and we therefore say that the longitude of a is 60° east. The arc pb being 105° in length, we mark the longitude of b 105° east, etc.

In determining the difference in longitude between places, we encounter three different conditions or cases; 1st, when both places are reckoned in the same direction from the prime meridian; 2d, when the places are reckoned in opposite directions from the prime meridian; and, 3d, when the sum of the longitudes reckoned in opposite directions exceeds 180°.

To find the difference in longitude between a and b, it is evident that we must subtract 60° from 105°, since both are reckoned in the same direction from the prime meridian p.

To find the difference in longitude between a and e, it is evident that we must obtain the sum of the arcs pa and pe, (150°), since these places are reckoned in opposite directions from p.

To find the difference in longitude between b and d, we proceed as follows; 105° (longitude of b) + 150° (longitude of d) = 255°, the length of the arc *depab;* but the difference is the length of the shorter arc, *dcb*, hence 255°

must be subtracted from the circle, 360°, which gives 105°, the correct result.

340. From the foregoing illustrations and explanations we deduce the following

RULE I. *If the longitudes of the two given places are both east or both west, find their difference by subtracting the less from the greater; but if one is east and the other west, add the two longitudes, and if the sum exceeds 180°, subtract such sum from 360°.*

II. *If the latitudes of the two given places are both north or both south, find the difference by subtracting the less from the greater; but if one is north and the other south, add the two latitudes.*

EXAMPLES FOR PRACTICE.

1. The longitude of Cincinnati is 84° 24′ west; and the longitude of Buffalo is 78° 55′ west. What is the difference in longitude between the two cities?

2. The latitude of Philadelphia is 39° 56′ 38″ north; Minneapolis is in north latitude 44° 58′ 12″. What is the difference in latitude between the two places?

3. The longitude of Boston is 71° 3′ 30″ west, and of Berlin 13° 23′ 48″ east. What is their difference in longitude?

4. The longitude of Sacramento is 121° 32′ 20″ west, and of Pekin 114° 30′ east. What is their difference in longitude?

5. The Tropic of Capricorn is in south latitude 23° 30′, and the Arctic Circle is in north latitude 66° 30′. What is the difference in latitude between these circles?

SOLAR TIME.

341. The daily revolution of the earth on its axis causes the sun to *appear* to pass from east to west around the earth (360°) in 24 hours. Hence it is evident that

ıe sun will apparently pass over $\frac{1}{24}$ of 360° or 15°, in 1 our. Everyone understands that it is noon (solar time),) all places on a given meridian when the sun's rays are ertical thereto. The sun, at the present instant, is irectly over some meridian of the earth, and to all places ı that meridian it is noon, and to all places 15° east it is ıe hour past noon, and to all places 15° west it lacks one ɔur of being noon, as it will require 1 hour for the meri- ian now 15° west to be brought under the sun's vertical ıys.

From the foregoing explanation we readily see that for difference of 15° in longitude there is a corresponding ifference of one hour in time; and that for a difference of $\frac{1}{60}$ of 15° (15′), there is a difference of $\frac{1}{60}$ of an hour 1 min.), in time; and that for a difference of $\frac{1}{60}$ of 15′ 15″), there is a difference of $\frac{1}{60}$ of a minute (1 sec.), in me. The relations above expressed are rendered more ɔncise in the following

TABLE.

Diff. in Long.	Diff. in Time.
15°	1 hr.
15′	1 min.
15″	1 sec.

Remark. Notice that the degrees, minutes, and seconds of the longi- ıde are 15 times as many as the hours, minutes, and seconds of time. ince the ratio of differences is 15, as shown by the table, it is evident that ıe difference in longitude, divided by 15, will give the corresponding dif- rence in time, and that the difference in time, multiplied by 15, will give ıe corresponding difference in longitude.

342. The *true* difference in longitude between two laces is the distance, expressed in degrees, minutes, and ɔconds, of the shorter arc of the parallel connecting their ıeridians (339); but when we wish to find the difference ı time between two places, one of which is in east, and ıe other in west longitude, we must regard the difference

in longitude, in the operation, as the length of the arc which crosses the prime meridian.

From the foregoing statements and explanations we deduce the following rules:

343. To find the difference of time between two places when both are in east, or in west, longitude.

RULE. *Divide as many hours, minutes, and seconds by 15 as there are degrees, minutes, and seconds of difference in the longitude. The quotient will be the difference in solar time.*

344. To find the difference of time between two places when one is in east, and the other in west, longitude.

RULE. *Divide as many hours, minutes, and seconds by 15 as there are degrees, minutes, and seconds in the arc of the parallel crossing the prime meridian and connecting the meridians marking the longitudes of the two places.*

345. To find the difference of longitude between two places when the difference of solar time is given.

RULE. *Multiply as many degrees, minutes, and seconds by 15 as there are hours, minutes, and seconds of difference in time; the product, if not in excess of 180°, will be the difference of longitude.*

REMARK. If the product exceeds 180° it must be subtracted from 360° to obtain the true difference of longitude.

EXAMPLES FOR PRACTICE.

1. The difference of longitude between two places, both of which are west of the prime meridian, is 42° 24′ 30″. What is the difference in solar time?

2. The difference in solar time between two places in east longitude is 4 hr. 12 min. 30 sec. What is the difference in longitude?

3. What is the difference in time between two vessels, one of which is in west longitude 179° 30′ and the other, at

the same instant, is in east longitude 179° 30′? If it is 8 o'clock p. m., May 9, according to the calendar kept by the master of the first mentioned vessel, what is the time and date according to the calendar kept by the master of the other vessel?

Ans. to 1st. 23 hr. 56 min.

Ans. to 2d. 7 o'clock 56 min. p. m. of May 10.

REMARK. Assuming that the vessels mentioned in the above example are in the same latitude, they are 1° apart, and, if the first one mentioned is sailing west, and the other is sailing east at the same rate of speed, it is evident that they will soon meet at the 180th meridian, and that their calendars will differ by 1 day; hence the west bound vessel's calendar must be moved forward 1 day to agree with east longitude reckonings, and the calendar of the east bound vessel must be moved backward 1 day to agree with the reckonings of west longitude.

4. The sun rises at Boston 1 hr. 11 min. 56 sec. earlier than at New Orleans; if the longitude of New Orleans is 89° 2′ 24″ west, what is the longitude of Boston?

5. A tourist starts from Philadelphia, longitude 75° 12′ west, on a journey westward; what longitude will he be in when his watch, which registers Philadelphia time, is 2 hours 15 minutes too fast?

6. The longitude of Cambridge, Eng., is 5′ 2″ east, and of St. Paul 93° 12′ west. When it is 2 o'clock p.m. at St. Paul, what is the time at Cambridge?

7. When it is 24 minutes past 8 o'clock a. m. at Pekin, whose longitude is 114° 30′ east, it is 39 minutes 49⅛ seconds past 4 o'clock p. m. at Sacramento, California. What is the longitude of the last named place?

8. A and B started from the prime meridian, London, at noon, July 4th, and traveled around the earth; A's course being eastward and B's westward. Each traveled at the rate of 15° in 24 hours, and therefore completed his journey at noon, July 28th. Each adjusted his watch to local time daily, but neither made any change in his

calendar. *Queries.* 1. How many times was it possible for each to see the sun rise while pursuing his journey? 2. What was the length of each day as calculated by A's calendar? 3. What was the length of each day as calculated by B's calendar? 4. What day of the month did they arrive in London, according to A's calendar? 5. What was the day according to B's calendar?

STANDARD TIME.

346. The inconvenience resulting from a constant change of time to those traveling east or west led certain railroad corporations operating lines of road in the United States to devise a system, in the year 1883, which is called "*standard time.*" This system has since been generally adopted. It provides that every change in time shall be one or more exact hours, all fractional parts of an hour having been eliminated. Four of the meridians traversing the United States are multiples of 15: namely, numbers 75, 90, 105, and 120. These are called "*standard time meridians*" or "*time centers.*" The country is conceived to be divided into belts 15° wide, as nearly as may be, and running north and south. Centrally located in each of these belts are the standard meridians or time centers. The most easterly of these belts is called "*the belt of eastern time*" and its time center is the 75th meridian. The 90th meridian is the time center of the next belt westward, which is called "*the belt of central time.*" The next belt which extends over the mountainous district west of the Mississippi, is called "*the belt of mountain time,*" and has the 105th meridian as its time center. The next belt extends to the Pacific coast and is called "*the belt of Pacific time,*" and the 120th meridian is its time center. The *lines of separation* between these belts are not straight lines

located exactly $7\frac{1}{2}°$ degrees each side of the standard meridians, neither do they in all cases extend exactly north and south, but are made to conform, as nearly as practicable, to state lines.

By the arrangement above explained, all places located in the eastern time belt take as their clock time the solar time of the 75th meridian; all places in the central time belt take as their clock time the solar time of the 90th meridian, which is exactly one hour later than the solar time of the 75th meridian, and one hour earlier than the solar time of the 105th meridian, etc. Hence throughout the United States, all clocks and watches, when registering correct standard time, can vary only by exact hours.

EXAMPLES FOR PRACTICE.

1. The difference in longitude between two American cities is 32° 18′ 36″; what is the difference in standard time?

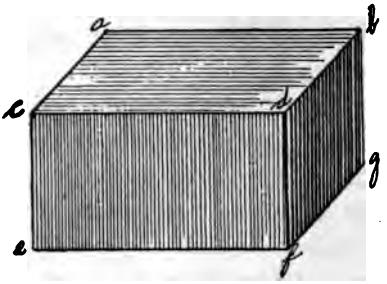
2. When it is 18 minutes past 1 o'clock p. m. in New York, what is the time in Portland, Oregon, the longitude of the former place being 74° 1′ 6″?

3. When it is 5 o'clock p. m. at San Francisco, what is the time at Indianapolis?

4. When the solar time in Stockholm, longitude 18° 3′ 30″ east, is 8 hours 15 minutes a. m., what is the standard time at Buffalo, longitude 78° 55′ west?

5. When the standard time is 6 o'clock a. m. at Galveston, Texas, longitude 94° 46′ 30″, the solar time at Constantinople is 1 hr. 55 min. 16 sec. p. m. What is the longitude of Constantinople?

MEASUREMENTS OF EXTENSION.



347. The accompanying figure is designed to represent a solid block. A close observation will reveal the fact that it has six faces, namely: two sides, two ends, a top, and a base. Three of these faces, one side, one end and the top, are exposed to view. These three faces all meet in one position, marked (indicated) by *d*. This mark of position is called a point. The corners of these three faces do not surround an open space, but they touch one another at the position *d*; hence, the point *d* has no extension. The place of meeting of the top face with the end face, *db*, is called a line; and, since there is no open space between these faces, it is evident that a line has no width; it has extension only in one direction, from *d* to *b*, or from *b* to *d*. Each of these faces illustrates a surface. The top face has extension from the line *ca* to the line *db*; this extension is its length; the extension from the line *cd* to the line *ab* is called width. Thus we see that a surface has two dimensions belonging to extension. When we examine the entire solid we discover that the line *cd* marks its length, that *db* marks its width, and that *df* marks its thickness.

REMARK. Since a solid is the only one of the extensions having all the dimensions belonging to space, it is evident that it is the only extension that can occupy space.

The foregoing illustrations and explanations suggest the following definitions:

348. A **Point** is a mark of position.

349. ***A Line*** *is* a one-dimensional extension.

350. A **Straight Line** is a line of unvarying direction.

REMARK. Since a straight line does not vary in direction, it marks the shortest distance between two points. The line, however, is not the *distance*, but the distance is an *attribute* of the line.

351. A **Curved Line** is a line whose direction varies at every point.

352. A **Broken Line** is a line whose direction varies at one or more fixed points.

353. A **Surface** is a two-dimensional extension. It has length and breadth, but no thickness.

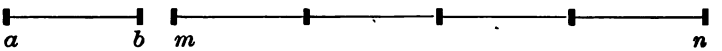
354. A **Plane Surface** or a **Plane** is a surface of such a character that if any two of its points are connected by a straight line, every point of the line will be in the surface.

355. A **Solid** or **Volume** is a three-dimensional extension. It has length, breadth, and thickness.

356. **Measurement** is the process of ascertaining how many times the thing to be measured will contain a measuring unit.

357. The **Measuring Unit of a Line** is a line of any desired length.

REMARK. It is customary, in arithmetical work, to select for the measuring unit of extension some *denomination* belonging to extension.



The measurement of the line *mn* is the process of ascertaining how many times it will contain the measuring unit *ab*. By applying this unit we discover that it is contained in the given line 4 times; hence we decide that the line *mn* is 4 units in length. If the measuring unit is 1 inch, the line *mn* is 4 inches long; if the unit is 1 foot, the line is 4 feet long, etc.

SURFACE MEASUREMENTS.

358. Surfaces usually take their names from their forms, hence they are known as *triangles*, *rectangles*, *parallelograms*, *rhomboids*, *pentagons*, *circles*, etc. The measurement of most surfaces requires a knowledge of geometry; a measurement of the most simple can be comprehended by the student of arithmetic, and such only are here presented.

359. A **Rectangle** is a surface bounded by 4 straight lines, each of which is perpendicular to the line adjacent.

360. A **Square** is a rectangle whose sides are equal.

361. The **Area** of a surface is the number of square units it contains.

362. The **Measuring Unit** of a surface is a square.

REMARK. Each side of the measuring unit selected for the purpose of obtaining the areas of rectangular surfaces should be of the same denomination as the two given dimensions of the surface to be measured. Let it be required to determine the area of a rectangle whose length is 5 feet, and whose width is 3 feet. Since the given dimensions are expressed in feet, we take 1 square foot for our measuring unit and proceed in the following manner:

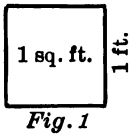


Fig. 1

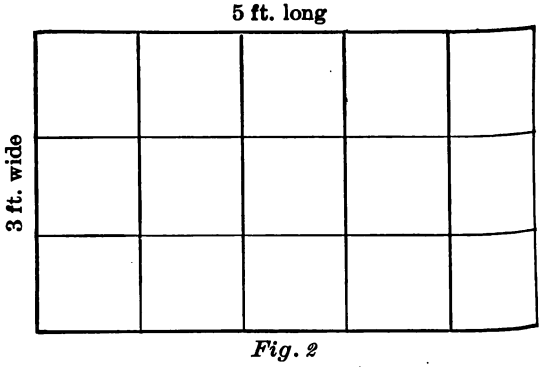


Fig. 2

Figure 1 is the measuring unit, and we are to determine how many times the rectangle represented by figure 2 will contain this unit. Since *the surface* to be measured is 5 feet long, it will take 5 of our measuring

(1 ft. long), to form a row reaching from one end to the other, and the width is 3 feet, the rectangle will contain 3 such rows; hence the ing solution:

sq. ft. (units in 1 row) × 3 (No. of rows) = 15 sq. ft. a).

63. To find the area of a rectangle. The explana- given suggest the following

ULE. *Multiply the number of units corresponding to one nsion by an abstract number equal in unit value to the dimension.*

EMARK. If necessary, reduce both dimensions to the same denom- n before applying the rule.

EXAMPLES FOR PRACTICE.

. How many square feet in the ceiling of a room 18 in. long and 14 ft. 3 in. wide?

. How much will it cost to plow a field 50 rods long 40 rods wide at $3.25 an acre?

. How many square feet of surface in a box 4 ft. 6 in. , 3 ft. wide, and 2 ft. 8 in. high?

. A city park is 20 rd. 8 ft. 6 in. long and 18 rd. 3 ft. e, and is surrounded by a cement walk 6 feet wide. v many square feet of surface does the walk contain?

. At $40 an acre, what is the value of a rectangular e of ground 48 ch. 60 l. long and 36 ch. 15 l. wide?

SOLUTION.

48.60 sq. ch. × 36.15 ÷ 10 = 175.689 A.
$40 × 175.689 = $7027.56 *Answer.*

. How many square yards in the walls and ceiling of om 22 ft. long, 18 ft. 6 in. wide, and 9 ft. 8 in. high?

. How much income will a farmer receive from a dow 24 ch. 12 l. long, and 18 ch. 8 l. wide, provided the average yield is 1½ tons per acre and he sells the at $8.25 a ton?

8. What is the area of a piece of land 12 rd. 2 yd. 1 ft. 5 in. long and 10 rd. 4 yd. 2 ft. wide?

364. To find either dimension of a rectangular surface, the area and the other dimension being given.

REMARK: Since the area is the product of the two dimensions, it is evident that a wanting dimension may be readily found by dividing the area (product) by the given dimension (one factor).

EXAMPLES FOR PRACTICE.

1. A rectangular field is 32 rods long and contains 688 square rods. What is its width?

2. In the purchase of a rectangular plat of ground 24 ch. 40 l. long, bought at $50 an acre, the sum of $2257 was paid. What was the width of the plat in rods?

3. A town lot containing $\frac{3}{20}$ of an acre is 49 ft. 6 in. long. What is its width?

4. A mason's bill for plastering the ceiling of a room 18 ft. 6 in. wide was $8.88. What was the length of the room, his price for plastering being 18 cents per square yard?

5. A farm containing 144 acres is 180 rods long. What is its width in chains?

SPECIAL CASES.

365. Application of one surface to another.

1. How many bricks 8 inches long and 4 inches wide will be required to pave a walk 64 feet long and 4 feet wide?

EXPLANATION. It is evident that as many bricks will be required as the area of one brick is contained times in the area to be covered; hence the following solution:

64 sq. ft. $\times 4 = 256$ sq. ft. (area to be covered).

8 sq. in. $\times 4 = 32$ sq. in. $= \frac{2}{9}$ sq. ft. (area of 1 brick).

256 sq. ft. $\div \frac{2}{9}$ sq. ft. $= 1152$ (No. of bricks).

NOTE. If thought advisable, require the student to formulate a rule, *after studying carefully* the explanation.

EXAMPLES FOR PRACTICE.

2. How many cement blocks 20 inches square will be equired to lay a walk 5 feet wide and 66 ft. 8 in. long?

3. How many marble blocks 8 inches square will be equired to pave the floor of an office 32 feet long and 24 eet 8 inches wide?

4. How many cement blocks 16 inches square will be equired to lay a walk 5 ft. 4 in. wide around a city park hat is 20 rods square?

BOARD MEASURE.

366. Board Measure is used in measuring boards, lanks, and other varieties of lumber, including sawed imber.

367. The **Unit of Board Measure** is a *board foot,* which s 1 ft. long, 1 ft. wide, and 1 in. thick.

Thus, a board 12 ft. long, 2 ft. wide, and 1 in., or less, hick, contains 24 board feet. A board, plank, or other iece of sawed lumber, exceeding 1 inch in thickness, conains as many times the number of board feet represented y its surface as it is inches thick.

368. The **Average Width** of a regularly tapering board s its width in the middle, which is one-half the sum of the vidth of its two ends.

1. How many board feet in a plank 18 ft. long, 10 in. vide, and 2 in. thick?

$\frac{\cancel{3}}{\cancel{18}\times 10\times\cancel{2}=30.}$
$\frac{\cancel{12}}{6}$

SOLUTION AND EXPLANATION. Since the width is 10 inches ($\frac{10}{12}$ of a foot), we multiply the two dimensions, 18 and $\frac{10}{12}$, together and ascertain the number of board feet in a board of the same length and width whose thickness is 1 inch; but since the lank is 2 inches thick, it will contain twice as many board feet as an inch oard; therefore, we multiply the result by 2. Hence the following solution.

RULE. *Multiply as many board feet as the board is feet in*

length by an abstract number equal to the number of inches in width, and divide the result by 12.

NOTE. If more than 1 in. thick, multiply the result obtained by the rule by the number representing the inches of thickness.

EXAMPLES FOR PRACTICE.

2. How many board feet in 5 planks, each of which is 16 ft. long, 14 in. wide, and 3 in. thick?

3. What is the cost of 25 joists 20 ft. long, 8 in. wide, and 2 in. thick, at $28 per thousand feet?

4. What is the cost of a stick of timber 24 ft. long, 20 in. wide, and 8 in. thick, at $31.50 per thousand feet?

5. How many board feet in a regularly tapering board 14 ft. long, 13 in. wide at one end, and 9 in. wide at the other (368)?

6. Find the cost of the following bill of lumber at $32.75 per thousand feet.

118	boards,	ea.	16	ft.	long,	10	in.	wide,	and	1	in. thick.
74	pieces,	"	18	"	"	4	"	"	"	2	" "
62	"	"	18	"	"	10	"	"	"	2	" "
64	"	"	16	"	"	6	"	"	"	2	" "
121	"	"	12	"	"	8	"	"	"	$1\frac{1}{2}$	" "
44	"	"	20	"	"	10	"	"	"	4	" "

369. To find a wanting dimension, the number of board feet and the other dimensions being given.

7. What is the width of a 2-in. plank, which is 18 ft. long and contains 42 feet?

SOLUTION AND EXPLANATION. Since 42 feet is the product of $\frac{\text{length} \times \text{width} \times \text{thickness}}{12}$, we have only to divide the product, 42, by the product of the given factors; thus, $42 \div \frac{18 \times 2}{12} = 14$.

8. What is the length of a 4-in. plank, which contains *80 ft.* and is 10 in. wide?

9. What is the thickness of a board which contains 28 ft. and is 16 ft. long and 14 in. wide?

10. I have a rectangular plat of ground 336 feet long and 248 feet wide. What will it cost to enclose it with a board fence, the price of posts being $24 per hundred, and the price of boards being $26.50 per thousand feet; the posts to be placed 8 feet apart, the boards to be 16 feet long, 6 inches wide, and 1 inch thick, and the fence 3 boards high?

MEASUREMENTS FOR CARPETING.

370. Since the carpeting necessary to cover a floor generally exceeds, in area, the floor space to be covered, the principles explained in 365 will not apply.

Before taking floor measurements for the purpose of determining the number of yards of carpeting, it will be necessary to decide whether the strips shall run lengthwise or widthwise of the room. It must also be borne in mind that carpeting must always be cut through the same design to secure a matching of the strips; hence it is frequently found necessary that the strips exceed the distance found by the floor measurement.

1. How many yards of carpeting, 1 yard wide, must be purchased to cover the floor of a room 18 ft. 6 in. long and 14 ft. 2 inches wide; the strips to run lengthwise and the design being such that the strips, in order to match, must be cut 3 inches longer than the room?

Operation.

$14\frac{1}{6}$ ft. $\div$ 3 ft. $=4+$, hence 5 strips.
18 ft. 6 in. $+$ 3 in. $=18\frac{3}{4}$ ft. (length of strips).
$18\frac{3}{4}$ ft. $\times 5 = 93\frac{3}{4}$ ft. $= 31\frac{1}{4}$ yds.

EXPLANATION. Since the strips are to run lengthwise, as many strips will be required as the quotient of $14\frac{1}{6}$ ft. (the width of the room) divided by 3 ft. (the width of each *strip*). This quotient exceeds 4, hence 5 strips must *be purchased and the surplus* be turned under in laying. Since each

377. The **Measuring Unit** of a volume is a cube.

378. The **Measurement of a Volume** is the act of determining the number of measuring units it contains.

REMARK 1. Volume measurement, in a mathematical sense, applies to the space occupied by a solid body. When we speak of a solid as containing 24 cu. ft., the true meaning of the expression is that it occupies 24 cu. ft. of space.

REM. 2. A cube of any size may be taken at will as the measuring unit; it is customary, however, and more convenient, to select a cube each edge of which is a unit of the same denomination as the given dimensions of the volume to be measured. Hence to find the volume of a rectangular solid 6 ft. long, 4 ft. wide, and 3 ft. deep, we select 1 cu. ft. as the measuring unit and proceed in the following manner:

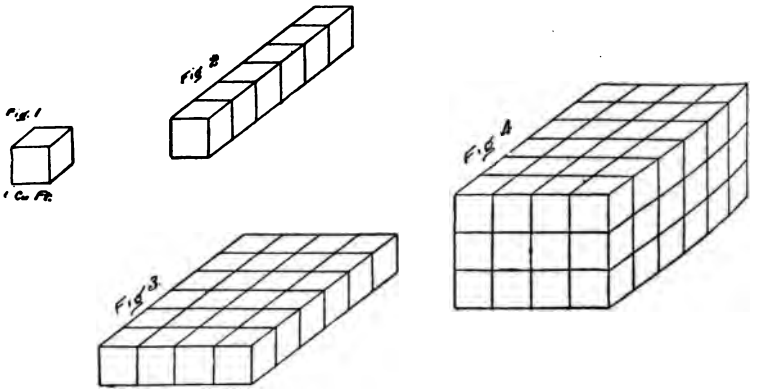


Fig. 1 is the measuring unit, which is 1 cu. ft. Since the volume is 6 ft. long, a row consisting of 6 of these measuring units will just reach from one end to the other. Fig. 2 represents such a row. Since the volume is 4 ft. wide, 4 such rows will just reach from one side to the other. These 4 rows are represented by fig. 3, which illustrates 1 layer containing (6 cu. ft. $\times 4$)=24 cu. ft. Since the volume is 3 feet deep (thick), it will contain 3 such layers, or (24 cu. ft. $\times 3$)=72 cu. ft., as shown in fig. 4.

The foregoing illustration will render clear the following solution:

$$6 \text{ cu. ft.} \times 4 \times 3 = 72 \text{ cu. ft.}$$

379. To find the contents of a rectangular volume, we *apply the following* rule as suggested by the foregoing *illustration:*

$$\frac{\cancel{15}}{\cancel{2}} \times \frac{\cancel{10}}{\cancel{3}} \times \frac{1}{\cancel{32}_{2}} = \frac{15}{4} = 3\tfrac{3}{4}.$$

RULE. *Multiply the length in feet, the height in feet, and* ⅓ *together. The result will be the number of cords.*

NOTE. The above rule applies to 4-foot wood only. If the length of ıe wood is less than 4 feet, take such a part of the result obtained by the ıle as the length of the wood is a part of 4 feet.

EXAMPLES FOR PRACTICE.

2. At $4.40 a cord, what is the value of a pile of 4-foot ɾood 43 ft. 4 in. long and 6 ft. high?

EXPLANATION. The price expressed as a common fraction is $\$\frac{22}{5}$. 'his number multiplied by the factors of the number expressing the quan- ty must give the total value; hence the following solution:

$$\$\frac{\cancel{22}^{11}}{\cancel{5}} \times \frac{\cancel{130}^{26}}{\cancel{3}} \times \frac{\cancel{6}^{2}}{1} \times \frac{1}{\cancel{32}_{\cancel{16}_{8}}} = \$\frac{286}{8} = \$35.75.$$

3. At $4 a cord, what is the value of a pile of 3-ft. ɾood 72 ft. long, and 5 ft. 2 in. high?

SUGGESTION. Apply the additional factor ⅔ to the gene.al formula.

4. At $6.50 a cord, what is the value of a pile of 16-in. tove wood, 47 ft. 5 in. long and 4 ft. 6 in. high?

5. A pile of 4-ft. wood contains 7½ cords. The height f the pile is 5 ft. 4 in. What is its length?

6. A fuel dealer sold a pile of 4-ft. wood 18 ft. 8 in. ɔng at $4 a cord, and received therefor $10.50. What ɾas the average height of the pile?

BRICK-WORK.

372. Brick-Work is usually estimated by the thousand ricks.

NOTE. An ordinary brick is 8 in. long, 4 in. wide, and 2 in. thick; ther sizes, however, are manufactured.

373. To find the number of bricks in a wall.

1. How many ordinary bricks are required for the walls of a building 40 ft. deep, 20 ft. high and 24 ft. front, allowing 200 sq. ft. of the surface for doors and windows, the walls being 2 bricks thick?

SOLUTION.

8 sq. in.$\times$2=16 sq. in., exposed surface of each brick.

144 sq. in.$\div$16 sq. in.=9, number of bricks to each sq. ft. of wall if laid solid, without mortar.

Deducting $\frac{1}{9}$ for mortar, (9—1=8), gives number of bricks to each sq. ft.

(40+24)$\times$2=128 ft., outside girth of wall.

Since each corner has been considered twice in determining length of wall, it will be necessary to deduct 4 times the thickness of the wall from the girth measure to obtain the correct length.

128 ft.—($\frac{2}{3}$ ft.$\times$4)=125$\frac{1}{3}$ ft., correct length of wall.

(125$\frac{1}{3}\times$20$\times$2)—200=4813$\frac{1}{3}$ sq. ft., net surface of walls including both thicknesses of brick.

8 (No. of bricks per sq. ft.) $\times$4813$\frac{1}{3}$ (No. of sq. ft.)=38506$\frac{2}{3}$ bricks.

RULE I. *Divide 144 by the exposed surface of 1 brick, and deduct $\frac{1}{9}$ of the result for mortar; the remainder will be the number of outside bricks per square foot of wall.*

II. *Multiply the number of bricks required for one square foot by the total number of square feet of surface belonging to all thicknesses of the wall.*

EXAMPLES FOR PRACTICE.

2. How many bricks 8$\frac{1}{2}$ in. long, 4 in. wide, and 2$\frac{3}{8}$ in. thick will be required for the walls of a store 120 ft. deep, 32 ft. wide, and 60 ft. high; the walls to be 3 bricks thick, and 620 sq. ft. to be allowed for openings?

3. How many ordinary bricks will be required to construct the walls of a dwelling 40 ft. front, 64 ft. deep and 48 ft. high, 2 bricks thick, and with gables 12 ft. high at the front and rear, 960 sq. ft. to be allowed for openings?

NOTE. The surface of any triangle is found by multiplying the base *(length), by one-half* the altitude (height).

LAND MEASURE.

374. To find the number of acres in a tract of land from a surveyor's dimensions.

1. How many acres in a plat of ground 16 ch. 35 l. long and 12 ch. 74 l. wide?

Operation.

16.35
12.74
6540
11445
3270
1635
20.82990

EXPLANATION. Since a link is $\frac{1}{100}$ of a chain, the dimensions are conveniently expressed decimally as chains. To find the number of sq. ch. we multiply length by width, and as 10 sq. ch. make an acre, we point off one additional place (5 places), and have the result of the computation in acres.

RULE I. *Express each dimension in chains and hundredths.*

II. *Multiply length by width and point off one more decimal place than are in both factors. The result will express the number of acres.*

EXAMPLES FOR PRACTICE.

2. How many acres in a meadow 48 ch. 82 l. long and 35 ch. 8 l. wide?

3. At $60 an acre, what is the value of a timber lot 20 ch. 54 l. long and 15 ch. 30 l. wide?

4. A tract of land containing 75.1338 acres is 31 ch. 15 l. long; what is its width expressed in chains and links?

MEASUREMENTS OF VOLUME.

375. A **Rectangular Volume** is one whose faces are rectangular surfaces.

A **Volume** or **Solid** contains all the dimensions belonging to space; *length*, *breadth*, and *thickness*.

376. A **Cube** is a rectangular solid bounded by six *equal square surfaces*.

377. The **Measuring Unit** of a volume is a cube.

378. The **Measurement of a Volume** is the act of determining the number of measuring units it contains.

REMARK 1. Volume measurement, in a mathematical sense, applies to the space occupied by a solid body. When we speak of a solid as containing 24 cu. ft., the true meaning of the expression is that it occupies 24 cu. ft. of space.

REM. 2. A cube of any size may be taken at will as the measuring unit; it is customary, however, and more convenient, to select a cube each edge of which is a unit of the same denomination as the given dimensions of the volume to be measured. Hence to find the volume of a rectangular solid 6 ft. long, 4 ft. wide, and 3 ft. deep, we select 1 cu. ft. as the measuring unit and proceed in the following manner:

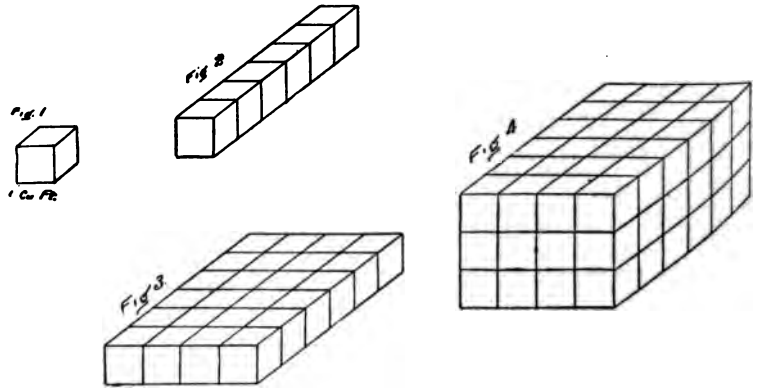


Fig. 1 is the measuring unit, which is 1 cu. ft. Since the volume is 6 ft. long, a row consisting of 6 of these measuring units will just reach from one end to the other. Fig. 2 represents such a row. Since the volume is 4 ft. wide, 4 such rows will just reach from one side to the other. These 4 rows are represented by fig. 3, which illustrates 1 layer containing (6 cu. ft. $\times$4)=24 cu. ft. Since the volume is 3 feet deep (thick), it will contain 3 such layers, or (24 cu. ft.$\times$3)=72 cu. ft., as shown in fig. 4.

The foregoing illustration will render clear the following solution:

6 cu. ft.$\times$4$\times$3=72 cu. ft.

379. To find the contents of a rectangular volume, we *apply* the following rule as suggested by the foregoing *illustration*:

RULE. *Multiply the number of measuring units corresponding to one dimension by the product of numbers equaling the other dimensions.*

REMARK. Reduce all compound numbers expressing denominations to simple denominate numbers of the same denomination before attempting to apply the rule.

EXAMPLES FOR PRACTICE.

1. What is the contents of a block of marble 3 ft. 8 in. long, 1 ft. 10 in. wide, and 9 in. thick?

2. How many cubic feet of air in a room 17 ft. 4 in. long, 14 ft. 5 in. wide, and 9 ft. 6 in. high?

3. What is the value of a pile of quarried stone 13 ft. 7 in. long, 6 ft. 4 in. wide, and 4 ft. high, at $8.25 per hundred cubic feet?

4. What is the cost of digging a cellar 28 ft. long, 22 ft. 6 in. wide, and 6 ft. 6 in. deep, at 36 cents per cubic yard?

5. How many gallons of water will a reservoir hold, its dimensions being 15 feet square and its depth 18 ft. 6 in., estimating $7\frac{1}{2}$ gallons to each cubic foot?

6. How many cubic feet of space in a coal bin whose dimensions are 3 yd. 1 ft. 6 in., 1 yd. 2 ft. 4 in., and 3 ft. 8 in.?

7. What is the weight of a piece of timber 10 in. by 12 in. and 30 feet long, at 42 pounds per cubic foot?

8. A block of granite containing 2 cu. ft. 1304 cu. in. is 1 ft. 8 in. long and 1 ft. 5 in. wide. What is its thickness?

Since the contents is the product of the three dimensions, it is evident that a required dimension may be found by dividing the contents by the product of the two given dimensions. Thus:

2 cu. ft. 1304 cu. in.=4760 cu. in.
1 ft. 8 in.=20 in., and 1 ft. 5 in.=17 in.
20×17=340, and 4760÷340=14 in. Ans.

9. The cost of digging a cellar 24 ft. long and $6\frac{1}{2}$ ft. deep at 40 cents per cubic yard was $41.60. What was the width of the cellar?

10. A man contracted to remove the dirt from an em-

bankment at 65 cents per cubic yard, and to take the same in part payment at 12 cents per load of 30 cubic feet. The length of the embankment was 2 rods, the width 25 ft. 6 inches, and the average depth 5 feet. What was the balance due on his contract?

SPECIAL CASES.

STONE-WORK.

380. Stone-work is estimated, in some localities, by the perch, which contains $24\frac{3}{4}$ cu. ft. In other localities a cord (100 cu. ft.), measured in the wall, is taken as the unit.

REMARK 1. In estimating *quantity of stone* to be used, allowance should be made for all openings, but in estimating the *masonry* no allowance is usually made for openings less than 3 feet wide; customs, however, vary in different localities, and allowances concerning openings will depend upon the contract.

REM. 2. The mason's measurement of length of wall is the *girth* or gross outside measurement. This exceeds the *true* length by 4 times the thickness of the wall. Such measurement is allowed, since the corners are more difficult to build.

381. To find the number of perches or cords for which masons are entitled to payment.

1. At \$4.20 per perch, what is the cost of erecting the stone walls of a building 40 feet long by 32 feet wide, the walls being 24 feet high and 1 ft. 8 in. thick, allowing $\frac{1}{9}$ the capacity for openings occasioned by 4 windows, each 4 ft. 8 in. wide by 10 feet high, and by 2 doors, each 6 feet wide and 8 feet high?

Operation.

$(40+32)\times 2=144$ ft., girth.

$144\times 24\times 1\frac{2}{3}=5760$ cu. ft., gross.

$4\frac{2}{3}\times 10\times 1\frac{2}{3}\times 4=311\frac{1}{9}$ cu. ft., windows.

$8\times 6\times 1\frac{2}{3}\times 2=160$ cu. ft., doors.

$$5760-\frac{311\frac{1}{9}+160}{2}=5524\frac{4}{9} \text{ cu. ft., net.}$$

$5524\frac{4}{9}\div 24\frac{3}{4}=223.21-$, perches.

$\$4.20\times 223.21=\$937.48$, cost.

EXPLANATION. The sum of the length and width $(40+32)\times 2=$ length of wall. The product of the three dimensions=5760 cu. ft., contents including openings. Deducting $\frac{1}{9}$ the contents of openings, as per contract, gives $5524\frac{4}{9}$ cu. ft.=223.21 perches,

for which the mason is to receive payment. The price per perch, $4.20 × 223.21, the number of perches, gives $937.48, cost of masonry.

RULE. *Find the number of cubic feet of masonry by 379, taking the girth measure for the length of the wall, and reduce the number of cubic feet thus found to perches or cords, according to the conditions of the contract.*

EXAMPLES FOR PRACTICE.

2. A mason contracted to construct the walls of a church at $8 per cord of 100 cu. ft. The size of the building was 80 by 56 feet, and the walls were 24 ft. high, with gables on each end wall 30 ft. 6 in. high. The thickness of the wall was 20 inches, and the openings were one door 7 by 10 feet, one door 4 by 8 feet, and 12 windows 4 by 6 feet each, and one window 6 by 10 feet. How much should the mason receive, $\frac{1}{2}$ the contents of openings being considered in the measurement?

3. I contracted with a mason to build a wall 50 ft. long and 1 ft. 6 in. thick, at $4.95 per perch, and paid him for his work $52.50. What was the height of the wall?

4. What will the stone cost me at $8 per cord of 100 feet, measured in the wall, for the foundation of a house 36 by 30 feet, the height of the wall being 7 feet, and the thickness 1 ft. 6 in.; the openings being one door 4 by $6\frac{1}{2}$ feet, and four windows, each 2 by 3 feet? What will the masonry cost at $5.75 a perch, no allowance being made for openings except that occasioned by the door?

MEASUREMENT OF CAPACITIES.

382. Measurement of Capacities, as here treated, relates to the volume of space contained in vessels in which grains, fruits, vegetables, liquids, etc., are stored.

NOTE 1. The term *Stricken Measure* implies that the measuring vessel is stricken off by a ruler and made even full. Grains, seeds, etc.,

are sold by *stricken measure*. The standard unit of stricken measure is the bushel, which contains 2150.42 cu. in.

NOTE 2. Fruits, vegetables, corn in the ear, coal, and other bulky articles, are sold by the *heaped bushel*, whose standard capacity is 2747.71 cu. in. It is customary to consider 5 stricken bushels equal to 4 heaped bushels.

NOTE 3. The standard liquid gallon contains a capacity of 231 cu. in.

383. To find the capacity of any enclosed space in bushels or gallons.

1. A grain bin is 12 ft. long, 4 ft. wide, and 3 ft. 6 in. deep. How many bushels of wheat will it hold without heaping?

Operation.

12 cu. ft. $\times 4 \times 3\frac{1}{2} = 168$, contents in cu. ft.
1728 cu. in. $\times 168 = 290304$, contents in cu. in.
$290304 \div 2150.42 = 134.905+$, stricken bushels.

RULE. *Divide the contents, expressed in cubic inches, by 2150.42 to find bushels stricken measure; by 2747.71 to find bushels heaped measure; or by 231 to find number of gallons liquid measure.*

2. What part of a bushel of grain will fill a box whose inside capacity is 1 cu. ft.? What part of a bushel of potatoes will fill the same box even full? Assuming the same box to be water-tight, how many gallons of water will it hold?

OPERATION.

$1728 \div 2150.42 = .8$ bu. *practically*
$1728 \div 2747.71 = .63$ bu. *practically*
$1728 \div 231 = 7\frac{1}{2}$ gal. *practically*

384. A careful inspection of the foregoing operations will reveal the fact that a *practical* solution may be reached according to the directions given in the following

RULE. *Multiply the capacity, expressed in cubic feet, by .8 to find stricken bushels; by .63 to find heaped bushels, and by $7\frac{1}{2}$ to find number of liquid gallons.*

NOTE. The logical relations would make the quantity contained in 1 cu. ft. of space the multiplicand, and the capacity the multiplier, but

results are more conveniently obtained by following the directions contained in the rule.

EXAMPLES FOR PRACTICE.

(Accurate answers to examples 3 to 8 inclusive are required. Apply 383.)

3. How many bushels of grain will exactly fill a box 5 ft. 8 in. long, 3 ft. 10 in. wide, and 3 ft. 2 in. deep?

4. A potato bin is 8 ft. 4. in long, 3 ft. 6 in. wide, and 2 ft. 3 in. deep. How many bushels of potatoes will it hold when even full?

5. A grain bin 6 ft. 4 in. long and 3 ft. 9 in. wide contains 45 bushels of oats. What is the average depth of the oats in the bin?

6. A watering trough is 8 ft. 6 in. long, 2 ft. wide, and 1 ft. 2 in. deep. How many gallons of water does it contain when full?

7. How many bushels of apples can a man carry to market in a single load, without heaping, if his wagon-box is 10 ft. 2 in. long, 3 ft. 8 in. wide, and 20 in. deep?

8. A rectangular water tank holds 1000 gallons of water. How many bushels of wheat will be required to fill it?

(Practical solutions (384) are required for the remaining examples.)

9. How many bushels of corn, in the ear, will fill a wagon-box just even full, the dimensions of the box being $9\frac{1}{2}$ ft. $\times$ $3\frac{1}{2}$ ft. $\times$ $1\frac{3}{4}$ ft.?

10. A grocer bought a sufficient quantity of blueberries to fill a box having a capacity of $6\frac{1}{2}$ cu. ft. at \$2.50 per bushel, and sold them at 10 cents per quart liquid measure. How much did he gain?

11. How many gallons of water will fill a rectangular cistern 6 feet *square* and $5\frac{1}{2}$ feet deep?

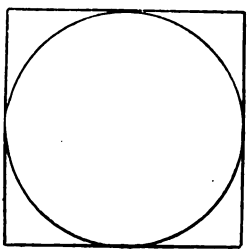
12. What must be the depth of a cistern 5 feet square to hold 1200 gallons of water?

13. How many pounds of shelled corn will fill a box whose inside measurements are 4 ft. 3 in. long, 3 ft. 8 in. wide, and 3 ft. deep?

14. How many bushels of corn in the ear will fill a crib 12 feet long, 3 ft. wide at the bottom, 5 ft. 4 in. wide at the top, and 6 ft. deep?

NOTE. The average width of a flaring crib is one-half the sum of top and bottom widths.

385. A cylindrical vessel is one whose ends are equal and parallel circles, and whose sides are uniformly round.



Inscribed Circle

REMARK. It can be demonstrated by geometry that the area of a circle inscribed in a square contains .7854 of the surface of the square; therefore it is evident that a cylindrical vessel, having a diameter equal to a side of a given square vessel, will contain .7854 of the capacity of the square vessel.

15. How many gallons of water will fill a cylindrical cistern $5\frac{1}{2}$ ft. deep and 4 ft. in diameter?

SOLUTION. $4\times4\times5\frac{1}{2}=88$ cu. ft., contents of a square cistern whose sides are each equal to the given diameter. $88\times.7854=69.1152$ cu. ft., contents of cylindrical cistern. $7\frac{1}{2}$ gal.$\times69.1152=518.364$ gallons.

16. A cylindrical reservoir is 36 ft. in diameter and 20 ft. deep; how many gallons of water will it hold when full?

17. How many gallons of water in a cylindrical cistern 5 ft. 3 in. in diameter, the depth of the water being 3 ft. 9 in.?

18. I have a round cask of uniform size, whose diameter is 32 inches and depth 3 ft. How many bushels of oats *will it* hold when even full?

19. How many cubic yards of earth were removed in *digging* a well 32 feet deep and 7 feet in diameter?

PERCENTAGE AND ITS APPLICATIONS.

PERCENTAGE.

386. Per Cent. is a contraction of the Latin phrase *per centum*, and signifies *by the hundred*.

The expression 5 per cent. signifies 5 of each hundred of the number to which it refers. 8 per cent. of $240 signifies $8 of each $100 contained in $240. 12 per cent. of 400 bushels signifies 12 bushels of each 100 bushels contained in 400 bushels, or 12 hundredths of 400 bushels.

387. Percentage is a term derived from *per cent.* and signifies a computation, or reckoning, *by the hundred*. The term is also employed (as shown hereafter) to name one of the elements of a percentage computation.

NOTATION.

388. In commercial transactions, the character, %, is generally employed to represent the words *per cent.*; thus, 5% = 15 per cent.

389. Since any per cent. is a corresponding number of hundredths, it may be properly expressed by a *decimal fraction;* thus, .24 = 24 per cent.

REMARK 1. When the commercial symbol is employed in the notation, the per cent. is said to be expressed *commercially*. When a decimal fraction is used, the per cent. is said to be expressed *decimally*.

REM. 2. In writing per cent. decimally, it is evident that neither more nor less than *two decimal places* can occur; hence any fractional part of one per cent. must be expressed as a common fraction. *Twelve and one-half per cent.* written as a simple decimal, .125, expresses *thousandths*, which is contrary to the meaning of the term *per cent.*

390. A Decimal Equivalent of any per cent. is a simple decimal fraction having the same value. The decimal equivalent of .05½ is .0525.

391. A Common Fraction Equivalent of any per cent. is a common *fraction* of the same value, expressed in its

most simple form. The common fraction equivalent of .25 is $\frac{1}{4}$.

Both forms of expression, together with decimal and common fraction equivalents, are presented in the following

TABLE.

	Commercial.	Decimal.	Dec. Equiv.	Com. Frac. Equiv.
1 per cent. =	1% =	.01	= .01	= $\frac{1}{100}$
2 per cent. =	2% =	.02	= .02	= $\frac{1}{50}$
5 per cent. =	5% =	.05	= .05	= $\frac{1}{20}$
8 per cent. =	8% =	.08	= .08	= $\frac{2}{25}$
10 per cent. =	10% =	.10	= .10	= $\frac{1}{10}$
$12\frac{1}{2}$ per cent. =	$12\frac{1}{2}$% =	$.12\frac{1}{2}$	= .125	= $\frac{1}{8}$
$16\frac{2}{3}$ per cent. =	$16\frac{2}{3}$% =	$.16\frac{2}{3}$	=	= $\frac{1}{6}$
25 per cent. =	25% =	.25	= .25	= $\frac{1}{4}$
$33\frac{1}{3}$ per cent. =	$33\frac{1}{3}$% =	$.33\frac{1}{3}$	=	= $\frac{1}{3}$
50 per cent. =	50% =	.50	= .50	= $\frac{1}{2}$
$62\frac{1}{2}$ per cent. =	$62\frac{1}{2}$% =	$.62\frac{1}{2}$	= .625	= $\frac{5}{8}$
100 per cent. =	100% =	1.00	= 1.00	= $\frac{100}{100} = 1$
125 per cent. =	125% =	1.25	= 1.25	= $\frac{5}{4} = 1\frac{1}{4}$
$\frac{1}{2}$ per cent. =	$\frac{1}{2}$% =	$.00\frac{1}{2}$	= .005	= $\frac{1}{200}$
$\frac{3}{4}$ per cent. =	$\frac{3}{4}$% =	$.00\frac{3}{4}$	= .0075	= $\frac{3}{400}$
1000 per cent. =	1000% =	10.00	= 10.00	= $\frac{1000}{100} = 10$
$\frac{1}{1000}$ per cent. =	$\frac{1}{1000}$% or .001%		$= .00\frac{1}{1000}$ = .00001	= $\frac{1}{100000}$

EXAMPLES FOR PRACTICE.

1. Express decimally 3 per cent.; 7 per cent.; 14 per cent.; 35 per cent.; 87 per cent.; 137 per cent.; 250 per cent.; 500 per cent.; 1250 per cent.

REMARK. To express any per cent. commercially, it is only necessary to substitute the commercial symbol for the words *per cent.*

2. Change to its decimal equivalent $.15\frac{3}{8}$.

SOLUTION. $\frac{3}{8} = .375$ (of the preceding order). Substituting this decimal for the common fraction gives .15375.

3. Change to decimal equivalents $.12\frac{2}{4}$, $.05\frac{1}{2}$, $.17\frac{5}{16}$, $1.34\frac{2}{4}$, $.21\frac{5}{32}$.

4. What per cent. is .1575?

SOLUTION. $.75 = \frac{75}{100} = \frac{3}{4}$. Substituting in place of the decimal reduced *gives* $.15\frac{3}{4}$ *or* $15\frac{3}{4}$%.

5. Reduce each of the following expressions to its quivalent per cent.: .18125, .43625, 24.12875.

6. Change 1.125% to the decimal form of expression.

Solution.

$.125\% \times 100 = 1.125$

$.125 \div 100 = .01125 = .01\frac{1}{8}$

EXPLANATION. Since the symbol % expresses 100ths, it is evident that its removal multiplies by 100. Dividing the result obained by the removal of the symbol by 100 restores value and gives ecimal equivalent .01125, which is equal to $.01\frac{1}{8}$.

7. Express the per cent. of each of the following values ecimally: 15.375%, 256.625%, 1.57%, .003%.

8. Express commercially the value of .0325.

EXPLANATION. Moving the decimal point two places toward the right ultiplies by 100 and gives for a result 3.25. Placing the commercial ymbol at the right restores value as it divides by 100, and we have the orrect result, 3.25%.

9. Express commercially each of the following values: 27, .32, $.33\frac{1}{3}$, .875, .003, .14375, .1025, $.06\frac{3}{7}$, $.00\frac{6}{11}$, $005\frac{2}{13}$.

PERCENTAGE COMPUTATIONS.

392. In the computations of percentage there are three arts or elements, namely: *Base*, *Rate per cent.*, and *Per-ntage*.

393. The **Base** is the number on which the number of undredths expressed is computed. In the expression, 6% f 120, 120 is the base, since it is the number on which the $\frac{6}{100}$ is computed.

394. The **Rate per cent.** is the number which denotes ow many hundredths of the base are to be taken.

NOTE. A rate may be any part of the base, as 12ths, 50ths, or thouandths, but a rate *per cent.* must express *hundredths*.

395. The **Percentage** is the number that denotes the alue of the hundredths taken of the base.

In the percentage computation, $120 \times .25 = 30$, a certain number of undredths are taken of 120, therefore 120 is the base. The multiplier xpresses the number of hundredths of the base to be taken, hence it is a

rate. The value of the hundredths taken is represented by 30, hence 30 is the percentage.

REMARK 1. An examination of the computation shown above will reveal the fact that the *percentage* is a *product* of which the *base* and *rate* are the *factors*. Those who understand the relations between product and factors, will easily comprehend the following formulas:

1. Base $\times$ Rate = Percentage.
2. Percentage $\div$ Base = Rate.
3. Percentage $\div$ Rate = Base.

REM. 2. Considering the analytical relations of percentage elements, the *base* corresponds to *unit value*, the rate to the *number of units*, and the *percentage* to *aggregate value*. (121.)

REM. 3. Since every base is a multiplicand, computations thereon must be by multiplication; and as percentages are products or dividends, all computations upon percentages must be by division.

REM. 4. A careful study of the language used in a question in percentage will enable the student to decide, in most cases, with little difficulty, what elements are given and what one is required. The base sustains such a relation to the rate, (which is readily known), as to make it easily identified; in the expression, "12% *of*", "12% *more than*", or "12% *less than*", an answer to the question, "12% of what?" or "12% more than what?" or "12% less than what?" as the case may be, will at once reveal the base; and when the base is known, it is evident that any other number, representing an element, except rate, is a percentage.

396. The formulas given, (Rem. 1), are sufficiently clear in direction, hence further rules are deemed unnecessary.

1. A clerk's monthly income was \$80, 15% of which he paid for a coat. What was the cost of the coat?

Operation.

$$\begin{array}{r} \$80 \\ .15 \\ \hline 400 \\ 80 \\ \hline \$12.00 \end{array}$$

EXPLANATION. The expression, "*monthly income*" furnishes an answer to the question, "15% *of what?*" and is therefore the base, whose value is \$80. Two related facts being known, namely, the *base* and *rate*, we ascertain the other element, the percentage, by formula 1.

REMARK. It frequently occurs that two or more percentages may have relation to the same base, and, since the rate expresses the mathematical relation between a base and its percentage, it is evident that as many different rates will exist as there are percentages. In the question solved and explained above, the *sum remaining* of monthly income constitutes *another* percentage, whose corresponding rate is 100%—15%=85%.

2. Wolves killed 12% of a flock of 250 sheep. What number of the flock remained?

Operation.

250
.88
2000
2000
220.00

EXPLANATION. Applying the test question, we readily see that the flock (250 sheep) is the base. Two percentages may be made to appear, the *number killed*, which is 12% of the flock, and the *number remaining*, which is 100%—12%=88% of the whole flock. Since the number remaining is asked for, it is evident that 88% is the normal rate.

3. The original capacity of a mill was 450 barrels per day, but by the addition of new machinery the capacity has been increased 18%. What is the present capacity of the mill?

Operation.

450
1.18
3600
450
450
531.00

EXPLANATION. The base is found, by previous instructions, to be the original capacity (450 brl.). Since present capacity is required, it is evident that 18% is not the normal rate, as it represents *increase*. The normal rate must represent *original capacity* + increase (100% + 18%), which is 118%, the rate by which we multiply the base.

REMARK. The normal rate is the rate *of* the base that will produce the required percentage. When the expression is a certain rate per cent. *more than* a given base, the normal rate is 100%+the expressed rate, and when the expression is a certain rate per cent. *less than* a given base, the normal rate is 100%—the expressed rate.

4. A farmer sold 400 bushels of wheat, which was 32% of his harvest. What was the number of bushels harvested?

Operation.

.32) 400.00 (1250
32
80
64
160
160
0

EXPLANATION By applying the test question to the rate, we find that the number of bushels harvested is the *base*, which is to be found by the operation; hence 400 bushels is a percentage, which must be divided by the normal rate, 32% (Formula 3).

5. A farmer harvested 520 bushels of corn and sold 130 bushels of the same. What per cent. of the amount harvested did he sell?

Operation.

520)130.00(.25
104 0
26 00
26 00

EXPLANATION. The test question applied to the expression *per cent.* reveals the fact that the number of bushels harvested (520) is the *base*, therefore we must divide the percentage, 130 bushels, by it to determine the *rate* (Formula 2).

EXAMPLES FOR PRACTICE.

6. Find 7% of $13.50. *7.* 32% of 460 miles. *8.* 17% of 1512 tons. *9.* 84% of 240 bushels. *10.* 125% of 800 men. *11.* 30% of 12 lb. 9 oz. 15 pwt. *12.* $\frac{1}{8}$% of 18.

13. What number is 13% more than $56? *14.* 45% more than 150 acres? *15.* .25 less than 280 lb.? *16.* .3% of $960? *17.* .02% less than 68000?

NOTE. Use decimal or common fraction equivalents in place of rates when more convenient.

18. What is the value of a farm that is worth to-day 16% more than one year ago, if its value at that time was $4800?

19. A capitalist is worth $62500, 42% of which is invested in stocks and the remainder in real estate. What is the value of his real estate?

20. A speculator lost 37% of his capital in a mining speculation and had $28350 remaining. How much did he lose?

21. What rate per cent. of 256 is 1024?

22. The number 292.95 is what rate per cent. less than the number 315?

23. A man sold a horse for $92, which was 15% more than cost. How much was his gain?

24. What per cent. of 48 yards is 12 yards?

25. What per cent. of 12 yards is 48 yards?

26. What per cent. more than 240 is 300?

27. What per cent. less than 300 is 240?

28. What per cent. of $\frac{3}{4}$ is $\frac{5}{32}$?

29. A man's salary is $1350 a year; if he pays 16% of the amount for board, 8% for clothing, 5% for books, and 7% for incidentals, what are his yearly expenses?

NOTE. 16%+8%+5%+7%=36%. In all cases where several rates refer to the *same* or *equal bases*, they may be added or subtracted, as the conditions of the question given may require.

30. A depositor had $4500 in a bank. He drew out 20% of it, then 15% of the remainder, and afterward deposited a sum equal to 5% of the amount withdrawn: how much had he then in bank?

31. What per cent. of a number is 40% of $\frac{7}{8}$ of it?

32. A man drew 30% of his bank deposits and invested 80% of the sum withdrawn in the purchase of a horse and carriage which cost him $270. How much had he remaining in bank?

33. A deposited 20% of $\frac{7}{8}$ of his money in a bank, which amount was 16% of $\frac{1}{3}$ of B's money. How much money had A, B's total amount being $1837.50?

34. After removing 15% of the grain from a bin, there remained 40 bu. 3$\frac{1}{8}$ pk. How many bushels were in the bin before the removal?

35. After selling 36% of my land, I had 224 acres left; how much land had I at first?

397. ADDITIONAL EXAMPLES FOR REVIEW:

1. What is 12$\frac{1}{2}$% ($\frac{1}{8}$) of 144? *2.* 33$\frac{1}{3}$% ($\frac{1}{3}$) of 96? *3.* 20% ($\frac{1}{5}$) of $375? *4.* 16$\frac{2}{3}$% ($\frac{1}{6}$) of 252 feet?

NOTE 1. When the rate per cent. is an aliquot part of 100, employ the common fraction equivalent.

5. What is 62$\frac{1}{2}$% more than ($\frac{1}{8}$ of) 720? *6.* 12$\frac{1}{2}$% less than ($\frac{7}{8}$ of) 560? *7.* 83$\frac{1}{3}$% ($\frac{5}{6}$) of 366? *8.* 16$\frac{2}{3}$% more than ($\frac{7}{6}$ of) 84?

9. A man paid $315 for a horse, carriage and harness. The carriage cost 25% more than the horse, and the harness cost 70% less than the carriage. What was the cost of each?

NOTE 2. When rates apply to different bases they must be brought to bear upon the same base before they can be added or subtracted. In the solution of the above question, the cost of the horse is the natural base upon which all rates must be brought to bear, as shown by the following solution:

Cost of horse= 100 % of cost of horse.
Cost of carriage= 125 % of cost of horse.
Cost of harness .30 of 1.25= $37\frac{1}{2}$ % of cost of horse.
.Cost of all= $262\frac{1}{2}$ % of cost of horse.
$315 (percentage) ÷ $2.62\frac{1}{2}$ (rate) = $120 (base).

10. A merchant's sales on Tuesday exceeded his Monday sales by 20%; his Wednesday sales were 10% more than his Tuesday sales; his sales on Thursday were 25% greater than on Wednesday; his sales on Friday were equal to his Thursday sales; and his sales for Saturday were equal to the sum of his sales for Monday and Tuesday. His sales for the week amounted to $338.25; what was the amount of his Monday sales?

11. A has $640, and $12\frac{1}{2}$% of his money equals 16% of B's money; how much money have both?

12. 28% of a man's wealth is in government bonds, 22% in real estate, 35% in mining stock, and the remainder in money. If his money amounts to $4875, what is the value of his real estate?

13. 30% of A's age equals $12\frac{1}{2}$% of B's age. The sum of their ages is 68 years; how old is A?

14. If a farmer harvests 510 bushels of wheat from a field containing 30 acres, and the rate of seed sown was $1\frac{1}{2}$ bushels per acre, what is the rate of increase?

15. A man bought a horse and carriage, paying 25% *more* for the horse than for the carriage; had the horse

cost 10% more, and the carriage 5% more, the cost of both would have been $291. What was the cost of each?

16. A freight train, whose running time is 24 miles per hour, starts from Chicago and runs on schedule time, which is 37½% less than its running time; a passenger train, whose running time is 75% greater than that of the freight, leaves Chicago 2 hours and 20 minutes later than the freight and follows after; if 16⅔% of the running time of the passenger train is consumed in stops at stations along the line, at what distance from Chicago will she overtake the freight train?

17. Of the population of a certain city, 76% are native born, and the remainder are of foreign birth; the foreign born are what per cent. of the native born?

18. A man engaged in the mercantile business, and at the close of each year he increased his capital by his gains; at the close of the fourth year he retired with double the capital he originally invested. Assuming that his gains amounted to the same sum each year, what was the rate per cent. of gain during the fourth year?

19. A grain dealer's shipments for October were 50% greater than for September; his shipments for November were 10% greater than during the former month, and his shipments for December were 40% less than for November: If his shipments during the last two months exceeded his shipments for September and October by 833 bushels, how many bushels did he ship during the month of November?

20. A and B rented 5 acres of land which they planted to potatoes, agreeing that 40% of the income should be divided equally and that the remainder should be divided in proportion to the sums respectively invested; A paid for seed and incidentals $5.60, and B paid the rental, and incidental expenses amounting to $22.40. The yield was

85 bushels per acre, and they sold the entire crop at 60 cents per bushel. How much should each receive?

21. A coal dealer received an order for a ton (2000 pounds) of coal, but, by mistake, a long ton was delivered; what was the per cent. of loss caused by the error?

22. If a piece of cloth 12 yards long and 46 inches wide shrinks $4\frac{1}{2}$ inches in length and 1 inch in width, what per cent. is it reduced in area?

23. A merchant bought a bill of goods invoiced at $1200 on 6 months' credit; the terms of the bill provided that if payment should be made within 10 days a discount of 10% would be allowed, and if payment should extend beyond ten days, but should be made within 30 days, a discount of 6% would be allowed. The bill was dated June 18, and payment was made July 14, thereafter; what sum paid the bill?

24. If flooring is reduced $16\frac{2}{3}\%$ in surface by matching, how many feet will be required for the floor of a room 18 ft. wide and 24 ft. long?

25. A speculator lost 15% of his money in a stock speculation, and sustained an additional loss of 20% of the remainder in a mining venture, when he found that he had but $4428.16 remaining. What was the sum total of his losses?

26. A dealer invested equal sums in the purchase of flour and bacon; he sold the flour at a profit of 8%, and the bacon at a profit of 12%. If his gain on the bacon exceeded the gain on the flour by $32.60, what sum was invested in the purchase of each?

27. B's age exceeds A's age by 50%, but is less than C's age by 10%. The sum of their ages is 50 years; how old is C?

28. A man owning $\frac{7}{8}$ of a ship sold 15% of his share

to B, and $\frac{1}{4}$ of his remaining interest to C at the same rate, for \$8181.25. What was the value of the ship?

29. Each of two men, A and B, desired to sell his horse to C. B's asking price was 50% more than A's; A then reduced his price 20%, and B his price 30%, at which prices C took both horses, paying for them \$148; what was each man's asking price?

30. If B can do 20% more work in a given time than A, how long a time will be required for both, working together, to perform a job of work that A can do alone in 9 days?

31. A's money is 5% less than B's, and 10% of A's added to 12% of B's is equal to \$23.80. How much has each?

LOSS AND GAIN.

398. Loss, in a commercial sense, is the excess of *cost* above *production.*

399. Gain, in a commercial sense, is the excess of *production* above *cost.*

REMARK 1. The seller of merchandise, or other property, is called the *vendor*, and the buyer the *vendee.*

REM. 2. The *prime cost* (first cost) is the sum paid to the vendor, and does not include expenses which may attend the purchase.

REM. 3. The *gross cost* is the prime cost plus all attending expenses. If a man buys a piano and pays the seller therefor \$450, and, in addition, pays a freight and dray bill amounting to \$15, the *prime cost* is \$450, and the *gross cost* is \$450 + \$15 = \$465.

REM. 4. It will be readily seen that all computations in loss and gain must depend upon the application of percentage principles, the *gross cost* being the *base*, and all other sums, concrete in character and representing percentage elements, being *percentages.* The foregoing explanations will render clear the following formulas, which will serve the purpose of rules:

CLASS 1—PERCENTAGES REQUIRED.

1. Gross cost × Rate of Loss = Loss.
2. Gross cost × Rate of Gain = Gain.
3. Gross cost × Rate of selling price = Selling price.

CLASS 2—RATES REQUIRED.

1. Loss ÷ Gross cost = Rate of Loss.
2. Gain ÷ Gross cost = Rate of Gain.
3. Selling price ÷ Gross cost = Rate of selling price.

CLASS 3—BASES REQUIRED.

1. Loss ÷ Rate of Loss = Gross cost.
2. Gain ÷ Rate of Gain = Gross cost.
3. Selling price ÷ Rate of selling price = Gross cost.

REM. 5. Notice that in all cases where a percentage is to be divided by a rate, no rate can be used as a divisor except it be of the same character as the percentage to be divided. *Loss* can be divided by no rate except *rate of loss*. *Selling price* can be divided only by *rate of selling price*, etc.

EXAMPLES FOR PRACTICE.

What is the loss on goods which cost

1. \$360 and are sold at 5% loss?
(Class 1, formula 1.)

2. \$315.75 and are sold at 3% loss?

3. \$52.16 and are sold at 12% loss?

4. \$4.50 and are sold at 35% loss?

What is the gain on goods which cost

5. \$324 and are sold at 8% profit?
(Class 1, formula 2.)

6. \$1532.50 and are sold at 15% profit?

7. \$628 and are sold at 25% profit?

8. \$13.20 and are sold at $12\frac{1}{2}$% profit?

9. \$212.25 and are sold at $33\frac{1}{3}$% profit?

10. What is the selling price of an article which cost \$220 and is sold at 6% loss?

REM. 6. When goods are sold at a loss, the *rate of selling price* is, 100%—the *rate of loss*. When goods are sold at a profit, the *rate of selling price* is 100% + the *rate of gain*. The loss or gain may be first found and subtracted from, or added to, the cost, as the conditions may require, but we believe the relations of the elementary parts will be more clearly understood by requiring a direct solution; thus:

\$220 (cost) × .94 (rate of selling price) = \$206.80 (selling price).
(Class 1, formula 3.)

11. What is the selling price of an invoice of sugar ɔught for $175 and sold at 5% profit?

12. In order to realize a gain of 16%, at what price ıust I sell a horse which cost me $90?

13. A merchant paid $560 for an invoice of carpeting, hich he afterward sold at a loss of 3½%; what was the ɜlling price?

14. A man paid $6500 for a farm, appropriated $720 ɔr improvements, and sold the property at a gain of 7½%; ow much did he receive?

15. A man invested $1350 in the purchase of wool, aid a commission agent $27 for making the purchase, and lso paid freight and other expenses amounting to $45. If e sell the wool at a gain of 18%, how much will he eceive?

What is the per cent. of loss on goods which cost

16. $840 and are sold at a loss of $75.60?
(Class 2, formula 1.)

17. $18.50 and are sold at a loss of $2.22?

18. $65.50 and are sold at a loss of $9.17?

19. $356 and are sold at a loss of $44.50?

20. $1520 and are sold at a loss of $4.56?

What is the per cent. of gain on goods which cost

21. $432.50 and are sold at a profit of $43.25?
(Class 2, formula 2.)

22. $23.75 and are sold at a profit of $4.75?

23. $960 and are sold at a profit of $320?

24. $1750 and are sold at a profit of $8.75?

What is the per cent. of selling price on goods which cost

25. $1600 and are sold for $1200?
(Class 2, formula 3.)

26. $1420 and are sold for $1462.60?

27. $36.25 and are sold for $43.50?

28. $4.50 and are sold for $13.50?

29. $90 and are sold for $89.91?

What is the per cent. of loss or gain on goods which cost

30. $820 and are sold for $779?

31. $1260 and are sold for $1423.80?

Selling price ÷ cost = rate of selling price,
and 100% — rate of selling price = rate of loss,
and rate of selling price — 100% = rate of gain.

Or, (cost — selling price) ÷ cost = rate of loss.
(selling price — cost) ÷ cost = rate of gain.

32. $3420 and are sold for $4343.40?

33. $560 and are sold for $559.30?

What is the gross cost of goods which are sold at a loss

34. Of $12, if the rate of loss is 3%?

(Class 3, formula 1.)

35. Of $84, if the rate of loss is $12\frac{1}{2}$%?

36. Of $136, if the rate of loss is 17%?

37. Of $1.80, if the rate of loss is .02%?

What is the gross cost of goods which are sold at a gain

38. Of $30.15, if the rate of gain is $33\frac{1}{3}$%?

(Class 3, formula 2.)

39. Of $12.40, if the rate of gain is 40%?

40. Of $1.50, if the rate of gain is $\frac{1}{6}$%?

41. Of $5, if the rate of gain is 1000%?

42. A man bought a house for $4250, paid $600 for repairs, and $24 for insurance, and then sold it at $8\frac{1}{2}$% profit. How much did he gain?

43. If a man pays $280 for a horse and carriage and sells the same for $315, how much does he gain per cent.?

44. A man sold a piano for $350 and lost $16\frac{2}{3}$%. What did the piano cost him?

45. A grain dealer bought barley at 45 cents per bushel and sold it at 10% loss. How many bushels did *he sell,* the total loss being $43.20?

46. If a merchant sells cloth at $4.50 per yard and gains 20%, at what price per yard was the cloth purchased?

47. If a grocer buys coffee at 15 cents per pound and sells the same at 20 cents, how much does he gain per cent.?

48. If the cost of coffee is 20 cents per pound, and the selling price 15 cents, what is the per cent. of loss?

49. If a coal dealer buys 50 long tons of coal at $7.50 per ton and sells the same at $8 per short ton, what is his gain per cent.?

50. If a stationer buys paper at $3.20 per ream, and retails it at 20 cents per quire, how much does he gain per cent.?

51. A dealer bought a quantity of hops and paid a commission merchant $25 for negotiating the purchase, and afterward paid freight and other attending expenses amounting to $65. He sold the entire invoice for $1539, and gained 14%. What was the prime cost?

52. A farmer purchased a flock of 80 sheep at $5 per head; 5% of the number were killed by wolves and 6 died from disease. If he sold the remainder at $6.40 per head, how much did he gain per cent.?

53. A grocer paid $250 for a quantity of potatoes and sold 60% of the purchase at 15% profit, and the remainder at 5% loss. What were his total receipts from sales?

54. A man sold a quantity of lumber for $3120 and thereby lost 4%. What was the amount of his loss?

55. A man sold a quantity of oats for $32.20, and gained $4.20. What was his rate per cent. of gain?

56. The invoice price of an organ bought of an agent was $95; the expenses attending the purchase amounted to $12. The organ was afterward sold for $115.56; what was the rate per cent. of gain?

57. If a city lot increase in value 15% per annum dur-

ing three successive years, and is then worth $3041.75, what was its value at the beginning of the first year?

58. A farmer sold two horses at $80 each; on one he gained 25%, and on the other he lost 25%. Did he gain or lose on the sale of both, and what amount?

59. A grocer bought 20 cases of eggs at 15 cents per dozen. If each case contained 30 dozen, and he sold 124 dozen at 18 cents per dozen, 380 dozen at 20 cents per dozen, and the remainder for $20.68, what was his rate per cent. of gain?

60. An apothecary forms a compound consisting of 1 gallon of brandy worth $4, 1 quart of water, and other ingredients amounting to 1 pint and worth $1.60. If he sells the mixture at 5 cents per fluid ounce, what is the rate per cent. of gain?

400. ADDITIONAL EXAMPLES FOR REVIEW:

1. A gain of 18% on 40% of an invoice is what rate per cent. of gain computed on the whole invoice?

2. A loss of 8% on 25% of an invoice is what rate per cent. of loss if computed on 80% of the whole invoice?

3. A dealer sold 50% of an invoice of sugar at a profit of 8%, and the remainder at a profit of 12%. What was his rate of gain on the entire invoice?

4. A provision merchant marked flour at a profit of $33\frac{1}{3}$%, but sold at a price $12\frac{1}{2}$% below the marked price and realized $7 per barrel. How much did the flour cost him per barrel?

5. A jobber shipped to a commission house in New York a quantity of sugar consisting of 150 brl. of New Orleans, each containing 280 lb., purchased at 5 cents per pound, and 250 brl. Cuban, each containing 320 lb., purchased at $5\frac{1}{2}$ cents per pound. The account of sales received from the commission house shows a loss of $\frac{1}{2}$% on

the New Orleans, and a profit of $\frac{1}{8}\%$ on the Cuban sugar. What was the per cent. of gain on the whole consignment?

6. If I make a profit of $18\frac{3}{4}\%$ by selling broadcloth for \$.75 per yard above cost, how much must I increase the selling price to realize a gain of $24\frac{1}{2}\%$?

7. A dealer sold 30% of an invoice at 20% profit, and $\frac{5}{7}$ of the remainder at 15% profit; the part remaining after the second sale was sold at 5% loss. If the net gain on the whole invoice amounted to \$88.45, what was the cost?

8. A wine merchant bought a cask of wine, containing 35 gallons, for \$87.50; a part having leaked out, he sold the remainder for \$3 per gallon, and found his loss to be 4% of the cost. How many gallons leaked out?

9. A fruit dealer sold 40% of an invoice of apples at 20% profit, 30% of the invoice at 12% profit, and 20% of the invoice at 10% profit; the remaining 10% proving worthless, he gave a drayman 4% of their cost to remove them from his cellar. Did he gain or lose on the whole invoice, and what rate per cent.?

10. A man sold his carriage at a gain of 25%, and with the procceds of the sale he bought another, which he sold for \$135 and lost 10%. What was the cost of each carriage?

11. A speculator bought government securities at a price 10% below their face value, and sold them at a price $3\frac{1}{2}\%$ above their face value. What was his rate per cent. of gain?

12. If I buy an article and sell it for \$54, and realize a profit of $11\frac{1}{9}\%$ of that sum, what is my gain per cent.?

13. If I sell $\frac{3}{4}$ of a barrel of flour for what I paid for all of it, what is my gain per cent.?

14. If I sell $\frac{3}{4}$ of a ton of coal for the sum paid for $\frac{5}{8}$ of a ton, how much do I lose per cent.?

15. A merchant invested equal sums in the purchase of Rio and Java coffee; he sold the Rio at 18% profit and

the Java at 15% profit. What were the proceeds of the sales if the difference in gain on the two grades amounted to $9.75?

16. An Indianapolis firm bought from a merchant in Liverpool, England, 225 yards of silk invoiced at £54 12s. 6d.; the duty amounted to 40% of the invoice price, and the transportation to $37.92; the bill at Liverpool was paid by a bill of exchange bought at $4.80 per £. At what price per yard must the Indianapolis firm sell the silk to make a net gain of 25%?

17. A produce merchant sold 20% of an invoice of potatoes at 10% gain, and the remainder at 5% loss. If the net loss on the invoice amounted to $8.40, what were the total sales?

18. A stationer sold an invoice of pens at 60% profit and with the proceeds purchased pencils, which he sold at 50% profit; he invested the sum received from the sale of the pencils in paper, which he sold at 33⅓% profit. If the gain on the paper was $9.60, how much did he gain on the pens?

19. A boy bought a sled, a pair of skates, and a book, and paid the same sum for each; he sold the sled at 10% profit, the skates at 20% profit, and the book at a gain of 8%, and received from the sale of the three articles $5.07. At what price did he sell the skates?

20. A grocer invested a certain sum in the purchase of spices, and a sum greater by 50% in the purchase of canned goods; if he sold each at a gain of 12% and realized $4.20 more profit from the sale of the canned goods than from the sale of the spices, what was the cost of the canned goods?

21. Allowing 5% for shrinkage and waste, and 10% for bad debts, at what rate per cent. above cost must goods *be sold* that a net profit of 25% may be realized?

COMMISSION AND BROKERAGE.

401. **Commission** and **Brokerage** are terms having reference to services performed by an agent for his principal; as, buying or selling property, collecting or investing money, etc.

REMARK 1. The *sum received* by the agent for transacting business for his principal is commonly called commission or brokerage, but the terms are not properly defined when we say they are the sums received by an agent for transacting business for others. The agent does not receive *commission;* he receives payment (usually money) *in consideration of commission.*

REM. 2. Payments made to commission agents or brokers are usually estimated at a per cent. of the gross proceeds of a sale or the prime cost of a purchase; but the charge made by the agent is sometimes a certain price per unit.

REM. 3. When credit sales, unauthorized by the principal, are made by the agent, the agent is lawfully responsible for amounts due upon such sales; but, when the principal authorizes credit sales, the agent is not responsible except by special contract, in which he charges a certain per cent. for assuming such responsibility. The charge thus made is called *guaranty.*

REM. 4. When the agent is placed in possession of the goods bought or sold, and acts in his own name, he is called a *commission merchant*; when he does not take possession, but simply contracts to buy or sell in the name of his principal, he is called a *broker.* The term *broker* is also applied to those who buy and sell stocks, bonds, and other securities for others in a similar manner.

REM. 5. A consignment is a shipment of merchandise to one party by another. The party making the shipment is called the *consignor*, and the one receiving the merchandise thus consigned is called the *consignee.*

402. The **Gross Proceeds** is a term applied to a sale or collection, and it is the total sum received by the agent.

403. The **Net Proceeds** is the amount belonging to the principal; it is the sum remaining after the commission and all other expenses are deducted from the gross proceeds.

REMARK. An *account sales* is a written statement rendered by an agent to his principal, specifying the goods sold, prices obtained, commission charged, *expenses incurred,* and exhibiting the balance due to or from *the principal, as the case may* be.

404. The **Prime Cost** of a purchase is the sum paid by the agent to the seller, and does not include commission or other charges.

405. The **Gross Cost** is the cost to the principal, and includes, in addition to the prime cost, commission and all other charges.

REMARK. An *account purchase* is a written statement rendered by an agent to his principal, specifying the property bought, prices paid, commission charged, expenses incurred, and exhibiting the gross cost of the purchase.

406. The computations in Commission and Brokerage, like those in Loss and Gain, are neither more nor less than the principles of percentage applied. The *gross proceeds of a sale*, or *the prime cost of a purchase*, being the *base*; and all other terms, concrete in character and representing percentage elements, being *percentages*.

REMARK. *Commission*, *Brokerage*, and *Guaranty* are percentage elements, since each is a value of some number of hundredths of the base. (395). Other expenditures, called *incidental* or *attending expenses*, such as freight, insurance, storage, advertising, etc., are not percentage elements, since they are not based upon values represented by either proceeds or cost. *Net proceeds* and *gross cost* are percentages when no expenditures occur other than commission, brokerage, or guaranty, but not otherwise. All expenses, other than those expressed by rates, must be added to the net proceeds or subtracted from the gross cost to obtain a result that is a percentage element. For convenience in analytical expression, we give this term the name of *commission proceeds* or *commission cost*, in harmony with the following definitions:

407. The **Commission Proceeds** is the result obtained by adding all expenses, except such as are expressed by rates per cent., to the net proceeds.

408. The **Commission Cost** is the result obtained by subtracting all expenses, except such as are expressed by rates per cent., from the gross cost.

REMARK. Since the commission proceeds and the commission are *together* equal to the gross proceeds (100%), it is evident that the rate of *commission* proceeds is equal to 100% — the rate of commission; and since

ɔmmission cost is the prime cost augmented by the commission, it is evident that the rate of commission cost is 100% + the rate of commission.

The foregoing explanation should furnish a clear understanding of the relations exhibited in the following formulas:

CLASS 1—SALES.

1. Gross proceeds×Rate of commission=Commission.
2. Gross proceeds×Rate of commission proceeds=Commission proceeds.
3. Commission÷Rate of commission=Gross proceeds.
4. Commission proceeds÷Rate of commission proceeds=Gross proceeds.
5. Commission÷Gross proceeds=Rate of commission.
6. Commission proceeds÷Gross proceeds=Rate of commission proceeds.

CLASS 2—PURCHASES.

1. Prime cost×Rate of commission=Commission.
2. Prime cost×Rate of commission cost=Commission cost.
3. Commission÷Rate of commission=Prime cost.
4. Commission cost÷Rate of commission cost=Prime cost.
5. Commission÷Prime cost=Rate of commission.
6. Commission cost÷Prime cost=Rate of commission cost.

EXAMPLES FOR PRACTICE.

1. What is the commission if the gross proceeds of a sale are $240 and the rate of commission 3%?

$240 (gr. pro.) × .03 (rate of com.) = $7.20 (com.)

What is the commission

2. If the gross proceeds are $1250 and rate of commission $2\frac{1}{2}$%?

3. If the gross proceeds are $75 and rate of commission 5%?

4. If the gross proceeds are $192 and rate of commission $1\frac{1}{4}$%?

5. If the prime cost is $3600 and rate of commission $\frac{3}{4}$%?

6. If the gross proceeds are $4800 and rate of commission $\frac{1}{8}$%?

7. If the prime cost is \$1200 and rate of commissio .3%?

What are the net proceeds of a sale if

8. The gross proceeds are \$480, the rate of commmis sion $2\frac{1}{2}\%$, and the other expenses \$8.75?

\$480 × .02½ = \$12. (com.)
\$480 — (\$12 + \$8.75) = \$459.25 (net proceeds).
Or \$480 × .97½ (rate of com. pro.) = \$468 (com. proceeds).
\$468 — \$8.75 (expenses) = \$459.25 (net proceeds).

9. The gross proceeds are \$560, the rate of commission $1\frac{1}{2}\%$, and the other expenses \$15.75?

10. The gross proceeds are \$12600, the rate of brokerage $\frac{5}{8}\%$, and the other expenses \$31.50?

11. The gross proceeds are \$3150, the rate of commission 3%, the rate of guaranty on $\frac{1}{3}$ the sale 2%, and the other expenses \$17.25?

What is the gross cost of the purchase if

12. The prime cost is \$24.50 and the rate of commission 4%?

13. The prime cost is \$420, the rate of commission $2\frac{1}{4}\%$, and the other expenses \$12.40?

14. The prime cost is \$8400, the rate of brokerage $\frac{3}{16}\%$, and all other expenses \$8.25?

15. The prime cost is \$35.20, the rate of commission $1\frac{3}{4}\%$, and the other expenses \$5.80?

16. The prime cost is \$14000, the rate of brokerage .15%, and the other expenses \$12?

17. What is the rate per cent. of commission if the prime cost is \$320, and the commission for buying \$12.80?

\$12.80 (commission) ÷ \$320 (prime cost) = 4% (rate of com.)

Remark. Since there are no expenses except commission, the *commission cost* is the *prime* cost.

What is the rate per cent. of commission if

18. The gross proceeds are \$450 and the commission \$5.40?

19. The gross proceeds are \$1350 and the commission \$31.50?

20. The prime cost is \$560 and the commission \$14?

21. The prime cost is \$760 and the commission \$19?

22. The gross proceeds are \$960 and the commission \$2.88?

23. The prime cost is \$840 and the commission \$7.35?

24. The prime cost is \$750 and the gross cost \$768.75?

25. The prime cost is \$660 and the gross cost \$669.90?

26. The gross proceeds are \$1480, and the net proceeds \$1476.30?

27. The gross proceeds are \$1500, expenses, other than commission, \$10, and net proceeds \$1460?

\$1500 (gross proceeds)—\$1460 (net proceeds)=\$40 (total expenses).
\$40 (total expenses)—\$10 (attending expenses)=\$30 (commission).
\$30 (commission)÷\$1500 (gross proceeds)=2% (rate of com.).

28. The prime cost is \$280, expenses, other than commission, \$5, and gross cost \$289.20?

\$289.20—\$280=\$9.20 (total expenses).
\$9.20—\$5 =\$4.20 (commission).

29. What are the gross proceeds of the sale if the commission is \$28 and the rate of commission 2%?

(Class 1, formula 3.)

What are the gross proceeds or prime cost if

30. The selling commission is \$13.50 and the rate 3%?

31. The selling commission is \$85.80 and the rate $2\frac{3}{4}$%?

32. The buying commission is \$18 and the rate $1\frac{1}{2}$%?

33. The brokerage for buying is \$20 and the rate $\frac{5}{16}$%?

34. The net proceeds are \$3152.50 and rate of commission 3%?

100%—3%=97% (rate of net proceeds).

35. The net proceeds are $1384.50 and rate of commission 2½%?

36. The gross cost is $659.80, expenses, other than commission, $15, and the rate of commission 4%?

$659.80 (gross cost)—$15 (expenses)=$644.80 (commission cost).
$644.80÷1.04 (rate of commission cost)=$620 (prime cost).

37. The net proceeds are $304.50, expenses, other than commission, $4.20, and the rate of commission 2%?

$304.50+$4.20=$308.70 (commission proceeds).
$308.70÷ .98 (rate of com. proceeds)=$315 (gross proceeds).

38. The net proceeds are $518, expenses, other than commission, $8.50, and the rate of commission 2½%?

39. The net proceeds are $12510, expenses, other than brokerage, $250, and the rate of brokerage $\frac{5}{16}$%?

40. The gross cost is $24235, freight $175, and rate of brokerage ¼%?

41. The gross cost is $698.50, expenses, other than commission, $15.40, and the rate of commission 3½%?

42. The gross cost is $126.20, the rate of commission 2½%, and the other charges $3.50?

43. The net proceeds are $536.65, the rate of commission 1½%, the rate of guaranty 2%, and the other charges $3.75?

44 A real estate agent sold a city lot on commission and remitted $2328 to his principal; how much did the agent receive for his services, his rate of commission being 3%?

45. An attorney received $34 for collecting a debt of $850; what was his rate of commission?

46. A grain dealer purchased in Chicago 5000 bushels of wheat at 90 cents, and shipped the same to an agent in Rochester, N. Y., who sold it at $1.15; what was the profit to the principal if he paid the agent 2% commission, *and paid* other expenses amounting to $240?

47. A speculator received $3990 as the net proceeds of a sale after allowing a commission of 5%; what was the sum paid by the purchaser?

48. A commission agent received from Gaylord & Phelps a consignment of merchandise which he sold on commission. The account sales rendered by the agent revealed the following facts: Cash sales, $1300; credit sales (sanctioned by principal), $500. $30 of the money received was refunded in exchange for damaged goods, which were resold for $10, cash. The agent paid a freight bill amounting to $45, and an advertising bill of $8.50; he charged $12 for storage, 2½% for commission, and 2% for guaranty. What was the principal's gain, if the goods cost him $1400?

49. A commission merchant sold a consignment of flour for $8540; he charged $84 for storage, and 4½% for commission. What were the net proceeds of the sale?

50. A retail dealer sent $510 to an agent in Boston to be invested in prints, after deducting his commission of 2%. If the price paid by the agent was 6¼ cents per yard, how many yards were purchased?

51. A commission merchant sells for Mott & Co. 540 barrels of flour at $6.50, on a commission of 2½%, and charges $7.80 for cartage; he invests the net proceeds, less his buying commission of 3%, in coffee at 15 cents. How many pounds of coffee do Mott & Co. receive?

52. A commission merchant, whose rate both for selling and buying is 5%, receives 24000 lbs. of sugar and $3000 in cash, with instructions to invest in cotton. If the price at which the commission merchant sells the sugar is 5 cents, what is his entire commission?

53. The net proceeds of a shipment of 300 tons of pressed hay, after deducting a commission of 3%, and

$250 for other charges, were $2078; what was the selling price per ton?

54. If a dealer sends a quantity of merchandise into the country to be sold at auction, on a commission of 6%, what amount of goods must be sold, that the agent may buy produce with the avails to the value of $2400 after retaining his buying commission of 4%?

55. A collector received a claim of $840, of which he collected 80%. What sum did he return to the principal, his rate of commission for collecting being 5%?

56. How much will a man realize from the sale of a farm if the real estate agent charges 2% commission, and the total charges, including $24 for an abstract, amount to $148?

409. ADDITIONAL EXAMPLES FOR REVIEW.

1. A commission merchant sold a consignment of 4500 bushels of corn at 40 cents per bushel, charged 4% commission for selling, paid freight and other expenses amounting to $150, and invested the net proceeds, less his purchasing commission of 2%, in coffee at 12 cents per pound; how many pounds did his principal receive?

2. Belden & Reid received from their agent a quantity of oats which they afterward sold at a profit of 12½%, thereby gaining 3⅜ cents per bushel. If the agent's commission of 3% amounted to $15.75, and the firm paid a freight bill, on receiving the oats, amounting to $26.25, how many bushels did the shipment contain?

3. A commission merchant received $858 to invest in flour after deducting commission and other charges. If the price paid for the flour was $5.40 per barrel, commission 5%, and drayage 50 cents per load of 10 barrels, how *many* barrels were purchased?

4. I sent $500 to a commission agent with instructions to invest in apples at $3.20, after deducting all expenses; the charges were, commission, 5%; drayage, 3 cents per barrel; freight, 15 cents per barrel; and all other expenses, $14.80. How many barrels did the agent purchase, and what was the unexpended balance of the remittance?

5. I sent to a commission agent in Charleston a sum of money with instructions to invest what remained, after deducting his commission of 4%, in cotton; after making the purchase as directed, he wired me that the market had advanced and that he could dispose of the purchase for a sum 20% in excess of the amount paid for it. I instructed him to sell and remit the net proceeds. If the agent charged 2% for selling, and the sum of his two commissions amounted to $128, how much did I gain on my investment?

6. A commission agent purchased for a retail fruit dealer 10 24-quart crates of strawberries and charged a commission of 5%. If the total expenses were $1.30 express, and 70 cents for commission, what will the retail dealer gain per cent. by selling the berries at 8 cents per quart?

7. A commission merchant sold a consignment of wool at 4% commission and invested the net proceeds, less his buying commission of 5%, in butter. What did he receive for the wool, if the sum of his two commissions was $144?

8. I received a carload of apples, bought by my agent at 5% commission; the freight and other expenses, in addition to commission, amounted to 45 cents per barrel. If I sold the apples at $3.78 per barrel and gained 5%, at what price did the agent buy them?

9. A broker in New York effected a purchase of tea for Knowlton & Kemp and charged for his services $84. The tea was consigned to a commission house at an expense

of $55.36, and sold at a commission of 2%. If the rate of brokerage for negotiating the purchase was $\frac{7}{8}$%, and Knowlton & Kemp's net profit was $974, what were the gross proceeds of the sale?

10. An agent sold a consignment of butter at 15 cents per pound; he charged 5% commission for selling, invested $956 in sugar and remitted $374 as the balance due his principal. If the agent charged 3% commission for purchasing the sugar, what was the total commission for both transactions?

11. The estimate for expenditures in a certain town is $28310; how large a tax must be levied if the collector is to receive $\frac{2}{3}$%, and 5% is estimated for delinquencies?

12. **ACCOUNT SALES.**

INDIANAPOLIS, Nov. 18, 1905.

Sold for account of G. L. RAYMOND,

By NELSON & JONES.

1905						
Oct.	24	5 cases eggs, 150 doz ..... 18¢	.....	....		
"	"	40 lb. Dairy Butter ..... 22¢	.....	....		
"	26	6 cases eggs, 180 doz.. ..... 20¢	.....	....		
"	30	25 lb. Dairy Butter ..... 23¢	.....	....		
Nov.	4	80 " Creamery Butter ..... 25¢	.....	....		
"	10	85 " Dairy Butter ..... 23¢	.....	....		
"	13	45 " Creamery Butter ..... 26¢	.....	....		
"	16	4 cases eggs, 120 doz ... . 25¢	.....	....	.....	....
		Charges				
		Freight	24	50		
		Drayage	3	75		
		Commission, 5%	.....	....	.....	....
		Net proceeds per draft.....			.....	....

13. **ACCOUNT PURCHASE.**

CHICAGO, March 20, 1906.

Bought for account and risk of L. C. HEAD & CO.,

By NORTHWESTERN COMMISSION CO.

25	bu. Seed corn	$1.10			
40	" " wheat	1.35			
75	" Snowflake potatoes	.70			
10	brls. Onions	3.50			
	Charges				
	Drayage		2	25	
	Commission, 3%				
	Charged to your account				

TRADE DISCOUNT.

410. Discount is an abatement or lessening.

REMARK 1. Discount, considered commercially, has relation to anything of a fluctuating value. Stocks, bonds, notes, and other commercial paper, prices of merchandise commodities, and of property generally, are objects to which discount may properly apply.

411. Trade Discount is an abatement from the list or marked price of articles belonging to *trade* or *commerce*.

412. A Discount Series is a succession of discounts having reference to the same article.

It is a custom with many manufacturers and dealers to list their goods and wares in published catalogs at prices sufficiently high as not to be overreached by any advance in price, to sell at such reductions as competition may demand, and to add additional discounts whenever prices decline.

It is also customary to allow a discount to the trade on all orders of a certain amount, a second discount on larger orders, *a third discount* on still larger orders, etc. It is

evident that either of the above conditions will create a *discount series*.

REM. 2. Most dealers add still another discount conditioned upon time of payment. Thus, the terms of the bill may be "60 days credit," and subject to a discount of 2% if paid within 10 days.

REM. 3. In manufacturing and wholesale houses, prices are usually listed in catalogs. Retail houses engaged in a business not exceedingly extensive mark their prices on the goods; hence the terms *catalog price*, *list price*, and *marked price*, have the same mathematical signification.

413. Computations in Trade Discount are neither more nor less than an application of percentage principles. The *list*, *catalog*, or *marked price* is the *base*, while all other amounts, concrete in character and representing percentage elements, are *percentages*.

The following formulas clearly illustrate the relations between Trade Discount and percentage:

1. List (or marked) price×Rate of discount=Discount.
2. List price×Rate of selling price=Selling price.
3. Discount÷Rate of discount=List price.
4. Selling price÷Rate of selling price=List price.

REMARK. Since the discount and selling price combined constitute the list or marked price (100%), it is evident that the rate of selling price is 100%—the rate of discount.

414. To find the selling price when goods are sold at a discount.

1. What is the selling price of organs, invoiced at \$200, and sold at a discount of 25%, 12%, and 10% from the invoice or catalog price?

Operation.

\$200	list price.
50	25% or ¼ off.
\$150	proceeds of 1st disc.
18	12% off.
\$132	proceeds of 2d disc.
13.20	*10%* or $\frac{1}{10}$ off.
\$118.80	*selling* price.

EXPLANATION. Subtracting the first discount (25% of \$200), gives \$150 as the proceeds of the first discount. Subtracting the second discount (12% of \$150), gives \$132, the proceeds of the second discount. Subtracting the third discount (10% of \$132), gives \$118.80, as the proceeds of the third discount, or the selling price.

Second Operation.

$200 \times .75 = \$150.$
$\$150 \times .88 = \$132.$
$\$132 \times .90 = \$118.80.$

EXPLANATION. We multiply the list price by 75%, the rate of proceeds of the first discount, and thus obtain the proceeds of the first discount by a direct operation; then by multiplying by the rate of the second proceeds, 88% (100%—12%), we obtain the proceeds of the second discount, etc.

EXAMPLES FOR PRACTICE.

Find the selling price:

	Invoice or List Price.	Discount.
2.	\$420	20%, 10%.
3.	\$336.50.....	10%, 10%, 5%.
4.	\$960	$12\frac{1}{2}$%, $16\frac{2}{3}$%, 10%.
5.	\$1200	10%, 10%, 10%.
6.	\$1642.75.....	40%, 5%, 5%.
7.	\$832.40.....	30%, 20%, 2%.

8. What is the cost to a purchaser of goods invoiced at the list price, \$875, and subject to a discount of 25%, 10%, and 5%?

9. The catalog price of hardware purchased by a retail firm is \$1680; if bought subject to a discount of 40%, 5%, and 5%, what is the cost to the purchaser?

10. A shoe dealer can buy from A at 30% discount from list price; B lists the same grade at the same price as A, but sells at 20% and 10% off. What amount will be saved on an invoice of \$1800 by buying from A?

11. An invoice of clothing amounting to \$2800 was sold Sept. 12, on 90 days' credit, and subject to a trade discount of 40% and 5%, with an additional discount of 3% if paid in 30 days, or 5% if paid in 10 days. What sum will pay the bill Oct. 1st?

12. The catalog price of a bill of crockery amounted to \$1260; \$720 of the bill was subject to a discount of 25%, 10%, and 5%; \$220 was subject to a discount of 15% and

10%, and the remainder was sold at net price. What was the cost to the purchaser?

13. A bill of merchandise amounting to $2400 was sold subject to the following "Terms: 60 days net; 5%, 10 days." If the purchaser paid $1824 one week after the purchase, how much credit should he receive?

The sum paid is part of the selling price. (Formula 4.)

415. To reduce a discount series to an equivalent single discount.

1. What single discount on the marked price is equivalent to the discount series 20%, 10% and 5%.

Operation.

20 %, 1st discount.
80 % × 10 % = 8 %, 2nd "
72 % × 5 % = $3\frac{3}{5}$ %, 3rd "
$31\frac{3}{5}$ %

EXPLANATION. The solution requires that each of the discounts be brought to bear upon the same base, so that they may be added together. The first rate (20%), is based upon the marked price, and after being deducted, 80% of the marked price remains; 10% of 80% of the marked price is 8% of the marked price, and being deducted, 72% remains; 5% of 72% of the marked price equals $3\frac{3}{5}$% of the marked price. Having brought the three rates to bear upon the same base, we combine them by addition, and obtain the single discount $31\frac{3}{5}$%, which is equal to the given series.

REMARK 1. Since the discount and selling price are together equal to the marked price (100%), it is evident that, in the above example, the rate of selling price is 100%—$31\frac{3}{5}$%=$68\frac{2}{5}$%.

REM. 2. The result of the computation is the same regardless of the order of the arrangement of the members of the series. Thus, 20%, 15%, and 5% off is the same as 15%, 20%, and 5%; or 5%, 15%, and 20%, etc.

2. What single discount on the marked or catalog price is equivalent to the discount series 40%, and 10%?

Business Method.

40 % + 10 % = 50 %
40 % × 10 % = 4 %
46 %

EXPLANATION. The addition of the two rates may not seem logical to the student as the 10% rate is based on a number which is 40% less than the base of the first discount; to correct the error we subtract 10% of 40%; which leaves 46%, the *correct result.* The student will observe that the correct result, in a series

of two discounts, is found by subtracting the product of the discounts from their sum.

REMARK. In a series of more than two discounts, combine the first two into one as explained; then combine the result with the third, etc.

EXAMPLES FOR PRACTICE.

Reduce each discount series in the following examples to an equivalent single discount.

3. 10% and 5%. *4.* 20%, 12%, and 10%. *5.* 30%, 10%, and 10%. *6.* 40% and 5%. *7.* 20, 16⅔%, and 3%. *8.* 33⅓%, 25%, and 2%. *9.* 30%, 5%, and 5%. *10.* 12½%, 10%, and 4%. *11.* 40%, 20%, and 2½%.

12. What single discount is equal to the difference between the series 15% and 10%, and the direct discount 25%?

13. What single discount is equal to the difference between the series 40%, 20%, and 10%, and the series 25%, 25%, and 20%?

Find the marked price

	Selling price.	Discount.
14.	$35.40	12½%.
15.	$5.44	15 %.
16.	$31.92	20% and 5 %.
17.	$24.30	40%, 10 %, and 10 %.
18.	$17.28	20%, 25 %, and 10 %.
19.	$9.60	20 % and 20 %.
20.	$1.12	25%, 33⅓%, and 10 %.
21.	$14.58	10%, 10 %, and 10 %.

22. Merchandise that cost $432 was sold at a profit of 25%; what was the marked price, the discount therefrom being 25% and 20%?

$432 (cost)+$108 (profit)=$540 (selling price).
$540 (selling price)÷60% (rate of selling price)=$900 (marked price).

23. I paid $108 for an organ and afterward sold it at a

discount of 20% and 10% off from the marked price. If my profit from the sale was $16\frac{2}{3}\%$, at what price was the organ marked?

24. A loss of 10% was sustained by selling an article at 5% and 10% off from the marked price; if the cost was $18.50, what was the marked price?

25. Goods costing $15 were marked $18 and sold at cost; what was the rate per cent. of discount?

26. If a dealer pays $24 per dozen for hats and marks them at $36 per dozen, at what rate per cent. must he discount them to gain 35%?

27. If chisels costing 50 cents are marked at 75 cents and sold at 10% loss, what is the rate of discount?

28. **CLOTHING.**

Subject to draft, without notice, if not paid when due.

INDIANAPOLIS, Oct. 12, 1905.

MARTIN & DAVIS,

Bought of GEO. BARTON & CO.

Terms: 90 days, net; 5% 10 days.

Quantity	Items		Subject to Discount		Net	
5	doz. suits, No. 1540	$120.				
3	" " " 2115	96.				
5	" pants " 840	36.				
$\frac{1}{2}$	" " " 960	48.				
2	" " " 980	30.				
$1\frac{1}{2}$	" suits " 1620	156.				
10	" overalls " 120	4.80				
8	" jackets " 160	3.75				
6	" linen hdks	2.40				
4	" sox	.96				
2	" " extra fine	1.25				
$3\frac{1}{2}$	" suits all wool underwear	10.50				
10	overcoats, No. 472	13.50				
	Less 10% and 5%					

What sum will pay the bill Oct. 21, 1905?

The solution requires that the bill be copied, the extensions made, and the columns footed.

COST MARKS.

416. Merchants usually employ some device for marking the cost of their goods, known only to themselves or to those in their employ; the object being two-fold,—to keep their customers in ignorance concerning their profits, and to prevent them from knowing what discounts can be afforded. Some word, or combination of words, consisting of 10 letters, no two of which are alike, is usually selected as a *cost word*, and the letters of this word are used in marking their goods instead of the numbers which mark their position. Thus, if the word selected is "*blacksmith*", *b* will represent 1; *l*, 2; *a*, 3; etc., as shown below:

b l a c k s m i t h
1 2 3 4 5 6 7 8 9 0

If the cost of an article is 50 cents, and the above word is used, the cost will be marked *kh*. An article costing $1.75 will be marked *bmk*. An additional letter is frequently employed, called a repeater, and is used to avoid repeating the same letter. If the repeater is *x*, an article costing 66 cents will be marked *sx*.

NOTE. Any word, or combination of words, containing 10 different letters, forms an appropriate cost mark; as, *corn basket*, *black horse*, *magnitudes*, *keg of nails*, *republican*, *winter days*, etc. The word selected for the following examples is

m a g n i t u d e s
1 2 3 4 5 6 7 8 9 0

What is the cost mark

1. *Of an article* costing 12¢?
2. *Of an article* costing 35¢?

3. Of an article costing $1.24?

4. Of an article costing 9¢?

5. Of an article costing 53¢?

6. Of an article costing $5.12?

7. An article marked at 30% profit is sold for $1.15. If the cost mark is *mas*, what is the rate of discount?

8. The cost mark of a cap is *ua*, and the marked price at which it is designed to be sold is 1.05. If sold at 16⅔% profit, what is the rate of discount?

9. A pair of shoes marked $4.50 can be sold at 20% discount and a gain of 20% realized. What is the cost mark?

417. ADDITIONAL EXAMPLES FOR REVIEW.

1. An agent, by purchasing sewing machines at 40% discount from the catalog price and selling them at 10% discount from the same catalog price, gains $12 on each machine. At what price are they listed in the catalog?

2. If pianos are bought at a discount of 40% and 10% off from list price, what is the rate per cent. of gain if they are sold at 20% and 5% off from the same list price?

3. An invoice of cutlery was bought June 28th at a trade discount of 20% from catalog price, and subject to an additional discount of 5% if paid within 10 days. The bill was paid July 6th, and the goods were afterward sold at 14% advance of prices named in catalog; what was the rate per cent. of gain?

4. A dealer was allowed 30% on an invoice of carpeting; in payment he transferred A's note for $100, which was $26 less than the discount, gave a check for $50 and drew on B for the remainder. What was the face of the *draft*?

5. If I mark plows at 40% profit, at what rate of discount can I sell them to net cost?

6. A dealer marked an invoice of apples at 20% profit and sold 60% of the same at the marked price; the second sale of 15% of the invoice was disposed of at 5% discount, and the remainder was sold at a discount of 10% from the former price. What was the rate of profit on the entire invoice?

7. If goods are sold at the marked price, 25% profit will be realized; what will be the rate of profit if sold at a discount of 10% from the marked price?

8. If a merchant marks goods at 20% profit and sells at 20% discount from marked price, does he gain or lose, and what rate per cent.?

9. A dealer bought 280 yards of silk from a house in Liverpool, England, billed at £120 8s. 6d., and subject to a discount of 10% and 5% off from the gross invoice price: he paid duty and transportation amounting to 40% of the net invoice price. If he marked the goods at 24% profit, and sold at a discount of 10% from the marked price, what was his selling price per yard, estimating the value of the pound sterling at $4.86 in United States money?

10. A stationer bought books and stationery on 90 days' credit; the books were invoiced at $84, and subject to a trade discount of 40% and 5%; the stationery was invoiced at $60 and subject to a trade discount of 15%. The terms of the bill provided for an additional discount of 5% if paid in 30 days, or 10% if paid in 5 days. The bill was dated October 8th, and paid November 1st. If the goods were marked 20% above the gross invoice price, and sold at 5% discount, what was the total amount of gain?

11. What rate per cent is gained if an article is marked at $33\frac{1}{3}$% profit and sold at $12\frac{1}{2}$% discount?

INTEREST.

418. Interest is a charge or an allowance made for the use of money.

419. The **Principal** is the sum for the use of which interest is charged.

420. The **Rate Per Cent.** of interest is the number of hundredths of the principal charged for its use for one year.

421. The **Time** is the period for which the interest is computed.

422. The **Amount** is the sum of the principal and interest.

423. Simple Interest is interest computed on the principal only.

424. Legal Interest is interest computed at the rate established by law.

425. Usury is illegal interest, or a greater per cent. than is sanctioned by law.

NOTE 1. The taking of usury is prohibited under various penalties in different states.

NOTE 2. In several states a contract rate somewhat above that fixed by statute is authorized, and in most states that have made provision for such contract rate, a maximum is fixed by law; in a few states, however, any rate fixed by contract is legal. In the absence of a specific contract rate, the legal rate prevails.

NOTE 3. Debts of all kinds are subject to interest from the time they become due, but not before, unless so specified.

426. The legal and maximum rates of interest allowed according to the laws of the different states and territories are shown in the following:

TABLE OF INTEREST RATES.

States and Territories	Legal Rate	Contract Rate	States and Territories	Legal Rate	Contract Rate
Alabama	8 %	8 %	Nevada	7 %	Any %
Alaska	8 "	12 "	New Hampshire	6 "	6 "
Arizona	6 "	Any "	New Jersey	6 "	6 "
Arkansas	6 "	10 "	New Mexico	6 "	12 "
California	7 "	Any "	New York	6 "	6 "
Colorado	8 "	Any "	North Carolina	6 "	6 "
Connecticut	6 "	6 "	North Dakota	7 "	12 "
Delaware	6 "	6 "	Ohio	6 "	8 "
Dist. of Columbia	6 "	6 "	Oklahoma	7 "	12 "
Florida	8 "	10 "	Oregon	6 "	10 "
Georgia	7 "	8 "	Pennsylvania	6 "	6 "
Hawaii	8 "	12 "	Philippines	6 "	Any "
Idaho	7 "	12 "	Porto Rico	6 "	12 "
Illinois	5 "	7 "	Rhode Island	6 "	Any "
Indiana	6 "	8 "	South Carolina	7 "	8 "
Iowa	6 "	8 "	South Dakota	7 "	12 "
Kansas	6 "	10 "	Tennessee	6 "	6 "
Kentucky	6 "	6 "	Texas	6 "	10 "
Louisiana	5 "	8 "	Utah	8 "	Any "
Maine	6 "	Any "	Vermont	6 "	6 "
Maryland	6 "	6 "	Virginia	6 "	6 "
Massachusetts	6 "	Any "	Washington	7 "	12 "
Michigan	5 "	7 "	West Virginia	6 "	6 "
Minnesota	6 "	10 "	Wisconsin	6 "	10 "
Mississippi	6 "	10 "	Wyoming	8 "	12 "
Missouri	6 "	8 "	Canada	5 "	Any "
Montana	8 "	Any "	Cuba	6 "	Any "
Nebraska	7 "	10 "			

427. Interest is a subject requiring percentage computations, the principal being the base, and the interest and the amount, percentages. Since an additional element, *time*, is introduced, the percentage is the product of three factors instead of two. The following formulas illustrate the necessary computations:

1. Principal × rate × time (in years) = interest.
2. Principal × rate × time (in months) = interest × 12.
3. Principal × rate × time (in days) = interest × 360.

428. In calculating interest for a fractional part of a year, it is customary to compute upon the basis of 360 days *to the year. This method* of computation has become a

law of custom, and has been legalized in a few states by statutory provision. Since the common year contains 365 days, this method of computing does not give *accurate* interest, but the approximation is sufficiently close for practical purposes. The interest on \$3600 for 1 year at 6% is \$3600×.06=\$216 (formula 1). For $\frac{1}{5}$ of a year (73 days) the interest is evidently $\frac{1}{5}$ of \$216, or \$43.20; but the interest, according to the ordinary computation is \$43.80. $\frac{360}{365}=\frac{72}{73}$ of this sum gives \$43.20; hence, if accurate interest is required, multiply the ordinary interest by $\frac{72}{73}$.

NOTE. The accurate interest on any sum of money for exact years is equal to the ordinary interest.

1. What is the interest on \$950 for 2 yr. 5 mo. 17 da. at 6%?

Operation.

\$950×.06×2=\$114.
(\$57×5)÷12= 23.75
(\$57×17)÷360= 2.69
\$140.44

EXPLANATION. Multiplying the principal by the rate gives \$57, the interest for one year; this sum multiplied by 2 gives the interest for 2 years, which is \$114. The interest for 5 months is $\frac{5}{12}$ of the interest for 1 year, hence we multiply \$57, the interest for 1 year, by 5 and divide the result by 12. The interest for 17 days being $\frac{17}{360}$ of the interest for 1 year, we multiply 1 year's interest by 17 and divide the result by 360. The sum of these results, \$140.44, is the interest for 2 years, 5 months, and 17 days.

EXAMPLES FOR PRACTICE.

2. What is the interest on \$720 for 1 year, 11 months at 8%?

3. What is the interest on \$3124.50 for 9 mo. 23 da. at 7%?

4. An interest bearing note of \$450 was given May 12, 1905, and paid September 3, 1905. What was the interest charged for the exact time in days, the rate being 6%?

5. A 6-months' note for \$650, with interest at 9%, was given February 23, 1905, and paid 14 days after maturity. What sum cancelled the debt?

6. The ordinary interest on a sum of money loaned for 8 months and 16 days is $68.45; what is the accurate interest?

7. What is the amount of $1250 for 7 mo. 12 da. at 10%.

8. What is the interest of $1600 for 3 years, 7 months, 12 days at $4\frac{1}{2}\%$?

SIX PER CENT. METHOD.*

429. We find by inspection that the interest on $1 for any given time at 6% is

6 times as many cents as there are years,

$\frac{1}{2}$ as many cents as there are months,

and $\frac{1}{6}$ as many mills as there are days. Hence to find the interest on $130 for 2 years, 7 months and 21 days, we proceed as follows:

Operation.

$.02	$\times 6 = \$.12$	interest on $1 for 2 years.
.07	$\div 2 = .035$	" " $1 " 7 months.
.021	$\div 6 = .003\frac{1}{2}$	" " $1 " 21 days.
	$\$.158\frac{1}{2}$	" " $1 " 2 yr. 7 mo. 21 days.

$\$.158\frac{1}{2} \times 130 = \$20.605$, interest on $130 for given time.

RULE I. *Multiply as many cents as there are years in the given time by 6; divide as many cents as there are months by 2; divide as many mills as there are days by 6; and add the results. The sum is the interest on $1 for the given time.*

II. *Multiply the interest on $1 by the principal. The product is the required interest.*

NOTE. To find the interest at any other per cent. by this method, first find it at 6%, and then increase or diminish the result by as many sixths of itself as the given rate is units greater or less than 6%. Thus, for 5% subtract $\frac{1}{6}$, for 8% add $\frac{1}{3}$, etc.; or, divide the interest at 6% by 6 and multiply the result by the units expressing the rate.

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*This method first appeared in the Scholar's Arithmetic, published by Daniel Adams, M. D., *1801.*

EXAMPLES FOR PRACTICE.

Find the interest at 6% per annum.

	Principal.	Time.
1.	$325	84 da.
2.	$724.50	132 "
3.	$576	7 mo. 13 "
4.	$2120	5 " 27 "
5.	$1850.80	17 " 12 "
6.	$384.40	11 " 17 "
7.	$5400	21 " 15 "
8.	$36.20	1yr. 3mo. 8 "
9.	$580	5 " 6 " 14 "
10.	$1575	3 " 9 " 10 "
11.	$17	8 " 23 "
12.	$28.10	3 mo. 16 da.
13.	$3500	11 " 1 "
14.	$2840	8 " 5 "
15.	$3870	10 " 25 "
16.	$8240	4 " 19 "
17.	$75.50	1 " 28 "
18.	$712	2 " 20 "
19.	$1735	63 "
20.	$1860	93 "
21.	$2160	123 "

22. What is the interest at 7% of $560 from May 13, 1904 to Aug. 17, 1904? (Exact time in days.)

23. What is the interest at 8% of $850 from Nov. 25, 1904 to March 7, 1905? (Exact time in days.)

24. Find the amount of $3200, at 5%, from June 10, 1903, to Apr. 26, 1906. (Time by compound subtraction.)

25. A note given Sept. 13, 1903, for $760, at 5% interest, was paid May 29, 1905. What amount cancelled the debt, the time being found by compound subtraction?

In the following examples, find exact time in days if less than 1 year; if more than 1 year, find the time by compound subtraction.

26. What sum will pay the interest, August 12, on $550 loaned Feb. 23, preceding, the rate being 10%?

27. What will $1625 amount to in 1 year, 5 months and 24 days at 7%?

28. What is the accurate interest on $2400, at 8%, for *3 yrs. 4 mo. 12* da.?

29. A note of $620 was given March 5, 1904, for 3

months, without interest. What sum was necessary to pay the debt August 20, 1904, the parties being residents of Minnesota? (425, note 3.)

30. A merchant residing in an Indiana town, bought goods invoiced at $2250, of a wholesale firm in Indianapolis, on 60 days' credit. What sum was required to balance the account 96 days after the date of purchase?

CONTRACTED METHODS OF COMPUTATION.

430. Many attempts have been made to incorporate time-saving methods into interest computations. A few of the various methods introduced have proven worthy of consideration, while others have been found to be time-savers only in special cases. Under the foregoing heading we present a few contracted methods which we deem worthy of notice.

REMARK. Most contractions concerning computations in interest are based upon the relation existing between *interest* and *time*; hence these relations, as here presented, should be carefully studied and analyzed.

431. The interest on $1, at 6%, is 6 cents for 360 days, or 1 cent for $\frac{1}{6}$ of 360, or 60 days; at 8%, $1 will earn 8 cents of interest in 360 days, or 1 cent in $\frac{1}{8}$ of 360, or 45 days; and, in any case, as many days will be required for a principal to earn $\frac{1}{100}$ of itself as the units expressed by the rate are contained times in 360. Therefore any principal will earn $\frac{1}{100}$ of itself

At 5%, in $\frac{1}{5}$ of 360 days, or 72 days.
At 6%, in $\frac{1}{6}$ of 360 days, or 60 "
At 8%, in $\frac{1}{8}$ of 360 days, or 45 "
Etc., etc., etc.

432. To find the interest on $1, at any given rate per cent. for any number of days.

1. What is the interest on $1 for 96 days at 5%?

Operation.

$\$.01 \times \frac{1}{72} = \$\frac{.01}{72}$

$\$\frac{.01}{72} \times 96 = \$\frac{.96}{72}$

$\$.96 \div 72 = \$.01\frac{1}{3}$

EXPLANATION. The interest on $1 for 72 days is $\frac{1}{100}$ of a dollar, or .01; multiplying this amount by $\frac{1}{72}$ gives the interest for 1 day, and the interest for 1 day multiplied by 96 gives the interest for 96 days (431). The operation reveals the fact that $\frac{1}{100}$ of the number of days divided by the time required for $1 to earn $\frac{1}{100}$ of itself, $.01, will, in every case, give the correct result.

NOTE. The time required for any principal to earn $\frac{1}{100}$ or $\frac{1}{1000}$ of itself is called the *unit of time*. The unit of time for a 6% rate may be either 60 days or 6 days; for a 9% rate, it may be either 40 days or 4 days.

RULE. *Divide $\frac{1}{100}$ of the number of days by the unit of time.*

REMARK. If the number of days required for $1 to earn $\frac{1}{1000}$ of itself is taken for the *unit of time*, the dividend should be $\frac{1}{1000}$ of the number of days.

Find the interest on $1

2. For 78 days at 6%.
3. For 243 days at 9%.
4. For 63 days at 8%.
5. For 184 days at 5%.
6. For 290 days at 10%.
7. For 23 days at 6%.
8. For 17 days at 8%.
9. For 315 days at 10%.
10. For 132 days at 12%.
11. For 5 days at 8%.

THE UNIT METHOD.

433. The **unit method** of computation (so called because the interest on a unit of the principal, $1, is first ascertained), is a favorite with many on account of its simplicity.

1. What is the interest on $260 for 264 days at 6%?

Operation.

$2.64 \div 60 = \$.044$, interest on $1.

$\$.044 \times 260 = \$11.44$, interest on $260.

We deem the operation sufficiently explanatory, and present the following

RULE. *Find the interest on $1 by 432, and multiply the same by the principal.*

For practice the teacher will assign problems from those *given on page* 240 indicating the rate per cent. to be used.

THE BANKERS' METHOD.

434. While the above term is somewhat indefinite in its application, and is misleading to some extent, since bankers at the present day compute interest altogether from tables, the historic origin of the term justifies its retention to designate the method here presented.

1. Following the line of reasoning presented in 431 we note that 1% of any principal will give interest thereon for such a part of one year (360 days) as 1% is a part of the rate. Hence the following

RULE. *Since moving the decimal point two places to the left in any principal indicates 1% of it, and shows at once the interest on that principal,*

For 120 days at 3 %,	For 48 days at 7½%,
For 90 days at 4 %,	For 45 days at 8%,
For 80 days at 4½%,	For 40 days at 9%,
For 72 days at 5 %,	For 36 days at 10%,
For 60 days at 6 %,	For 30 days at 12%,

Point off as above, and to find interest for longer or shorter time than that given, take aliquot parts of the result obtained by pointing. For 7% compute at 6% and add $\frac{1}{6}$ to the result; for 11% compute at 10% and add $\frac{1}{10}$, or at 12% and subtract $\frac{1}{12}$.

NOTE. The *Varied Application* of the method is illustrated by the following examples, but they by no means present all the short cuts to which the method lends itself. If the time be one or more years, pointing off *one* decimal place instead of two will give the interest for *ten* times the number of days named in the rule; if the time be still longer, the entire principal, *unpointed*, may be taken as the interest for *one hundred* times the number of days named in the rule; or if the time be very short, pointing off *three* places will give the interest for *one-tenth* of the time named in the rule. This latter pointing could be used only for rates where the days given in the rule end with a cipher. When the time includes years, call each full year 360 days.

EXAMPLES.

(*a*) Required the interest on \$125 for *90 days at 6%.*

	Operations.
Pointing off,	\$1.25=int. for 60 ds.
Taking ½,	.63=int. for 30 ds.
Adding,	\$1.88=int. for 90 ds.

EXAMPLES.

(*b*) Required the interest on \$500 for 4 mos. 15 ds. at 8%.

NOTE. Time equals 135 days.

Operations.

Pointing off, \$5.00=int. for 45 ds.
Multiplying by 3 3
\$15.00=int. for 135 ds.

(*c*) Required the interest on \$90 for 26 days at 7%.

NOTE. Result will be the same as interest on \$26 for 90 days.

Int. on \$26 for 60 ds. @ 6% = \$0.26
Int. on \$26 for 30 ds. @ 6% = .13
Int. on \$26 for 90 ds. @ 6% = .39
Adding $\frac{1}{6}$, .07
Gives int. on \$26 for 90 ds. or on \$90 for 26 ds. @ 7% } \$0.46
which is the interest required.

(*d*) Required the interest on \$540 for 28 days at 6%.

Pointing off gives int. for 60 ds., \$5.40
Taking $\frac{1}{2}$ gives int. for 30 ds., 2.70
Subtracting $\frac{1}{30}$ of int. for 60 ds. .18
Gives int. for 28 days required, \$2.52

EXAMPLES FOR PRACTICE.

IMPORTANT SUGGESTION. The following problems should be used for rapid class drill in mental computation, first at 6%, then at the other rates named in the rule, and lastly at 7% and 11%. At 6% all the problems are easily solved mentally, and to guide the student somewhat in method while working at 6%, the first column is planned to point off two places for 60 days, the second three places for 6 days, and the third, one place for 600 days for the basic figures of the interest. Further, the pupil will do well to note that the problems are numbered horizontally, and that the horizontal lines of problems in pairs are planned to work out along the same lines as the illustrative examples which bear the same letters, (*a*), (*b*), (*c*), or (*d*). Problems 25-30 are miscellaneous, and when working at other rates than 6%, all will be miscellaneous, and the pupil must exercise his own ingenuity both in pointing and in selecting aliquots, to obtain results quickly and without pencil or paper, if possible. Home study will find short methods.

	Principal.	Time.	Principal.	Time.	Principal.	Time.
(*a*)	*1.* \$1850	60 ds.	*2.* \$185	6 ds.	*3.* \$2500	600 ds.
	4. \$1572	72 ds.	*5.* \$75	8 ds.	*6.* \$3500	1 yr. 8 mos.*
(*b*)	*7.* \$1375	240 ds.	*8.* \$300	18 ds.	*9.* \$2285	5 yrs.
	10. \$1060	180 ds.	*11.* \$250	24 ds.	*12.* \$4500	3 yrs. 4 mos.
(*c*)	*13.* \$90	87 ds.	*14.* \$12	133 ds.	*15.* \$1800	2 yrs. 3 mos. 23 ds.
	16. \$63	123 ds.	*17.* \$18	125 ds.	*18.* \$600	1 yr. 2 mos. 13 ds.
(*d*)	*19.* \$1985	54 ds	*20.* \$120	5 ds.	*21.* \$2300	1 yr. 6 mos.
	22. \$1088	45 ds.	*23.* \$270	4 ds.	*24.* \$3890	1 yr. 4 mos. 20 ds.
(*m*)	*25.* \$900	193 ds.	*26.* \$36	223 ds.	*27.* \$3600	1 yr. 5 mos. 13 ds.
	28. \$800	80 ds	*29.* \$545	54 ds.	30. \$1200	1 yr. 1 mo. 15 ds.

* *Reduce all time to days.*

BUSINESS METHOD.

435. The principles and relations upon which this (or any other method) is based, are as old as arithmetical science, but we are not aware that the method of solution as here presented, has ever before been published.

1. What is the interest of $1563 for 27 days at 10%?

Operation.

	15.63	
4	3	
36	27	
	46.89	
	11.72	

EXPLANATION. On the right of a vertical line we write the interest for 36 days (unit of time), which is $\frac{1}{100}$ of the principal (431). The interest for 27 days is evidently $\frac{27}{36}$ of the interest for 36 days; hence we should multiply by 27 and divide the result by 36, but as 9 is a common factor in both dividend and divisor, we remove it and divide the product of the remaining factors of the dividend, 46.89, by the remaining factor of the divisor, 4, and obtain $11.72, the correct interest.

RULE. *Write $\frac{1}{100}$ of the principal and the number of days of time in a column, on the right hand of a vertical line, as factors of a dividend, and the unit of time on the left hand as a divisor; cancel all factors common to the dividend and divisor, and divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor. The result will be the required interest.*

NOTE. In cases where the divisor is entirely removed by cancellation, the product of the uncancelled factors of the dividend is the correct result.

2. What is the interest on $852 for 78 days at 7%?

Operation.

	$8.52
36	.71
3	26
	78
	426
	142
	18.46
	.7
	$12.922

EXPLANATION. We first find the interest at 10%, as previously explained, and multiply the result by .7, the reason for which is obvious.

EXAMPLES FOR PRACTICE.

Find the interest by Business Method:

	Principal.	Time.	Rate.		Principal.	Time.	Rate.
3.	$540	73 days	8 %	*13.*	$90	156 days	7 %
4.	$12.60	93 "	6 "	*14.*	$274	89 "	6 "
5.	$1550	214 "	6 "	*15.*	$1320	123 "	8 "
6.	$990	23 "	8 "	*16.*	$5250	246 "	12 "
7.	$2520	113 "	9 "	*17.*	$872.50	13 "	6 "
8.	$175.30	63 "	10 "	*18.*	$2000	51 "	5 "
9.	$6000	189 "	7 "	*19.*	$3600	173 "	6 "
10.	$960	225 "	5½ "	*20.*	$8500	76 "	9 "
11.	$4560	312 "	4 "	*21.*	$15.80	148 "	10 "
12.	$56.75	135 "	8 "	*22.*	$3750	310 "	4 "

23. A merchant buys a bill of goods invoiced at $2400, on 60 days' credit, and subject to a discount of 3 % if paid in 10 days; not having ready money, he fails to take advantage of the discount and pays the bill at maturity. How much would he have gained by borrowing the money at 6 % and paying the bill at the end of 10 days?

436. To find the rate per cent., the other elements, principal, time, and interest, being given.

1. If the interest of $560 for 93 days is $8.68, what is the rate per cent.?

Operation.

$$
\begin{array}{r|l}
 & 560 \\
 & 14 \\
3 & 93 \\
9 & 31 \\
360 & 14 \\
 & 42 \\
 & 3)4.34 \\
 & 1.44\tfrac{2}{3}
\end{array}
$$

$8.68 \div \$1.44\tfrac{2}{3} = 6$

$1\% \times 6 = 6\%$.

EXPLANATION. We first find the interest at 1%, which is $1.44⅔. Since the given interest is $8.68, the rate is as many times 1% as $1.44⅔ (the interest at 1%) is contained times in $8.68. The quotient shows that the interest is 6 times 1%, or 6%.

RULE. *Divide the given interest by the interest on the given principal for the given time at 1%; the quotient will show how many times 1% the required rate is.*

EXAMPLES FOR PRACTICE.

Find the rate per cent.:

	Principal.	Time.	Interest.
2.	$840	76 days	$26.60.
3.	$1440	127 "	$35.56.
4.	$3760	51 "	$23.97.
5.	$1872	270 "	$56.16.
6.	$56.50	212 "	1.99\frac{6}{7}$.
7.	$720	38 "	$7.60.
8.	$2520	130 "	$72.80.
9.	$660	117 "	$6.435.
10.	$1560	98 "	$38.22.
11.	$1080	220 "	$37.40.

12. At what rate per cent. must $3600 be loaned to earn $568.80 in 2 yrs. 7 mo. 18 da.?

13. A note of $2400 was dated June 23, 1904, and paid March 5, 1906. If the principal and interest amounted to $2604, at what rate per cent. did the note draw interest?

437. To find the time when the other elements, principal, rate and interest, are given.

1. At 8% per annum, in what time will $750 produce $72.50 interest?

Operation.

$750 × .08 = $60.

60) 72.50 (1 yr.
60
12.50
12
60) 150.00 (2 mo.
120
30
30
60) 900 (15 da.
60
300
300

EXPLANATION. We divide the given interest ($72.50) by the interest on the principal for 1 year at the given rate ($60), obtaining 1 year and a remainder of $12.50, or $\frac{12.50}{60}$ of a year; reducing this fractional part of a year to months by multiplying the numerator by 12 and dividing by the denominator, we obtain 2 months and a remainder of 30. ($\frac{30}{60}$ of a month); reducing this fraction of a month to days, gives 15 days. Hence our answer is 1 yr. 2 mo. 15 da.

PROOF. Interest of $750 at 8% for 1 yr. 2 mo. 15 da. = $72.50.

RULE I. *Divide the given interest by the interest of the given principal for 1 yr. at the given rate.*

II. *Reduce any fractional part of a unit of time to the next lower denomination.*

NOTE. Since the day is the lowest unit considered in interest computations, regard any fraction of a day that equals or exceeds $\frac{1}{2}$ as a day, and ignore fractions of a less value.

EXAMPLES FOR PRACTICE.

Find the time:

	Principal.	Rate.	Interest.
2.	$1200	4%	$59.16.
3.	$375	6%	$5.13.
4.	$960	10%	$93.87.
5.	$2500	7%	$145.50.
6.	$85.75	8%	$13.20.
7.	$8460	3%	$315.45.
8.	$280	6%	$140.
9.	$576	$4\frac{1}{2}$%	$288.
10.	$25	12%	$5.
11.	$3250	5%	$16.50.

12. In what time will a principal double itself at 7%?

438. To find the principal, the other elements, rate per cent., time, and interest, being given.

1. What principal loaned at 6% will produce $23.25 interest in 93 days?

Operation.

$.93 ÷ 60 = $.0155.
$23.25 ÷ .0155 = $1500.

EXPLANATION. We first find the interest on $1 for the given time, at the given rate, by dividing $\frac{1}{100}$ of the number of days by the unit of time, as previously explained; then, since the required principal is as many dollars as the interest of $1 is contained times in the given interest, we perform the division and obtain $1500 as the required principal.

RULE. *Divide the given interest by the interest of $1 for the given time and rate.*

EXAMPLES FOR PRACTICE.

Find the principal:

	Rate.	Time.	Interest.
2.	5%	190 days	$9.69.
3.	7%	73 "	$20.44.
4.	6%	212 "	$31.80.
5.	8%	144 "	$36.16.
6.	9%	110 "	$8.525.
7.	4½%	220 "	$9.625.
8.	10%	320 "	$109.33.
9.	4%	56 "	$3.75.

439. To find the principal when the rate per cent., time and amount are given.

1. What principal will amount to $583.36 if loaned for 115 days at 4%?

Operation.

$1.15÷90=$.012⅞.
$1.+$.012⅞=$1.012⅞.
$583.36÷$1.012⅞=576.

EXPLANATION. First find the interest on $1 for the given time and rate, as in the former case. $1 + the interest thus obtained is the amount of $1; then, since the required principal is as many dollars as the amount of $1 is contained times in the given amount, we perform the division and obtain the correct result.

440. The division performed to ascertain the interest on $1 must be fractionally exact, as a small deviation will materially affect the value of the quotient in the division which follows. Fractional divisors may be eliminated by following the plan illustrated in the following operation, in which the example is the same as the one above explained:

Operation.

(1) $\frac{1}{90}\times 1.15$=interest on $1.
(2) $1+\frac{1}{90}\times 1.15$=Amt. of $1.
(3) 90+1.15=90 times amt. of $1.
(4) 90+1.15=91.15.
(5) $583.36×90=$52502.40.
(6) $52502.40÷$91.15=576.

EXPLANATION. In (1) we have expressed the interest on $1 by the indicated division of $\frac{1}{100}$ of the number of days by the unit of time. In computation (2) we represent the amount of $1 by adding thereto the interest. No. (3) is the result of the multiplication of (2) by 90 (the unit of time); the sum of the separate parts in (3)

is exhibited in (4). Then, by the multiplication of the amount, $583.36 by 90, as shown in (5), it will be seen that both the dividend and the divisor of the former operation have been multiplied by the same number, (the unit of time), hence the quotient will be the same as by the former method of solution.

REMARK. The divisor (the unit of time + $\frac{1}{100}$ the number of days), will never contain a common fraction; however, we do not claim that this method is superior, in all cases, to the one first explained. It will be found convenient in many instances.

RULE I. *Divide the given amount by the amount of $1 for the given time and rate, or*

II. *Multiply the given amount by the unit of time and divide the result by the unit of time increased by* $\frac{1}{100}$ *the number of days.*

EXAMPLES FOR PRACTICE.

2. What sum of money, loaned at 6% for 4 mo. 23 da. will amount to $1628.68?

3. What principal will amount to $2662.50 in 8 mo. 20 da., if loaned at 9%?

4. What principal loaned at 10% for 5 mo. 12 da. will amount to $1968.78?

5. A sum of money, loaned at 8% for 2 mo. 11 days, amounted to $13.713; what was the sum loaned?

6. What principal will amount to $3663.99 if loaned for 1 yr. 10 m. 13 da. at 7%?

7. On May 5, 1904, I loaned a sum of money at 5% per annum, and on Sept. 23, 1904, I received $326.27 in full of principal and interest. What was the sum loaned? (Find exact time in days.)

8. What principal loaned Aug. 12, 1903, at 6% per annum, will amount to $1322.37 on June 5, 1904? (Find time by compound subtraction.)

GENERAL FORMULA.

441. Any wanting element of an interest computation may be readily found, when the other three elements are known, from the following formula:

(1.)

Principal	
Rate	
Time	
(in years)	Interest

(2.)

Principal	
Rate	12
Time	
(in months)	Interest

(3.)

Principal	
Rate	360
Time	
(in days)	Interest

Since the product of the principal, rate, and time, in years, is equal to the interest (Formula 1, 427), we arrange the factors on the left of a vertical line and place the interest, (the product), on the right, as shown in No. 1; and since the product of the principal, rate, and time, in months, is equal to the interest multiplied by 12 (Formula 2, 427), we arrange the two sets of factors as shown in No. 2; and again, since the product of the principal, rate, and time, in days, is equal to the interest multiplied by 360 (Formula 3, 427), we arrange the two sets of factors as shown in No. 3.

Since the numbers on opposite sides of the vertical line, in each case, are factors of equal products, we can readily determine the value of any wanting element by the formula, as shown in its application to the following example:

1. At what rate per cent. will \$1480 produce \$53.28 interest, if loaned for 5 mo. 12 da.?

Operation.

370	
~~1480~~	
	~~3~~
	~~12~~
(?)	
	~~53.28~~
.2 .~~8~~	5.92
~~8~~.~~4~~	

EXPLANATION. We arrange the different elements in the order shown in formula (2), placing (?) in the place made vacant by the wanting element. Since the products from the two sets of factors are equal, we reject equal factors by cancellation. Since the product of the three factors on the left of the line is equal to the product on the right (5.92), we divide it by the product of the given factors ($370\times.2$), and obtain the wanting factor, .08, the rate.

$5.92\div(370\times.2)=.08$

NOTE. When the number of days is a multiple of 3, it is generally preferable to express such time as tenths of a month, as every 3 days is one-tenth of a month.

EXAMPLES FOR PRACTICE.

Find the rate per cent.:

	Principal.	Time.	Interest.
2.	$1260	87 da.	$30.45.
3.	$1242	7 mo.	$65.205.
4.	$84.50	153 da.	$2.873.
5.	$660	217 da.	$23.87.
6.	$540	96 da.	$7.92.
7.	$1520	34 da.	$12.92.

Find the principal:

	Rate.	Time.	Interest.
8.	5%	37 da.	$74.15.
9.	6%	118 da.	$9.80.
10.	4½%	71 da.	$5.60.
11.	9%	82 da.	$26.855.
12.	8%	112 da.	$4.32.
13.	7%	9 mo.	$18.20.

In the following examples, if the interest of the given principal for 1 year exceeds the given interest, place the factor 12 or 360 on the right of the line, and thus secure an answer in months or days as may be desired.

A fractional part of a day less than ½ should not be considered; a fractional part of a day that equals or exceeds ½ should be regarded as a day.

Find the time:

	Principal.	Rate.	Interest.
14.	$1530	6%	$19.38.
15.	$1820	5%	$19.97.
16.	$87.30	10%	$10.11.
17	$125	8%	$1.65.
18.	$345	9%	$5.30.
19.	$94.50	4%	$2.15.

NOTES AND DRAFTS.

442. A **Note** is a written promise made by one party, called the *maker*, to pay to a second party, called the *payee*, a certain sum of money on demand or at a specified time.

Note Payable to Order, Without Interest.

$450\frac{00}{100}$. ST. PAUL, MINN., May 8, 1904.

Sixty days after date I promise to pay to

.....John D. Boyd.....or order,

Four Hundred Fifty.....$\frac{00}{100}$ Dollars,

Value received.

No. 1. Due July 7, 1904. GEO. KASPER.

Note Payable to Bearer, With Interest.

$1200\frac{xx}{100}$. FORT WORTH, IND., Aug. 23, 1905.

Two months after date, I promise to pay to

.....J. S. Raymond & Co.,.....or bearer,

Twelve Hundred.....$\frac{no}{100}$ Dollars,

with interest at 6% per annum from date until paid.

G. W. WILKES.

No. 2. Due Oct. 23/26, 1905.

443. A **Draft** is a written order issued by one party, called the *drawer*, to a second party, called the *drawee*, to pay a third party, called the *payee*, a certain sum of money, at sight, or at some specified time.

Sight Draft.

No. 1. MINNEAPOLIS, MINN., Oct. 12, 1905.

At.....sight, pay to

.....E. B. Jordan.. ....or order,

Four Hundred Twenty and $\frac{50}{100}$.....Dollars ($420\frac{50}{100}$).

Value received, and charge to account of

HENRY C. MORR.

To Western Supply Co.,
Chicago, Ill.

Time Draft.

No. 2. MILWAUKEE, WIS., June 12, 1905.

At.....Thirty days.....sight, pay to

..... S. J. Wilkins.....or order,

Six Hundred..... .....$\frac{00}{100}$ Dollars (600\frac{xx}{100}$),

Value received, and charge to account of

H. B. NELSON.

To Harper & Jones,
Detroit, Mich.

444. The **Holder** of a note or other commercial instrument, is the party who owns it.

445. The **Face of a Note** or other commercial paper is the sum stipulated therein, exclusive of interest. The face of Note No. 1, is $450; of Draft No. 2, $600.

An **Indorsement** is that which is written on the back of a note or other commercial instrument.

446. Grace is extra time (3 days), added, by the statutes of some states, to the time specified in the note or draft.

447. The **Maturity of a note or draft** is the time at which it becomes legally due, and includes "days of grace," when allowed.

REMARK. If a legal obligation matures on Sunday or any legal holiday, it is not legally due until the day following, except in the states of Arkansas and Mississippi, whose statutes make payment due the day preceding.

448. A **Negotiable Note or Draft** is one that places the purchaser in possession of a legal title.

REMARK 1. A legal title gives the purchaser an unconditional claim against the maker or drawee.

REM. 2. A *non-negotiable note*, if bought, does not give the purchaser a *legal* title; he secures an *equitable* title only. If the maker has an offset against the payee, it will operate also against the buyer of the *non-negotiable* paper, as the buyer's interest is neither more nor less than *the payee's equities*.

REM. 3. A note or draft is negotiable until the date of maturity; it

made payable to *bearer*, or to the *order* of the payee; a note or draft bought *after* maturity secures to the purchaser an equitable title only.

REM. 4. An obligation payable to *bearer* may be transferred to another simply by delivery; but, if payable to *order*, the seller must indorse his name on the back of the instrument as his order.

449. Specimen Forms of Indorsement.

(*In Blank*)	(*In Full*)	(*Qualified*)
No. 1	No. 2	No. 3
G. R. BATES.	Pay to A. J. Clark or order. G. R. BATES.	Pay to H. C. Day or order, without recourse. G. R. BATES.

The indorsement on No. 1 is a transfer by the payee to the person to whom he delivers it, and obligates the maker or drawee to pay whoever presents it at maturity. The payee also becomes security for payment, if legal requirements are observed, in case of a failure to collect from the maker or drawee.

The indorsement on No. 2 is a transfer by the payee to A. J. Clark, but by the expressed conditions Clark may order it paid, by indorsement, to any other person. The indorsement and all subsequent indorsements become security as explained in No. 1.

The indorsement on No. 3 is a transfer by the payee to H. C. Day, who, by indorsement, may order that payment shall be made to anyone he may designate, or, if indorsed by him in blank, to anyone presenting it; the words "*without recourse*" relieves the payee from responsibility otherwise occasioned by a failure on the part of the maker or drawee to make payment. He is responsible only concerning the legality of the paper and his legal right to transfer it.

REMARK 1. Payment should be demanded at maturity, and if refused,

all indorsers must be notified of the demand and refusal within 24 hours (except when conditions make a longer time necessary), otherwise the indorsers will be released from their obligations of surety.

REM 2. Indorsements may be made *conditional* or *restrictive*, but such indorsements are not common among business men.

450. On presentation of a sight draft by the holder, the drawee is expected to make immediate payment; but, when a time draft is presented, the drawee is required to write the word "*Accepted*," with the date of the transaction, across the face of the draft. The writing should be in red ink and across the left hand end; this act is called *accepting the draft*, and the draft is afterwards called an *acceptance*, and is a promise by the drawee to pay the same at maturity. If the drawee refuses to pay or accept a draft as may be required, the same is said to have been *dishonored*.

REMARK 1. When the time for payment is expressed in days, all computations should be made upon the basis of *exact time in days*. A note given Jan. 15, for 30 days, will fall due Feb. 14, unless grace is allowed, in which case the date of maturity will be Feb. 17. If the time is expressed in months or years, computations should be made upon the basis of *compound subtraction*.

REM. 2. In estimating the time by calendar months, if it should be found by the computation that the note falls due on the 29th, 30th, or 31st of a month containing less than that number of days, the note would be due on the last day of such month, except in cases where grace is allowed, which would extend the time to the 3d day of the following month.

451. A **Protest** of a note or acceptance is a formal statement by a notary public, at the request of the holder, that said obligation was presented to the maker thereof for payment, and by him refused.

REMARK. Protest, although customary in bank transactions, is not a legal requisite, except in cases of foreign exchange.

452. To find the date of maturity of a note or an acceptance.

1. A note dated June 13, 1904, was payable by its terms *90 days after* date. What was the date of maturity, allowing grace?

Operation.

=93 days, time to run.
17 days remaining in June.
76
31 days in July.
45
31 days in August.
t. 14, 1904, maturity.

EXPLANATION. Subtracting from the full time (93 days) the days remaining in June (17), gives 76 days; subtracting the days belonging to July, and we find that the note has yet 45 days to run; subtracting August and there are still 14 days belonging to Sep-

, hence the date of maturity is September 14.

other plan is to consider the time (90 da. + 3 da.) as 3 mo. 3 da., l it to the date (6 mo. 13 da.); this gives 9 mo. 16 da. or September since July and August have each 31 days, the addition has carried ays beyond maturity, therefore 2 days must be deducted, giving ıber 14, as before.

EXAMPLES FOR PRACTICE.

nd the date of maturity:

(With Grace.)

	Date.	Nominal Time.
2.	May 3, 1902	60 days.
3.	May 3, 1902	2 months.
4.	Aug. 24, 1903	90 days.
5.	Nov. 18, 1901	100 days.
6.	Dec. 31, 1904	2 months.

(Without Grace.)

	Date.	Nominal Time.
7.	June 21, 1901	80 days.
8.	Jan. 18, 1904	60 days.
9.	Mar. 31, 1903	1 yr. 5 mo. 20 da.
10.	Apr. 5, 1900	90 days.
11.	Sept. 29, 1903	5 months.

2. A draft, payable 90 days from sight, was dated 17, 1904, and accepted 5 days later. What was the of maturity, without grace?

3. To *find the number* of days to maturity.

A note dated Oct. 14, 1902, was made payable 3

months after date. Find the number of days to maturity, allowing grace.

Operation.

Yr.	Mo.	Da.	
1902	10	14	(date)
	3	3	
1903	1	17	maturity.

Days remaining in October	17
Days in November	30
Days in December	31
Days in January	17
From date to maturity	95 da.

EXPLANATION. Oct. 14, 1902 + 3 mo 3 da. = Jan. 17, 1903, date of maturity. The number of days from Oct. 14, 1902, to Jan. 17, 1903, as shown by the operation, is 95.

Another method is to consider each month 30 days and make the proper additions; thus, 90 da. + 3 da. (grace) + 2 da. (for 2 31-day months)= 95 days as before.

EXAMPLES FOR PRACTICE.

Find the number of days to maturity.

(With Grace.)

	Date.	Nominal Time.
2.	July 7, 1901	6 months.
3.	Sept. 15, 1903	8 "
4.	Feb. 12, 1902	3 "
5.	Aug. 26, 1902	4 "
6.	Jan. 29, 1902	1 "
7.	May 11, 1902	3 "
8.	Nov. 3, 1903	7 "
9.	Oct. 18, 1904	1 yr. 2 mo.

454. To find the former of two dates when the last date and the number of days between the dates are given.

1. A 60-day note, with grace, was paid at maturity, Apr. 22, 1905. What was the date of the note?

Operation.

60+3=63 days (time).
22 " in Apr.
41 "
31 " in Mar.
10 " in Feb.
28−10=18. Feb. 18.

EXPLANATION. Subtracting the 22 days belonging to April, and the 31 days belonging to March, we find that the last 10 days of February are included in the time; hence to ascertain the date these 10 days must be deducted from the number of days belonging to February.

EXAMPLES FOR PRACTICE.

Find date of note.

(The Time Includes Grace.)

	Date of Maturity.	Time.
2.	June 7, 1905.....	93 days.
3.	Aug. 4, 1905.....	63 days.
4.	Dec. 12, 1904.....	57 days.
5.	Nov. 25, 1905.....	69 days.
6.	Dec. 13, 1905.....	123 days.
7.	May 5, 1904	6 mo. 3 da.

8. A retail dealer settled with a wholesale firm 43 days after balance was due by giving an interest-bearing note for 3 months. If the date of settlement was July 16, what should be the date of the note?

PARTIAL PAYMENTS.

455. A **Partial Payment** is a payment of only a part of a debt and its accrued interest.

456. The Supreme Court of the United States has adopted a method for determining the unpaid balance due on interest bearing notes, which has been made the law of most of the states. This method is known as the

UNITED STATES RULE.

1. A note of $2500 with interest at 6%, was given Apr. 18, 1904. Upon the back of the note were indorsed the following payments: Aug. 12, 1904, $450; May 30, 1905, $50; Sept. 6, 1905, $20; June 24, 1906, $1200; Oct. 12, 1906, $200. How much was due Apr. 18, 1907?

Operation.

Face of note, or original principal		$2500.
Interest from date, Apr. 18, 1904 to Aug. 12, 1904 (3 mo. 24 da.)		47.50
Amount due at time of first payment (Aug. 12, 1904)		$2547.50
First payment		450.
New principal, (Aug. 12, 1904)		$2097.50
(Since the 2d and 3d payments were each insufficient to pay the accrued interest, we compute interest to the time of the 4th payment.)		
Interest from Aug. 12, 1904 to June 24, 1906 (1 yr. 10 mo. 12 da.)		234.92
Amount June 24, 1906		$2332.42
Second payment, May 30, 1905 (less than int.)	$ 50.	
Third payment, Sept. 6, 1905 (less than int.)	20.	
Fourth payment, June 24, 1906	1200.	
		1270.
New principal, June 24, 1906		$1062.42
Interest to Oct. 12, 1906 (3 mo. 18 da.)		19.12
Amount Oct. 12, 1906		$1081.54
Fifth payment, Oct. 12, 1906		200.00
New principal, Oct. 12, 1906		$881.54
Interest from Oct. 12, 1906, to Apr. 18, 1907 (6 mo. 6 da.)		27.33
Amount due Apr. 18, 1907		$908.87

RULE I. *Find the amount of the principal from the date of the note to such time as the payment, or payments, indorsed on the note are sufficient to pay the accrued interest, and from such amount subtract all payments then or previously made; the remainder will be a new principal.*

II. *Similarly find the amount of this, and all subsequent new principals, as in* I; *the new principal corresponding in time to the date of settlement is the sum due on the note.*

REMARK. A careful study of the foregoing rule will render the fact obvious that the accrued interest must first be paid before any part of a payment can be applied to the reduction of the principal; hence, if a partial payment is less than the interest then accrued, no reckoning regarding the unpaid balance will be made until such time as some subsequent payment will meet interest demands.

EXAMPLES FOR PRACTICE.

(2)

$800. CHICAGO, ILL., May 12, 1902.

Two years after date I promise to pay John G. Briggs, or order, Eight Hundred Dollars, with interest at 6%. Value received. HENRY DAYMON.

What was the balance due at maturity, the following partial payments having been previously made: Oct. 5, 1902, $250; June 18, 1903, $175?

(3)

$1200. BUFFALO, N. Y., Oct. 15, 1899.

Two years after date we promise to pay E. B. Preston, or order, Twelve Hundred Dollars, with interest. Value received. HAMMOND & BRIGGS.

What was the balance due at maturity, indorsements being as follows: Oct. 15, 1900, $1000; Apr. 15, 1901, $200?

(4)

$1600. MADISON, WIS., June 1, 1902.

Six months after date I promise to pay S. B. Jones, or order, Sixteen Hundred Dollars, with interest at 6%. Value received. RICHARD ROE.

A payment of $5 was made on the first day of each of the following four months, and on Nov. 16, 1902, a payment of $1200 was made. What sum was due at maturity?

(5)

$500. ST. PAUL., MINN., Feb. 1, 1904.

On demand I promise to pay Claud Hemingway, or order, Five Hundred Dollars, with interest at 7%. Value received. WM. JENKINS.

Indorsed as follows: May 1, 1904, $40; Nov. 14, 1904, $8; Apr. 1, 1905, $12; May 1, 1905, $30. How much was due Sept. 16, 1905?

PARTIAL PAYMENTS BY THE BANKERS' METHOD.

457. The advantage of using this method in computing interest when partial payments have been made on obligations, lies not only in its general brevity, but especially in the ease with which interest for an additional period is computed at any time that the payment does not equal the interest due.

(1) By the Bankers' Method find the balance due Jan. 16, 1910, on the following note:

$1100. WOODSTOCK, CT., Jan. 1, 1904.

For value received, I promise to pay Frank Wells, or order, Eleven Hundred $\frac{\text{no}}{100}$ Dollars, on demand, with interest at 6%. C. D. WENTWORTH.

Indorsements on the above note are as follows: Sept. 1, 1904, $30; April 1, 1905, $200; Dec. 1, 1905, $180; March 1, 1907, $195; Sept. 16, 1907, $250; May 16, 1909, $100; July 16, 1909, $170.

Operation.—1st. Finding the Time Between Dates.

Date of Principal and Payments.		yr.	mo.	da.	Time Between Dates. (Compound subtraction, upper date from lower.)			Total Time in Days.
Of principal	$1100,	'04	1	1				
Of 1st paym't	30,	'04	9	1	— yrs.	8 mos.	— ds.	240 ds.
Of 2d paym't	200,	'05	4	1	— yrs.	7 mos.	— ds.	210 ds.
Of 3d paym't	180,	'05	12	1	— yrs.	8 mos.	— ds.	240 ds.
Of 4th paym't	195,	'07	3	1	1 yr.	3 mos.	— ds.	450 ds.
Of 5th paym't	250,	'07	9	16	— yrs.	6 mos.	15 ds.	195 ds.
Of 6th paym't	100,	'09	5	16	1 yr.	8 mos.	— ds.	600 ds.
Of 7th paym't	170,	'09	7	16	— yrs.	2 mos.	— ds.	60 ds.
Of settlement	$,	'10	1	16	— yrs.	6 mos.	— ds.	180 ds.
Proof of Subtractions,		6	00	15	2 yrs.	48 mos.	15 das.	2175 ds.
					or 6 yrs.	00 mos.	15 das.	

NOTE. Subtracting the date of the note from the date of settlement in the first column, and adding the other two columns in the operation *above, gives* three footings, which are found to be equal, thus proving the *work to this point*. This is a practice that the student should never neglect.

2d. Computing the Interest.

pal	=	$11|00.00	Pointing gives interest for 60 days.
ɔlying $11 by 4	=	44.	interest for 240 days.
e paym't is only $30,			hence we compute for next period,—
s ½ of $11 (30ds. int.)	=	38.50	interest for the next 210 days.
g	=	$1182.50	amount due April 1, 1899.
$ 30.			
tsum of paym'ts 200.		230.	
pal after April 1, 1899	=	$ 9|52.50	Pointing gives interest for 60 days.
ɔlying 9.52½ by 4	=	38.10	interest for 240 days.
g	=	$ 990.60	amount due December 1, 1899.
t 3d payment		180.	
pal after Dec. 1, 1899	=	$ 81|0.60	Pointing gives interest for 600 days.
ng 81.06 by 2	=	40.53	interest for 300 days.
ng 40.53 by 2	=	20.27	interest for 150 days. 450 days.
g	=	$ 871.40	amount due March 1, 1901.
t 4th payment		195.	
pal after Mar. 1, 1901	=	$ 6|76.40	Pointing gives interest for 60 days.
ɔlying 6.764 by 3	=	20.29	interest for 180 days.
ng 6.76 by 4	=	1.69	interest for 15 days. 195 days.
g	=	$ 698.38	amount due Sept. 16, 1901.
t 5th payment		250.	
pal after Sept. 1, 1901	=	$ 44|8.38	Pointing gives interest for 600 days.
ting to add		44.84	interest for 600 days.
g	=	$ 493.22	amount due May 16, 1903.
t 6th payment		100.	
ıfter May 16, 1903	=	$ 3|93.22	Pointing gives interest for 60 days.
ting to add		3.93	interest for 60 days.
g	=	$ 397.15	amount due July 16, 1903.
t 7th payment		170.	
ıfter July 16, 1903	=	$ 2|27.15	Pointing gives interest for 60 days.
ɔlying 2.27 by 3	=	6.81	interest for 180 days.
g	=	$ 233.96	amount due Jan. 16, 1904.

EXAMPLES FOR PRACTICE.

(*2*)

) San Jose, Cal., June 1, 1902.

ne day after date, I promise to pay to Geo. W. Curtis, der, Fifteen Hundred Dollars, in U. S. gold coin, with ɔst at 9%. Value received. J. G. Hill.

he following payments were endorsed: Oct. 1, 1902, Feb. 1, 1903, $30; Aug. 1, 1903, $500; April 1, 1904, ; Feb. 1, 1905, $350. How much was due Jan. 1, 1905?

ɣte. The *above is* a common form of demand note in the locality . *Such notes are often* permitted to run over due until the statutory *four years.*

458 MERCHANTS' RULE

The following rule is often used by merchants in determining the balance due on obligations when settlement is made within a year after interest commences. The obligation may or may not be in note form:

I. *Compute the amount of the principal to the time of settlement.*

II. *In like manner compute the amount of each payment from the time it was made to the time of settlement.*

III. *Subtract the sum of the amounts of the payments from the amount of the principal; the remainder will be the balance due.*

NOTE. In computations under this rule, it is customary to *count the exact time in days*, and this custom is followed in the problems following. Observe, however, that this rule is not governed by any statute, but is a matter of custom or agreement. When it is applied to interest-bearing debts running more than a year, it is usual to compute a settlement at the end of each year and take the balance for a new principal. In this case it is more equitable to find the time by compound subtraction. The underlying principle is that each of the parties shall receive interest on the money paid to the other.

EXAMPLES FOR PRACTICE

(*1*)

By the Merchants' Rule, find the balance due May 1, 1903, on an obligation bearing interest at 6%, dated June 12, 1902, and on which are the following endorsements: Oct. 24, 1902, $150; April 6, 1903, $225.

Operation.

Face of note, dated June 12, 1902		$600.
Interest to May 1, 1903 (323 days)		32.30
Sum of principal and interest at date of settlement		$632.30
First payment, made Oct. 24, 1902	$150.	
Interest on $150 to May 1, 1903 (189 days)	4.73, $154.73	
Second payment made Apr. 6, 1903	$225.	
Interest on $225 *to* May 1, 1903 (25 days)	.94, 225.94	
Sum of payments and interest at date of settlement		$380.67
Balance due on settlement		$251.63

(*2*) INDIANAPOLIS, IND., Jan. 1, 1909.

Thirty days after date, for value received, I promise to pay to H. G. McFarland, or order, One Thousand Dollars. $1000. GEO. CUMMINGS.

Indorsed as follows: April 1, 1909, $80; Aug. 1, 1909, $30; Oct. 1, 1909, $9; Dec. 1, 1909, $600. What was due Jan. 1, 1910? (NOTE: Interest after maturity at legal rate.)

(*3*) TOLEDO, OHIO, May 10, 1909.

On demand, I promise to pay David E. Nash, or order, Two Thousand Dollars ($2000) with interest at 6%. Value received. FRANK IVES.

Indorsed as follows: July 19, 1909, $800; Sept. 19, 1909, $400; Feb. 1, 1910, $300. How much was due April 9, 1910?

459. In states where interest is allowed on deferred interest payments, if payments have been made at irregular intervals, the previous rules regarding partial payments are subject to the following modifications:

If, at the end of any interest period, the payments are insufficient to pay the accrued interest, the principal will remain unchanged, but the interest will be diminished by the payments, and the part remaining unpaid will be treated as deferred interest which will draw interest until paid or until the note matures; and subsequent payments will apply as follows: first, to interest on deferred interest; second, to deferred interest, and last to accrued interest due, and principal.

NOTE. A few states have special rules for computing interest in cases of partial payments on obligations. Teachers in those states can, by selecting the problems from those given, arrange for special drill under these special rules to any extent desired.

ANNUAL INTEREST.

460. Annual Interest signifies, literally, interest payable annually, but the term, as generally applied, is made to mean an extra payment of interest as a penalty for not paying interest annually or at the end of some other stated period as promised in the note.

461. If B gives A his note for $1000 on 4 years' time with *interest at 6%*, payable annually, A will have a legal *claim against B at the* end of the first year for one year's

interest, $60. The laws of some states, in case of non-payment, would give A but one remedy, to bring action and thus force collection. Other states, by a decision of their courts, would give A the right to wait until the end of the four years and collect from B interest on deferred interest payments, as a penalty for non-fulfillment of the contract.

462. In states where the collection of interest upon interest is not sanctioned, money loaners frequently require their debtors to give coupon notes, of which the following is a sample:

COUPON NOTE.

$600.00 WINONA, MINN., May 1, 1902.

Two years after date I promise to pay Hemingway & Wright, or order, Six Hundred and $\frac{00}{100}$.....Dollars.
Value received.

JOHN D. HILL.

$18.00 WINONA, May 1, 1902. Two years after date I promise to pay Hemingway & Wright, or order, Eighteen Dollars. Value received. JOHN D. HILL.	$18.00 WINONA, May 1, 1902. Eighteen months after date I promise to pay Hemingway & Wright, or order, Eighteen Dollars. Value received. JOHN D. HILL.
$18.00 WINONA, May 1, 1902. One year after date I promise to pay Hemingway & Wright, or order, Eighteen Dollars. Value received. JOHN D. HILL.	$18.00 WINONA, May 1, 1902. Six months after date I promise to pay Hemingway & Wright, or order, Eighteen Dollars. Value received. JOHN D. HILL.

On payment of the coupon notes they are detached and delivered to the maker. In case they are not paid until the expiration of the time for which the original note was

given, interest can be collected from date of maturity, which is equivalent to the sum ascertained by an annual interest computation.

1. A note for $2000 was given for 5 years with interest at 6% per annum, payable annually. What sum was due at the expiration of the five years, provided no payment of either principal or interest had been previously made?

Operation.

Amount at simple interest=$2600.
Annual interest payment 120.
$\frac{(4+0)\text{ yrs.}}{2}\times 5=10$ yrs., deferred time.
$\$120\times.06\times10=\$72$, additional interest.
$\$2600+\$72=\$2672$, sum due.

EXPLANATION. It is evident that the debtor must pay the amount of the note at simple interest, together with the interest on the deferred interest payments. By the conditions of the note, $120 of interest money should have been paid at the end of each year, but since these interest payments were not made when due, they should draw interest for the time deferred, which is equivalent to the interest on one payment, $120, for the full time. The first interest payment was made four years after due; the second was deferred 3 years, the third, 2 years, the fourth, 1 year, and the fifth was paid when due. The time which marks these deferred interest payments is as follows: 4 yrs. + 3 yrs. + 2 yrs. + 1 yr. + 0 yrs. = 10 yrs. Since the difference in this series of years is a constant number, the sum may be found by multiplying ½ the sum of the extremes (the average time), by 5 (the number of interest payments), as shown in the operation. The interest on $120 for the full time, 10 years, is $72. This sum, added to the amount at simple interest, gives $2672, the sum due.

RULE. *To the amount of the principal at simple interest add the interest of each deferred interest payment from the date it falls due to the date of payment. The result will be the amount due at maturity.*

EXAMPLES FOR PRACTICE.

2. A note was given for 8 years with interest payable *quarterly.* *Nothing* was paid till 10 days after date of

maturity. For what time should interest be computed on deferred interest payment?

SUGGESTION. The number of interest periods is 32. The first interest payment was deferred 31 periods, 10 days, and the last interest payment was deferred 10 days. The sum of the extremes is $31\frac{2}{3}$ periods.

3. What amount will be required to pay a note of $2400 on Oct. 12, 1905, the date of the note being Sept. 20, 1902, and the rate of interest being 8%, payable semi-annually, no previous payments having been made?

4. A speculator purchased a section of western land and gave therefor his note of $12000, without interest, payable in 4 years from date. To secure the payment of interest on the credit given, the seller required the purchaser to give 4 additional non-interest-bearing coupon notes, due respectively in 1, 2, 3, and 4 years from date. If the rate of interest for the credit given was 6%, and nothing was paid until the expiration of 4 years, what amount was then due?

5. A note having a face value of $1800 was given March 15, 1900, with interest at 5%, payable quarterly. The first three interest payments were paid when due, but no subsequent payments were made until May 15, 1902, at which time note and interest were paid in full. Required the amount.

Find the amount due at maturity:

	Principal.	Rate per annum.	Terms.	Time.
6.	$1500	5%	Annually	4 yrs.
7.	$4000	6%	"	3 "
8.	$3600	4%	"	8 "
9.	$6000	7%	Semi-annually	2 yrs. 6 mo.
10.	$480	10%	"	4 " 2 " 20 da.
11.	$200	8%	"	5 " 0 " 15 "
12.	$8400	$4\frac{1}{2}$%	Quarterly	3 " 2 "

COMPOUND INTEREST.

463. Compound interest is interest which is computed upon the principal and accrued interest.

NOTE 1. The simple interest may be added to the principal annually, semi-annually, quarterly, or at shorter intervals, as the parties may agree; but the taking of compound interest is illegal in most states, except upon special conditions.

1. What is the compound interest of \$1500 for 3 yrs. at 5%?

Operation.

	\$1500	Principal for 1st year.
\$1500	×1.05=\$1575	" " 2d "
\$1575	×1.05=\$1653.75	" " 3d "
\$1653.75	×1.05=\$1736.4375	Amount for 3 years.
	1500	Given principal.
	\$236.44	Compound interest for 3 yrs.

RULE I. *Find the amount of the given principal at the given rate for one period, and make it the principal for the second period.*

II. *Find the amount of this new principal, and make it the principal for the third period, and so continue the work for the given time.*

III. *Subtract the given principal from the last amount; the remainder will be the compound interest.*

NOTE 2. When the time contains a fractional part of a period, find the amount for the full number of complete periods, upon which compute the interest for the fractional part of a period, and add it to the last amount before subtracting the principal.

EXAMPLES FOR PRACTICE.

2. What is the compound interest of \$720 for 4 yrs. at 6%?

3. What will \$500 amount to if loaned at compound interest for 3 years, 4 months, at 6%?

4. At 8% compound interest, compounded quarterly, what will $2000 amount to in 1 year, 3 months, and 15 days?

5. Find the compound interest of $1800 for 2 years, 1 month, and 6 days at 7%, the interest period being 6 months.

464. By an investigation of principles and their application as exhibited in the illustrative example, it will be seen that the principal for the second year is found by multiplying the given principal by the rate of amount, (100% + rate of interest); that the principal for the third year is found by multiplying the given principal by the square of the rate of amount; the principal for the fourth year by the cube of the rate of amount, etc. An investigation of the foregoing facts reveals the following:

$\$1 \times 1.06 =$ Principal for 2d year at 6%.
$\$1 \times \overline{1.06}^{2} =$ " " 3d " " "
$\$1 \times \overline{1.06}^{3} =$ " " 4th " " "
$\$1 \times \overline{1.06}^{4} =$ " " 5th " " "
etc., etc., etc.

465. The foregoing indicated computations illustrate the method to be observed in the formation of compound interest tables, which are a source of great convenience in computation.

TABLE

Showing the amount of $1, at compound interest for any number of periods from 1 to 50 inclusive.

P	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	P
1	1.015000	1.020000	1.025000	1.030000	1.035000	1.040000	1
2	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600	2
3	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864	3
4	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859	4
5	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653	5
6	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319	6
7	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932	7
8	1.126493	1.171659	1.218403	1.266770	1.316809	1.368569	8
9	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312	9
10	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244	10
11	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454	11
12	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032	12
13	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074	13
14	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676	14
15	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944	15
16	1.268986	1.372786	1.484506	1.604706	1.733986	1.872981	16
17	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901	17
18	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817	18
19	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849	19
20	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123	20
21	1.367058	1.515666	1.679582	1.860295	2.059431	2.278768	21
22	1.387564	1.545980	1.721571	1.916103	2.131512	2.369919	22
23	1.408377	1.576899	1.764611	1.973587	2.206114	2.464716	23
24	1.429503	1.608437	1.808726	2.032794	2.283328	2.563304	24
25	1.450945	1.640606	1.853944	2.093778	2.363245	2.665836	25
26	1.472710	1.673418	1.900293	2.156591	2.445959	2.772470	26
27	1.494800	1.706886	1.947800	2.221289	2.531567	2.883369	27
28	1.517222	1.741024	1.996495	2.287928	2.620172	2.998703	28
29	1.539981	1.775845	2.046407	2.356566	2.711878	3.118651	29
30	1.563080	1.811362	2.097568	2.427262	2.806794	3.243398	30
31	1.586526	1.847589	2.150007	2.500080	2.905031	3.373133	31
32	1.610324	1.884541	2.203757	2.575083	3.006708	3.508059	32
33	1.634479	1.922231	2.258851	2.652335	3.111942	3.648381	33
34	1.658996	1.960676	2.315322	2.731905	3.220860	3.794316	34
35	1.683881	1.999890	2.373205	2.813862	3.333590	3.946089	35
36	1.709140	2.039887	2.432535	2.898278	3.450266	4.103933	36
37	1.734777	2.080685	2.493349	2.985227	3.571025	4.268090	37
38	1.760798	2.122299	2.555682	3.074783	3.696011	4.438813	38
39	1.787210	2.164745	2.619574	3.167027	3.825372	4.616366	39
40	1.814018	2.208040	2.685064	3.262038	3.959260	4.801021	40
41	1.841229	2.252200	2.752190	3.359899	4.097834	4.993061	41
42	1.868847	2.297244	2.820995	3.460696	4.241258	5.192784	42
43	1.896880	2.343189	2.891520	3.564517	4.389702	5.400495	43
44	1.925333	2.390053	2.963808	3.671452	4.543342	5.616515	44
45	1.954213	2.437854	3.037903	3.781596	4.702359	5.841176	45
46	1.983526	2.486611	3.113851	3.895044	4.866941	6.074823	46
47	2.013279	2.536344	3.191697	4.011895	5.037284	6.317816	47
48	2.043478	2.587070	3.271490	4.132252	5.213589	6.570528	48
49	2.074131	2.638812	3.353277	4.256219	5.396065	6.833349	49
50	2.105242	2.691588	3.437109	4.383906	5.584927	7.106683	50

NOTE 1. The columns marked "P" contain the numbers of interest periods. If the time is 4 years with interest at 8%, payable quarterly, there are 16 periods with a rate of 2% a period.

TABLE

Showing the amount of $1, at compound interest, fo
any number of periods from 1 to 50 inclusive.

P	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	P
1	1.045000	1.050000	1.060000	1.070000	1.080000	1.090000	1
2	1.092025	1.102500	1.123600	1.144900	1.166400	1.188100	2
3	1.141166	1.157625	1.191016	1.225043	1.259712	1.295029	3
4	1.192519	1.215506	1.262477	1.310796	1.360489	1.411582	4
5	1.246182	1.276282	1.338226	1.402552	1.469328	1.538624	5
6	1.302260	1.340096	1.418519	1.500730	1.586874	1.677100	6
7	1.360862	1.407100	1.503630	1.605782	1.713824	1.828039	7
8	1.422101	1.477455	1.593848	1.718186	1.850930	1.992563	8
9	1.486095	1.551328	1.689479	1.838459	1.999005	2.171893	9
10	1.552969	1.628895	1.790848	1.967151	2.158925	2.367364	10
11	1.622853	1.710339	1.898299	2.104852	2.331639	2.580426	11
12	1.695881	1.795856	2.012197	2.252192	2.518170	2.812665	12
13	1.772196	1.885649	2.132928	2.409845	2.719624	3.065805	13
14	1.851945	1.979932	2.260904	2.578534	2.937194	3.341727	14
15	1.935282	2.078928	2.396558	2.759032	3.172169	3.642488	15
16	2.022370	2.182875	2.540352	2.952164	3.425943	3.970306	16
17	2.113377	2.292018	2.692773	3.158815	3.700018	4.327633	17
18	2.208479	2.406619	2.854339	3.379932	3.996020	4.717120	18
19	2.307860	2.526950	3.025600	3.616528	4.315701	5.141661	19
20	2.411714	2.653298	3.207136	3.869685	4.660957	5.604411	20
21	2.520241	2.785963	3.399564	4.140562	5.033834	6.108808	21
22	2.633652	2.925261	3.603537	4.430402	5.436540	6.658600	22
23	2.752166	3.071524	3.819750	4.740530	5.871464	7.257874	23
24	2.876014	3.225100	4.048935	5.072367	6.341181	7 911083	24
25	3.005434	3.386355	4.291871	5.427433	6.848475	8.623081	25
26	3.140679	3.555673	4.549383	5.807353	7.396353	9.399158	26
27	3.282010	3.733456	4.822346	6.213868	7.988062	10.245082	27
28	3 429700	3.920129	5.111687	6.648838	8.627106	11.167140	28
29	3.584036	4.116136	5.418388	7.114257	9.317275	12.172182	29
30	3.745318	4.321942	5.743491	7.612255	10.062657	13.267679	30
31	3.913857	4.538040	6.088101	8.145113	10.867669	14.461770	31
32	4.089981	4.764942	6.453387	8.715271	11.737083	15.763329	32
33	4.274030	5.003189	6.840590	9.325340	12.676050	17.182028	33
34	4.466362	5.253348	7.251025	9.978114	13.690134	18.728411	34
35	4.667348	5.516015	7.686087	10.676582	14.785344	20.413968	35
36	4.877378	5.791816	8.147252	11.423942	15.968172	22.251225	36
37	5.096860	6.081407	8.636087	12.223618	17.245626	24.253835	37
38	5.326219	6.385477	9.154252	13.079271	18.625276	26.436681	38
39	5.565899	6.704751	9.703508	13.994820	20.115298	28.815982	39
40	5.816365	7.039989	10.285718	14.974458	21.724522	31.409420	40
41	6.078101	7.391988	10.902861	16.022670	23.462483	34.236268	41
42	6.351615	7.761588	11.557033	17.144257	25.339482	37.317532	42
43	6.637438	8.149667	12.250455	18.344355	27.366640	40.676110	43
44	6.936123	8.557150	12.985482	19.628460	29.555972	44.336960	44
45	7.248248	8.985008	13.764611	21.002452	31.920449	48.327286	45
46	7.574420	9.434258	14.590488	22.472623	34.474085	52.676742	46
47	7.915268	9.905971	15.465917	24.045707	37.232012	57.417649	47
48	8.271456	10.401270	16.393872	25.728907	40.210573	62.585237	48
49	8.643671	10.921333	17.377504	27.529930	43.427419	68.217908	49
50	9.032636	11.467400	18.420154	29.457025	46.901613	74.357520	50

NOTE. 2. To find the amount of $1 for more than 50 years, separat
the time into such parts as are shown by the table; then find by the tabl
the amount of $1 for each separate part and multiply these amounts t

ıer. Thus, the amount of $1 for 69 years at 5% is equal to the amount 1 for 30 years ($4.321942), multiplied by the amount of $1 for 39 years 704751), which is $28.977545.

466. To find the compound interest or the amount by ı tables.

1. What is the compound interest of $400 for 9 years 6%, if compounded quarterly?

EXPLANATION. Since the interest is compounded quarterly, there are ɔeriods, and since the rate is 6% per annum, it is 1½% per period. On a ı with 36 in the column headed P, we find in the 1½% column, $1.709140, ich is the amount of $1 for the given time and rate; this number minus principal, $1, leaves $709140, the compound interest on $1, and)9140×400=$283.656, the required answer.

EXAMPLES FOR PRACTICE.

2. What will $1350 amount to in 27 years at 5%, if the terest is compounded annually?

3. What is the amount of $2700 for 16 yrs. 7 mo. 6 da., compounded annually at 5%? (463, Note 2.)

4. Find the interest of $2430, compounded semi-annuly for 15 years at 8%.

5. Find the amount of $3500, compounded semi-annuy at 7% for 13 years, 9 months.

6. A man deposited $2500 in a savings bank, which ıid interest semi-annually, at the rate of 4% per annum; interest was placed on deposit at the end of each interest rm, what sum would be due the depositor at the end of years?

7. How much more will I receive on a $1200 loan comɔunded semi-annually at 6% than if compounded annuly at the same rate, for 10 years?

467. To find other elements than interest by the tables.

1. At what rate per annum will $640 amount to 033.9392 in 23 years?

Operation and Explanation.

$3033.9392 (amt. in 23 yrs.) ÷ $640 (prin.) = 4.74053 (amt. of $1).

Passing from the 23d interest period in a horizontal line we find this amount of $1 (4.74053) in the 7% column; hence 7% is the required rate.

2. In what time will $3000 amount to $8694.834 if compounded semi-annually at 6% per annum?

Operation and Explanation.

$8694.834 (amt. for required time) ÷ $3000 (prin.) = 2.898278 (amt. of $1).

Since the rate is 6% per annum, the rate per period is 3%. Glancing down the 3% column, we find this amount (2.898278) on a line with the 36th period; but since there are two periods to each year, the time is 18 years.

3. What principal will amount to $1511.2475 if compounded annually for 10 years at 8%?

Operation and Explanation.

$1511.2475 (amt. of required principal) ÷ 2.158925 (amt. of $1 for given time and rate) = $700 (required principal).

The facts stated in the example show that the divisor is to be found in the 8% column on a line with the number 10 in the period column.

EXAMPLES FOR PRACTICE.

4. At what rate per cent. per annum will $1750 amount to $4875.43525, if compounded 21 years annually?

5. If $80, compounded quarterly for a period of 7 yrs. 3 mo., amounted to $163.71256, what was the rate per cent. per annum?

6. In what time will $300 amount to $1187.778, if compounded semi-annually at 7% per annum?

7. At what rate per cent. per annum will $8000 amount to $594860.16, if compounded annually for 50 years?

8. What principal will amount to $106.63344, if compounded annually for 25 years at 4%?

9. How long a time will be required for $100 to amount to $6258.5237, if compounded annually at 9%?

10. At what rate per cent. per annum will $5000 amount to $8673.885, if compounded quarterly for 9 yrs. 3 mo.?

11. In what time will $360 amount to $2188.11636, if compounded annually at $4\frac{1}{2}\%$ per annum?

12. If $821.1414 was received as the amount of a loan compounded at 4 per cent. per annum for a period of 8 years, what was the principal?

468. REVIEW EXAMPLES IN INTEREST.

1. A broker's interest collections for a single day were as shown by the following statement. What was the total sum of interest money collected?

Principal.	Time.	Rate.	Interest.
$840	35 days	6%	$
$1300	126 "	7%	$
$1231	16 "	9%	$
$1080	17 "	$3\frac{1}{2}\%$	$
$1350	24 "	6%	$
$800	52 "	$4\frac{1}{2}\%$	$
$315	108 "	10%	$
$5400	67 "	8%	$
$1440	71 "	5%	$
$960	220 "	6%	$
$1310	28 "	9%	$
	Total		$

NOTE. A reasonable amount of time for the solution of the above example by "Business Method" of computation, for one who is reasonably quick and accurate in addition, is 2 min. 30 sec.

2. What is the accurate interest of $850 for 2 years, 5 months, and 20 days, at 6%?

3. A note of $975, given for 90 days, without grace, was dated April 18th and paid August 5th. Required the amount, the rate of interest being 7%.

4. If the ordinary interest on a sum of money loaned for a given time is $14.60, what is the accurate interest?

5. A speculator contracted with an eastern syndicate to loan money in North Dakota, agreeing to pay interest at the rate of 5% per annum on all sums loaned. The first loans under the contract were made May 12, 1904, with interest at 8% per annum. Collections on these loans were made June 18, 1906. If the speculator's profits were $1162.35, what sum should he remit to the eastern syndicate?

6. What is the interest of $135.75 for 3 mo. 24 da. at 4½%?

7. What is the amount of $514.90 for 3 yr. 4 mo. at 5½%?

8. What is the interest on a note of $580 given Feb. 13, 1904, for 90 days with interest at 7%? (No grace.)

9. A note was given Nov. 30, 1905, for 3 months, without grace. What was the date of maturity?

10. What sum of money loaned for 7 mo. 18 da. at 10% will amount to $1250?

11. The interest collected on a loan of $960 for 9 mo. 20 da. was $8.40. At what rate per cent. was the money loaned?

12. If $1600 produced $32.50 interest, what was the time for which it was loaned, the rate being 5% per annum?

13. How much is the interest on a note for $516.25, dated March 5, 1905, and payable August 19, 1906, at 7%?

14. A man sold a farm for $11450; the terms were cash $4500, and the remainder in two equal installments due respectively in 4 months and 10 months with interest at 6% per annum. What was the whole sum paid by the purchaser?

15. August 12, 1904, a grain dealer borrowed $8528, with which he purchased wheat at 82 cents a bushel. If *the wheat* was bought Sept. 18 and sold May 3, following,

at 98 cents a bushel, cash, how much did he gain by the transaction, the storage bill which he paid being $\frac{1}{4}$ of a cent a bushel per month, and the rate of interest being 6% per annum?

16. A merchant's net gain in business is 12% per annum on a capital of $22500. If he discontinues business and loans the money at 8% per annum, in how long a time will his losses by the change amount to $2400?

17. A merchant buys a bill of goods on 6 months' credit, subject to a discount of 5% if paid at the end of the first month. How much will he gain by borrowing the money at 6% with which to take advantage of the discount, the invoice price of the goods being $4500?

18. If goods are bought on 90 days' credit, with a discount of 5% for cash, what annual rate of interest does the purchaser really pay by not taking advantage of discount privilege?

19. A note for $1400, with interest at 6% per annum, was dated May 4, 1903. What sum was due at maturity, May 4, 1905, if the following payments were thereon indorsed: Aug. 15, 1903, $320; May 4, 1904, $475; June 19, 1904, $5; Aug. 19, 1904, $15; Jan. 4, 1905, $150? (U. S. Rule.)

20. If the accurate interest on a sum of money loaned for a given time is $79.20, what is the ordinary interest?

21. A merchant bought a bill of goods invoiced at $1800 on 4 months' credit, but subject to a discount of 3% if paid within 10 days. If he remits $1200 before the expiration of the 10 days, what sum will be required to pay the balance of the account 6 months after date of purchase, money being worth 6%?

22. A note for $1320 was given for a specified number of days, with interest at 6% per annum. The amount due

at maturity, September 14, was $1333.86. What was the date of the note?

23. A note for $1620, given for 5 years, contains the promise to pay interest at 6% per annum quarterly. Provided nothing was paid until 12 days after maturity, what sum was then due? (460.)

24. A tenant, who occupies a residence at a monthly rental of $16, buys the property at a cost of $3000. If he pays $25 a year for repairs, $21.50 for taxes, and $10 for insurance, how much will he lose in 3 years if the property advances during that time $100, money being worth 6%?

25. What will $4200 amount to in 21 years at 7%, semi-annual compound interest?

26. Two young men receive the same legacy at the age of 21 years. A wastes $50 each year in foolish extravagance, and B deposits each year the same amount in a savings bank, which pays 4% semi-annual compound interest. What will be the difference in their resources at the end of 5 years, the earnings and legitimate expenses of each being the same?

27. What will a loan of $2000 amount to in 20 years if the interest be compounded quarterly at the rate of 8% per annum?

28. In what time will $500 amount to $1213.63 if loaned at 6% per annum, and the interest be compounded semi-annually?

COMMERCIAL DISCOUNT.

469. Commercial Discount is an abatement from the value at maturity of commercial paper bought or paid for before due.

NOTE. The term *commercial*, as here employed, should not be confounded with the word as used in a mercantile sense. As shown by the definition, it is here used with special reference to commercial paper.

470. The **Present Worth** of commercial paper is its value in present time.

471. Commercial discount may be classified as *True Discount* and *Bank Discount*.

TRUE DISCOUNT.

472. Considering money worth 6%, it is evident that an obligation of $106, due in one year, has the same value as $100 of ready money, since the $100, if loaned for a year, will have the same value at the end of the year as the future obligation.

473. The **True Present Worth** of an obligation or debt due in future time is such a value as will amount to the obligation if loaned at the legal rate of interest for the intervening time.

474. The **True Discount** is the difference between the value of a debt at maturity and its present worth.

475. From the foregoing definitions and explanations it will be seen that the true present worth is simply an interest element (principal), which may be easily found from principles already explained (438). The time and rate are given, and the sum due at maturity is the amount.

1. What is the true present worth of a $600 note, dated March 5, 1906, with interest at 6%, and maturing Sept. 5, 1906, assuming the present date to be June 7, 1906, and money worth to the purchaser 5% per annum?

Operation.

$600 × 1.03 = $618, value at maturity
Time from June 7 to Sept. 5, 90 days
.90 ÷ 72 = .0125, interest on $1
$1 + .0125 = $1.0125 amt. of $1
$618 ÷ 1.0125 = $610.37, present worth
$618 — $610.37 = $7.63, true discount

EXPLANATION. We first find the amount due at maturity by adding interest to principal. This amount divided by the amount of $1, for the given time and rate, gives $610.37, the true

present worth. The value at maturity diminished by the true present worth gives the true discount, which is $7.63. (473.)

NOTE. The time constituting the discount period for any fractional part of a year is the exact number of days.

RULE I. *To find the true present worth, divide the amount due at maturity by the amount of $1 for the given time and rate.*

II. *To find true discount, subtract the present worth from the amount of the debt at maturity.*

EXAMPLES FOR PRACTICE.

NOTE. In the solution of the following ten examples grace is to be considered.

Find true present worth and true discount:

	Face of Paper.	Date.	Time.	Rate of Int.	Date Discounted.	Rate of Disc.
2.	$1200	May 4, 1905	8 mo	7%	Oct. 12, 1905	6%
3.	$120.50	Aug. 29, 1905	1 yr		Dec. 18, 1905	8%
4.	$750	Feb. 20, 1906	6 mo	5%	June 7, 1906	6%
5.	$720.80	July 10, 1906	90 da		Aug. 17, 1906	7%
6.	$1500	Nov. 13, 1905	60 da	5%	Jan. 5, 1906	6%
7.	$2250	Apr. 3, 1904	2 yr	5%	Jan. 31, 1906	10%
8.	$180	Sept. 28, 1905	10 mo		May 17, 1906	6%
9.	$225	June 10, 1906	30 da		June 10, 1906	7%
10.	$450	May 12, 1906	90 da	8%	May 30, 1906	6%
11.	$1300	Jan. 3, 1906	3 mo	7%	Mar. 1, 1906	6%

12. A merchant bought 200 barrels of apples on 4 months' credit, at $3.10 per barrel. One month from the date of purchase he paid the present worth of the bill. Considering money worth 6%, what amount did he pay?

13. I can buy coffee at 13 cents a pound cash, or at 15 cents on 90 days' credit. How much will I save on a purchase of 720 pounds by dealing on a cash basis, if money is worth 6%?

14. I hold A's note for $1800, payable by its terms in 4 equal installments, due respectively in 3 months, 6 months, 9 months, and 1 year, without interest; he being desirous of paying the note at once, I agree to accept the

true present worth. If money is considered worth 5% per annum, what sum should I receive?

15. A broker holds my obligation for $7200, due April 10, 1907, without interest. Being in need of ready money, he offers to receive whatever sums I am willing to advance, and to appropriate the same according to the principles of true discount. February 9, 1907, I paid him $4500, and 15 days later I paid the remainder of the debt at its present worth. How much was the remainder at that time, money being considered worth 8%?

NOTE. The foregoing method of application of commercial discount has passed into such disuse during the last quarter of a century, that it may be regarded, at the present time, as nearly obsolete. The *Bankers' Method*, used in an earlier day by bankers and brokers only, is now employed in computation in nearly all departments of business life. In comparison with true discount, it will be found to lack equity, as it gives the creditor undue advantage over the debtor.

BANK DISCOUNT.

476. The **Bank Discount** on an obligation due in future time is the simple interest computed on the amount due at maturity for the given time and rate.

477. The **Bankable Present Worth**, or proceeds of an obligation due in future time, is the difference between the value at maturity and the bank discount.

REMARK. If an individual invests money in the purchase of a future obligation according to the principles governing true discount, he receives simple interest on the money invested; but if the purchase be made according to the principles governing bank discount, he receives not only the interest on the sum invested, but the interest on the interest of that sum also.

1. I hold a note which will be worth, at maturity, $1830. What will a bank that charges 8% per annum for the use of money pay for it 75 days before due?

Operation.

$\$1830 \times \frac{.75}{45} = \$30.50$

$\$1830 - \$30.50 = \$1799.50$

EXPLANATION. We first find the bank discount, which is the simple interest on the value at maturity, $30.50 (476). Subtracting the bank discount from the sum due at maturity, gives the bankable present worth, $1799.50.

RULE I. *Compute the simple interest on the sum due at maturity for the given time and rate. The result will be the bank discount.*

II. *From the sum due at maturity subtract the bank discount. The result will be the bankable present worth or proceeds.*

EXAMPLES FOR PRACTICE.

2. I hold a note for $650, dated June 7, and payable, with interest at 5%, 8 months after date, without grace. What will be the proceeds or bankable value, if discounted at a bank at 8% on October 12th?

NOTE 1. The expression, "*discounted at a bank*," signifies, sold to a bank at a discount.

3. A note, worth $925 at maturity, was discounted at a bank 112 days before due. Find the bank discount and proceeds at 10% per annum.

NOTE 2. Other examples, requiring an application of the principles of bank discount, are given in banking.

BANKING.

478. A **Bank** is an institution incorporated, either under national or state law, for the purpose of performing a part or all of the following functions: To receive and properly care for money placed in custody by depositors; to discount notes and other commercial paper; to loan money; to make collections; to issue bank notes or bills as a circulating medium, to be used in place of legal money; to facilitate the transmission of money from one place to another by means of drafts, bills of exchange, or other bank paper.

479. A **National Bank** is a bank incorporated under *national* law.

480. A **State Bank** is a bank incorporated under the laws of a state.

481. A **Bank of Deposit** is a bank whose leading function is the receiving and loaning of money taken into custody from depositors.

REMARK. Money deposited in a bank becomes at once the legal property of the bank. A deposit, in a legal sense, is a loan of money to the bank and is due to the depositor, in most cases, on demand.

482. A **Bank of Issue** is a bank that issues bank notes or bills which are placed in circulation as money.

REMARK. State banks were formerly banks of issue, but, by the passage of the National Banking Law, the privilege of issuing bank bills was practically destroyed by an excessive taxation placed upon their circulation; hence, at the present time, only national banks are banks of issue. The United States government, by a provision of the National Banking Act, becomes security for the redemption of all national bank bills, in consideration of government bonds placed in the treasury department as collateral security.

483. The different functions of banking to which the principles of arithmetic mainly apply are the following: Discounting commercial paper; interest on deposits and loans, and the purchase and sale of drafts and bills of exchange. Bank discount has already been considered (476), hence, only an application of the principles already explained will be found necessary in connection with our present subject.

484. Computations in Bank Discount are neither more nor less than percentage principles applied. The sum due at maturity is the *base*, and the bank discount and proceeds are *percentages*.

REMARK 1. The *absolute rate* is the rate that corresponds to the given time. If the annual rate is 6%, the absolute rate for 8 months is 4%; for 90 days it is .90÷60=.015, etc.

REM. 2. Since the proceeds plus the bank discount equals the value at maturity, it is evident that the absolute rate of bank discount plus the absolute *rate of proceeds* equals 100%; therefore, when either rate is

known, the other may be instantly determined by subtracting the given rate from 100%.

485. The following formulas, which are given instead of rules, will be found to cover the whole ground of computation.

1. Value at maturity×Absolute rate of disc.=Discount.
2. Value at maturity×Absolute rate of proceeds=Proceeds.
3. Discount÷Absolute rate of disc.=Value at maturity.
4. Proceeds÷Absolute rate of proceeds=Value at maturity.
5. Discount÷Value at maturity=Absolute rate of discount.
6. Proceeds÷Value at maturity=Absolute rate of proceeds.

486. Since the bank discount is the simple interest on the sum due at maturity for the given time and rate, we recommend, as a solution for obtaining it, the Business Method of interest computations (435). The proceeds may then be found by subtracting the bank discount from the sum due at maturity. The above formulas are given more especially for the purpose of exhibiting the direct relations that the different elements sustain to one another.

EXAMPLES FOR PRACTICE.

Find bank discount and proceeds:

	Val. at maturity.	Time.	Rate.		Val. at maturity.	Time.	Rate.
1.	$485.	84 da.	6%.	*5.*	$1650.	27 da.	7%.
2.	$1232.	73 da.	5%.	*6.*	$840.	39 da.	8%.
3.	$432.50	123 da.	8%.	*7.*	$925.	212 da.	6%.
4.	$720.	117 da.	10%.	*8.*	$1120.	65 da.	6%.

9. A note for $960, dated May 12, was given for 6 months, with interest at 6%. Allowing grace, what were the proceeds if discounted at a bank Sept. 3, the bank rate of interest being 8%?

487. The definition and general requisites of notes have already been given (442); but for the benefit of the learner, we append specimens of leading note forms.

10. **Time Note Without Interest.**

400\frac{00}{100}$ ROCHESTER, N. Y., June 5, 1906.

Ninety days after date, I promise to pay Geo. D. 'alsingham,..... or order,
ourteen Hundred.....Dollars.
alue received.

E. B. WENTWORTH.

REMARK. The above note is payable 93 days from June 5, or on Sept. 1906. Since no interest is mentioned, it cannot draw interest until aturity, after which, if it remains unpaid, it will draw interest at the gal rate in New York.

11. **Time Note With Interest.**

'48$\frac{50}{100}$ DAYTON, OHIO, Feb. 18, 1905.

Four months after date, I promise to pay Frank S. arden,.....or bearer,
even hundred forty-eight & $\frac{50}{100}$.....Dollars.
'ith interest. Value received.

H. G. GILMORE.

REMARK. The above note is payable 4 mo. 3 da. from Feb. 18, or on ne 21, 1905. No indorsement is necessary to transfer it, as it is payable bearer. It will draw interest at the legal rate in Ohio.

12. **Demand Note.**

350$\frac{xx}{100}$ CEDAR RAPIDS, IOWA, Aug. 8, 1905.

On demand, for value received, I promise to pay Nelson ros. & Brown,..... or order,
hree hundred fifty &.....$\frac{xx}{100}$ Dollars.
'ith interest at 8%.

S. B. JENKINS.

REMARK. The above note is due at any time the holder sees fit to esent it and make demand for payment. It will draw interest according its terms until paid.

13. **Joint Note.**

$1380\frac{xx}{100}$ SPARTA, WIS., Nov. 28, 1904.

Eight months after date, we promise to pay Mortimer & Phelps,.....or order,
Thirteen hundred eighty & $\frac{no}{100}$.....Dollars.
Without grace. Value received.

S. B. PHILLIPS.
C. E. LANGDON.

REMARK. The above note is called *joint* because two persons have joined in an agreement to pay it, which makes them jointly responsible. In case of non-payment the holder's right of action is against both parties jointly, as each of the makers is responsible only for his proportionate part (one-half) the face of the note. The note matures July 28, 1905.

14. **Joint and Several Note.**

$754\frac{75}{100}$ DIXON, ILL., Jan. 15, 1906.

Two months after date, we jointly and severally promise to pay to F. C. Clayton,.....or order,
Seven hundred fifty-four & $\frac{75}{100}$.....Dollars,
at Merchants' bank, without grace. Value received.

JOHN D. WILKES.
E. C. HALL.

REMARK. The above note differs from an ordinary *joint note* by holding each of its makers individually responsible for payment. Action may be brought, in case of non-payment, against the makers *jointly*, or against either of them *individually*. Presentation, and demand for payment must be made at Merchants Bank.

15. What are the proceeds of note No. 10, if discounted at bank Aug. 20, 1906, at 8%?

16. What are the proceeds of note No. 11, if discounted at bank May 1, 1905, at 9%?

17. What are the proceeds of note No. 13, if discounted at bank Feb. 13, 1905, at 6%?

18. What are the proceeds of note No. 14, if discounted at bank Jan. 30, 1906, at 7%?

19. The true discount on a note, for 98 days, is \$14.38; what is the bank discount, considering money worth 6%?

20. The bank discount on a note for 112 days is \$9.36; what is the true discount, considering money worth 8%?

488. To find the face of a note, or the sum due at maturity.

1. For what sum must I draw my note, payable in 100 days, without grace, that the proceeds, when discounted at a bank at 8%, will be \$850?

Operation.

$1.00 \div 45 = .02\frac{2}{9}$, rate of discount
$1.00 - .02\frac{2}{9} = .97\frac{7}{9}$, rate of proceeds
$\$850 \div .97\frac{7}{9} = \$869.32$, face

or

$(\$850 \times 45) \div (45 - 1.00) = \$869.32$

EXPLANATION. We first find the rate of discount by (433), which is $2\frac{2}{9}$%; by subtracting the rate of discount from 100% we have the rate of proceeds, or the proceeds of \$1.00 of the note, $97\frac{7}{9}$%, or, regarded as the proceeds of \$1, \$.$97\frac{7}{9}$; dividing the proceeds by the rate of proceeds, (formula 4), gives \$869.32, the face of the note, or the sum due at maturity. For an explanation of the second operation, see (440).

RULE. *Divide the given proceeds by the proceeds of \$1 at the given rate and for the given time.*

REMARK. In states where grace is sanctioned by law, the term of discount is 3 days in excess of the time expressed in the note.

EXAMPLES FOR PRACTICE.

Find face of note:

	Proceeds.	Term of Disc.	Rate of Disc.
1.	\$500	90 days	6%
2.	\$780	84 "	8%
3.	\$1547.50	115 "	5%
4.	\$946.30	67 "	7%
5.	\$1215	108 "	10%
6.	\$1348.45	17 "	6%
7.	\$430.75	72 "	7%
8.	\$1150	6 mo.	$5\frac{1}{2}$%
9.	\$375	4 "	8%
10.	\$872	3 "	9%

11. A bank, whose rate of discount is 8%, gave me, as the proceeds of a 90-day note, $532.10. What was the face of the note? (With grace.)

12. A man, owing a Chicago firm $5000, gave his note for $3200, payable in 30 days, without grace, which was accepted at its bankable value; he paid the remainder of the debt in cash, which he procured at a bank by discounting his 90-day note, without grace. If the bank rate of discount regarding each note was 6%, what was the face of the second note?

13. A banker charged me $9.84 for discounting my note for $820, which matured in 54 days. What was the annual rate of discount?

14. A banker, whose rate of discount is 9%, gave me, as the proceeds of a $1600 note, $1564.80. What was the term of discount?

SAVINGS BANKS.

489. A **Savings Bank** is a bank having for its leading function the receiving of money from its depositors for safe-keeping.

REMARK. Savings banks usually pay interest on deposits according to certain rules and regulations established by the directors. These rules are somewhat different in different banks; some banks will not pay interest on deposits if any portion is withdrawn before the end of some fixed period of time; others pay interest on the smallest balances remaining during the period. The interest, if not withdrawn at the end of the period, is credited to the depositor as an additional deposit, and draws interest during the next period. Thus it will be seen that savings banks allow compound interest on deposits.

490. The **Interest Term** is the period of time at the close of which depositors' accounts are balanced and the interest credited if not withdrawn.

REMARK 1. The length of the interest term is not uniform among the *different banks.* The term with some savings banks is one year; with *others it is 6* months, beginnin and July 1st. Still another class

divide the year into 4 terms, beginning Jan. 1st, Apr. 1st, July 1st, and Oct. 1st.

REM. 2. Some savings banks do not allow interest on deposits until the beginning of the following term, or the following month after they are made, except when made the first day of the term or month.

REM. 3. In computing interest on deposits, fractional parts of a dollar are disregarded.

491. To find the balance of a deposit account when there are no withdrawals.

1. A depositor's balance in a savings bank Jan. 1, 1903, was $50. He afterward made the following deposits: Jan. 18, $40; Feb. 8, $65; Apr. 9, $20; Apr. 21, $120; June 16, $80; Aug. 7, $60; Sept. 19, $200; Nov. 3, $10; Dec. 14, $30. If the interest term was 6 months, and interest was to begin on the first day of the calendar month following the deposit, in each case, what was the balance of the account Jan. 1, 1904, the rate of interest being 4%?

Operation.

1903									
Jan.	1	Balance	50		On Int.	6	mo. or	$50×6=	$300 for 1 mo.
Jan.	18	Deposit	40		"	5	"	40×5=	200 "
Feb.	8	"	65		"	4	"	65×4=	260 "
Apr.	9	"	20	}	"	2	"	140×2=	280 "
Apr.	21	"	120	}					
June	16	"	80		"	0	"	80×0=	0 "
June	30	Interest	3	47				.01 of $1040÷3=	$3.47
July	1	Balance	378	47	"	6	"	378×6=	2268 for 1 mo.
Aug.	7	Deposit	60		"	4	"	60×4=	240 "
Sept.	19	"	200		"	3	"	200×3=	600 "
Nov.	3	"	10		"	1	"	10×1=	10 "
Dec.	14	"	30		"	0	"	30×0=	0 "
Dec.	31	Interest	10	39				.01 of $3118÷3=	$10.39
1904									
Jan.	1	Balance	688	86					

EXPLANATION. The balance left over from the preceding term was in the bank 6 full calendar months of first interest term, hence drew interest for 6 mo., which is equivalent to the interest of $300 ($50×6), for 1 month. The first deposit, $40, commenced drawing interest Feb. 1, hence was on interest 5 months, which is the same as the interest on 5 times that sum for one month. Reasoning in the same manner concerning the other deposits, we find that the total deposits for the first term are (in use) the same as a single deposit of $1040 for 1 month. The interest as found by

Business Method is $10.40÷3=$3.47. Adding this sum to the other deposits for the term, gives, for the balance at the beginning of the second term, $378.47. Proceeding with the deposits of the second term in the same manner, we find that the amount due the depositor at the end of the year is $688.86.

EXAMPLES FOR PRACTICE.

2. A depositor opened an account with a savings bank Feb. 24, 1904, and deposited $150. He afterward made the following deposits: March 5, $90; May 17, $50; June 10, $225; Sept. 29, $100; Nov. 8, $60; Dec. 16, $130. If the interest term was 6 months and interest was to begin, in each case, on the first day of the quarter following the deposit, what was the balance due the depositor Jan. 1, 1905, the interest rate being 4% per annum?

3. If the depositor mentioned in the former example, had received interest on smallest quarterly balances, what would have been the balance standing to his credit on Jan. 1, 1905, the interest term and rate being the same?

4. On Jan. 1, 1905, a depositor's balance in a savings bank, whose interest term is 3 months, was $280. During the first six months of the year he made the following deposits: Feb. 22, $145; May 1, $200; May 29, $40; June 1, $125. Allowing interest on smallest monthly balances at 5% per annum, what was the credit balance of his account July 1, 1905?

492. To find the balance of a deposit account when there are withdrawals.

1. A depositor's balance in a savings bank Jan. 1, 1905, was $230. During the year he made the following deposits: Feb. 19, $85; March 25, $165; Apr. 13, $90; June 3, $310; June 22, $15; July 30, $320; Aug. 18, $250; Nov. 9, $60; Dec. 24, $120. His withdrawals were: Jan. 29, $65; March 31, $50; Apr. 3, $20; May 5, $210; July 20, $100; Aug. 2, $250; Aug. 14, $40; Oct. 15, $400. If the depositor *received interest* at the rate of 4% per annum on smallest

quarterly balances, what was the balance to his credit Jan. 1, 1906, the interest term being 6 months?

Operation.

Date			Withdrawals	Deposits	Balance	Sm. Bal.	
1905							
Jan.	1	Balance	.....	230.	230.		Interest on $390 (165+225) for 3 months is $3.90.
"	29	.....	65.	.....	165.		
Feb.	19	.....	.....	85.	250.	165.	
Mar.	25	.....	.....	165.	415.		
"	31	.....	50.	.....	365.		
Apr.	3	.....	20.	.....	345.		
"	13	.....	.....	90.	435.		
May	5	.....	210.	.....	225.	225.	Since 90 days is the unit of time, the interest for 1 quarter is $\frac{1}{100}$ of the principal. (433.)
June	3	.....	.....	310.	535.		
"	22	.....	.....	15.	550.		
"	30	Interest	.....	3.90	553.90		
"	"	Balance	553.90				
			898.90	898.90			
July	1	Balance	.....	553.90	553.90		
"	20	.....	100.	.....	453.90	453.90	Interest on $786 (453+333) for 1 quarter is $7.86. (See 490, Rem. 3.)
"	30	.....	.....	320.00	773.90		
Aug.	2	.....	250.	.....	523.90		
"	14	.....	40.	.....	483.90		
"	18	.....	.....	250.	733.90		
Oct.	15	.....	400.	.....	333.90		
Nov.	9	.....	.....	60.	393.90	333.90	
Dec.	24	.....	.....	120.	513.90		
"	31	Interest	.....	7.86	521.76		
"	"	Balance	521.76				
			1311.76	1311.76			
1906							
Jan.	1	Balance	.....	521.76	521.76		

EXPLANATION. We arrange the deposits and withdrawals in separate columns as shown in the operation, recording them in the order of their respective dates. We add to the former balance each deposit, and subtract therefrom each withdrawal, and, in each case, record the result; by this means the smallest balance is easily discovered. For convenience we separate interest periods by double lines, and quarters by single lines. The interest due the depositor, at the end of the first interest term, according to the terms expressed in the example, is $3.90, and since it is not withdrawn, *it must be credited to* the depositor's account, as shown, on the

last day of the term. Balancing the account and bringing down the balance for the beginning of the new term, we continue the work of the second term in the same manner and find the balance to be $521.76.

EXAMPLES FOR PRACTICE.

2. In the preceding example, what would have been the amount due the depositor Jan. 1, 1906, if the interest term had been 3 months, and interest had been allowed on smallest monthly balances at the same rate per annum?

3. What was due on the following savings-bank account Jan. 1, 1904, interest at 4% per annum being allowed on smallest quarterly balances, and credited at the end of each quarter? Account opened Jan. 1, 1903, and $325 deposited. Deposits: Jan. 28, $50; Feb. 11, $180; Nov. 15, $215. Withdrawals: Jan. 12, $95; Dec. 21, $80.

4. The balance to a depositor's credit Jan. 1, 1905, was $500, and his deposits for the first term of 6 months were: Jan. 1, $200; Feb. 28, $85; Apr. 1, $370; May 19, $25; June 7, $40. His withdrawals were: Jan. 25, $110; March 18, $65; Apr. 4, $20; Apr. 20, $30; June 13, $95. What was the balance July 1, 1905, computing interest at 3% per annum on smallest monthly balances?

EXCHANGE.

493. Exchange is a system pertaining to commerce whereby payments are made to individuals in distant places by written orders called *Drafts* or *Bills of Exchange*.

REMARK 1. Exchange, as usually conducted, is a function belonging to banking, although the system may be carried on independent of banks. Those engaged to any extent worthy of mention in the business of banking, have deposits to their credit in New York and other commercial centers; hence, if A, living in St. Paul, wishes to pay B, living in New York, or vicinity, $500, he can go to any bank in St. Paul and purchase a draft or bill of exchange on some New York bank, and send the same by mail to *B, who,* by endorsement, can transfer it to any bank suited to *ence for* cash. By this means the inconvenience, *sending* the money direct is removed.

REM. 2. The person to whom payment is made may be named in the draft as the payee, but a better method for an individual to observe in buying a draft is to have it written payable to his own order and then indorse it in full to the party to whom he sends it. By this method a more complete record is secured.

NOTE. For definitions and general information concerning drafts, see Notes and Drafts (442 to 449, inclusive).

494. A **Bank Draft** or **Bill of Exchange** is a draft drawn on a bank.

495. A **Domestic Bill of Exchange** (usually called a draft) is one in which the drawer and drawee reside in the same country.

496. A **Foreign Bill of Exchange** is one in which the drawer and drawee reside in different countries.

497. Bank drafts and bills of exchange are subject to the same law respecting negotiability, indorsements, protests, etc., as individual drafts and notes (442 and following sections).

498. The **Face or Par of a Bill of Exchange** is the sum expressed upon its face. It is the original obligation.

REMARK 1. When conditions are such that a bill of exchange sells for *more* than its face value it is said to be at a *premium*, or *above par*. If it sells for *less* than its face, it is at a *discount*, or *below par*.

REM. 2. Exchange is subject to the same law of supply and demand as controls the commercial values of other commodities. When there is an equality of debts, or *nearly* an equality, between two cities, as Indianapolis and Baltimore, the course of exchange between those cities will be at par; but if conditions are such that a greater demand exists in Indianapolis for exchange on Philadelphia than there exists in Philadelphia for exchange on Indianapolis, the tendency will be to advance the price in Indianapolis and place exchange at a premium, while the effect on Philadelphia will be the reverse.

499. The principles of percentage apply to computations in domestic exchange. The face of the draft is the *base;* and all other sums concrete in character and representing percentage elements are *percentages*.

REMARK 1. The unit method of solution, hereafter explained, will be found, in many cases, *to possess* an advantage over the percentage method.

1. What is the cost of a sight draft of $560 at $\frac{3}{4}\%$ premium?

Operation.

$560 \times \frac{3}{4}\% = \$4.20$

$\$560 + \$4.20 = \$564.20$

or

$\$560 \times \$1.00\frac{3}{4} = \$564.20$

EXPLANATION. We first multiply $560, the face of the draft (base) by the rate of premium, $\frac{3}{4}\%$, which gives $4.20, the premium; and the face of the draft plus the premium is evidently the cost. The second operation is simply the face of the draft multiplied by rate of gross cost (base×rate of gross cost= gross cost).

REM. 2. In the daily purchase of drafts at nearly all banks, except it be those of the large cities, which are commercial centers, premium and discount are almost wholly ignored. The charge by the bank is usually a small fee for the transaction of the business performed. The sum of 10 cents for each hundred dollars of the face of the draft; or any fractional part of a hundred, is usually charged.

EXAMPLES FOR PRACTICE.

2. What is the total cost of the following sight drafts bought at a bank whose charges are 10 cents for each hundred dollars or fractional part thereof? $579.40, $28.25, $716, $375.75, $1148.60, $826, $1315, $1723.35, and $1500?

Find the cost of each sight draft described in the following examples:

3. A draft of $375 bought at $\frac{5}{8}\%$ premium. *4.* $876 bought at $\frac{1}{4}\%$ discount. *5.* $1472 at $\frac{1}{2}\%$ discount. *6.* $972.50 at $\frac{7}{8}\%$ premium. *7.* $2500 at $\frac{5}{8}\%$ premium. *8.* $1800 at $1\frac{1}{8}\%$ discount. *9.* $1365.80 at $1\frac{1}{8}\%$ discount. *10.* $2450 bought at $\frac{5}{8}\%$ premium with brokerage at $\frac{1}{8}\%$.

REM. 3. Drafts are sometimes bought and sold through brokers' offices, in which case the broker charges a small rate, based on the face of the draft, the same as other percentages.

11. What is the cost of a 90-day draft of $1200 at $\frac{1}{4}\%$ premium, the rate of interest being 6%?

PERCENTAGE METHOD.

$1200 (face of draft)×.00¼ (rate of premium)=$3 (premium).

$1200×$\frac{93}{60}$ or (.0155)=$18.60 (interest for 90+3 days).

$1200+$3 (premium)—$18.60 (interest)=$1184.40 (cost).

UNIT METHOD.

$1 (unit of draft)+$.0025 (premium)—$.0155 (interest)=$.987 (cost of unit).

$.987×1200 (number of units)=$1184.40 (aggregate cost).

REM. 4. Premium and brokerage are always addends. Since the bank has the use of the money received from the sale of time drafts, the purchaser is given the benefit of interest; hence interest and discount are both subtrahends.

REM. 5. Grace is allowed on time paper in nearly all the states, and on sight drafts in a few states. In the following examples, grace is to be allowed on all time drafts.

12. What is the cost of a 60-day draft of $600, bought at a discount of $\frac{1}{10}\%$; interest 4% per annum and brokerage $\frac{1}{8}\%$?

13. Find the cost of a 30-day draft of $1525, bought at a premium of $\frac{1}{4}\%$; brokerage $\frac{1}{8}\%$ and interest 5% per annum.

14. What is the face of a sight draft, which can be bought for $821.64, if the premium and brokerage are each $\frac{1}{10}\%$?

$1 (unit)+$.001 (premium)+$.001 (brok.)=$1.002 (cost of unit).
$821.64 (gross cost)÷$1.002 (gross cost of unit)=820 (face).

15. What is the face of a 90-day draft which can be bought for $238.93, at $\frac{1}{4}\%$ discount; brokerage $\frac{1}{8}\%$ and interest 6% per annum.

$1 (unit)+$.00125 (brok.)—$.0025 (disc.)—$.0155 (int.)=$.98325 (cost of unit).
$238.93 (gross cost)÷$.98325 (cost of unit)=$243, hence face is $243.

16. What must be paid in Baltimore for a draft on Chicago drawn at 90 days for $3500, the course of exchange being $101\frac{1}{8}$? (Interest at the legal rate.)

17. A sight draft was purchased for $324.80; exchange being at a premium of $1\frac{1}{2}\%$, what was the face?

18. An agent in Madison, Wis., having $875.50 due his principal, is instructed to remit the same by a draft drawn at 30 days; what will be the face of the draft, exchange being $\frac{1}{8}\%$ premium, and interest 4%?

19. A bank in Boston having declared a dividend of $5\frac{1}{2}$ per share, a stockholder in Detroit drew on the bank for the *sum due him*, and sold the draft at a discount of

$\frac{1}{2}\%$, thus realizing \$262.68 from his dividend; how many shares did he own?

20. A resident of Minneapolis found that it would cost him \$1197.90 to remit a 60-day draft in payment of a debt in Philadelphia, premium on exchange being $\frac{7}{8}\%$, and interest 6%. What sum did he owe in Philadelphia?

21. An attorney, having made collections for a firm in Chicago, to the amount of \$3250, is instructed to remit by sight draft. If the attorney charges a commission of $1\frac{1}{2}\%$, and exchange is $\frac{1}{10}\%$ premium, how much will the firm in Chicago receive?

FOREIGN EXCHANGE.

500. Foreign Exchange is that which is carried on between people of different countries.

REMARK. The terms "short exchange" and "long exchange," as employed in relation to exchange with foreign countries, have the same significance as the terms "sight exchange" and "time exchange" as used with reference to exchange between different places of the same country. Three days of grace are usually allowed on foreign bills of exchange.

501. The **Par of Exchange** is the estimated value of the coins of one country expressed in the monetary unit of another. This value may be either *intrinsic* or *commercial.*

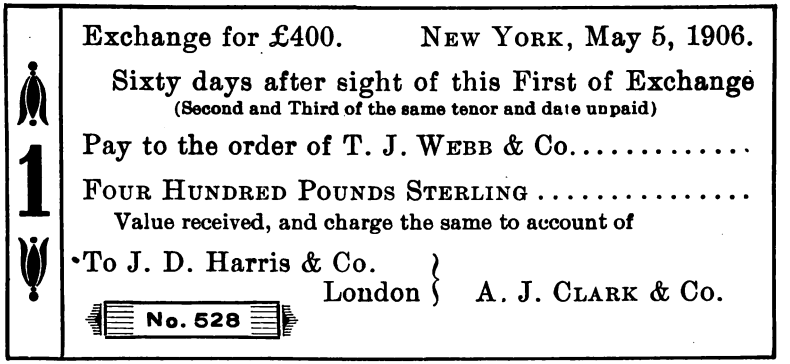
REMARK 1. The *intrinsic par of exchange* is the real value of coins as estimated by weight and purity. Thus, the relation between the sovereign or £ of Great Britain and the gold dollar (the monetary unit of the United States), is 4.8665. The sovereign contains 113 grains of pure gold and the gold dollar 23.22 grains, and $113 \div 23.22 = 4.86649+$. The relation (practically), is 4.8665, hence the intrinsic par of exchange with Great Britain is \$4.8665.

REM. 2. The *commercial par of exchange* is the market value, which is liable to fluctuate by virtue of trade relations.

502. A Set of Exchange is a bill of exchange consisting of three copies, alike in all respects excepting number. The payment of one renders each of the others void.

REMARK. The object of a set of exchange is to guard against loss in *sending from* one country to another. The bills are sent either by different *routes,* or by the same route at different dates.

SET OF EXCHANGE.



1

Exchange for £400. NEW YORK, May 5, 1906.

Sixty days after sight of this First of Exchange
(Second and Third of the same tenor and date unpaid)

Pay to the order of T. J. WEBB & Co.....

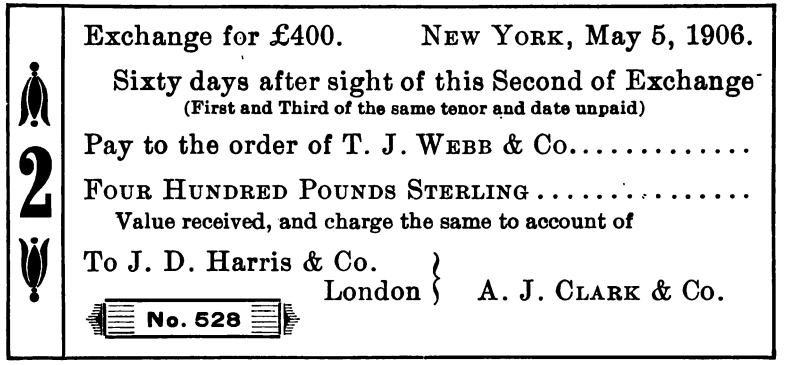
FOUR HUNDRED POUNDS STERLING .....

Value received, and charge the same to account of

To J. D. Harris & Co.
London

A. J. CLARK & Co.

No. 528



2

Exchange for £400. NEW YORK, May 5, 1906.

Sixty days after sight of this Second of Exchange
(First and Third of the same tenor and date unpaid)

Pay to the order of T. J. WEBB & Co.....

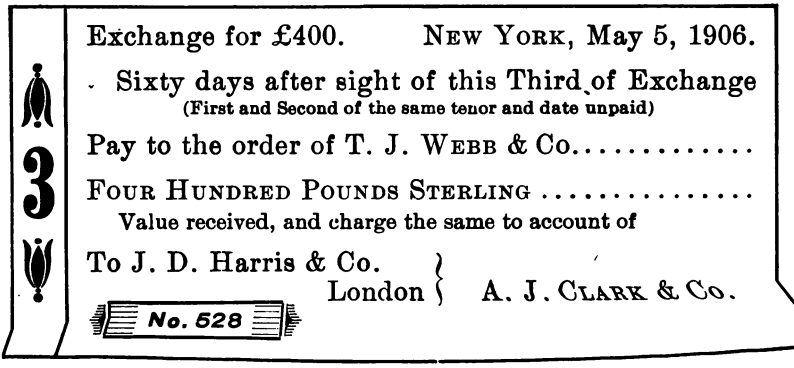
FOUR HUNDRED POUNDS STERLING .....

Value received, and charge the same to account of

To J. D. Harris & Co.
London

A. J. CLARK & Co.

No. 528



3

Exchange for £400. NEW YORK, May 5, 1906.

Sixty days after sight of this Third of Exchange
(First and Second of the same tenor and date unpaid)

Pay to the order of T. J. WEBB & Co.....

FOUR HUNDRED POUNDS STERLING .....

Value received, and charge the same to account of

To J. D. Harris & Co.
London

A. J. CLARK & Co.

No. 528

503. GOLD COINS OF THE UNITED STATES.

Denominations	Coinage Commenced	Standard Weight Grains	Abrasion Allowed in Grains	Least Current Wt. in Grs.	Estimated Time before subject to disc. from abrasion
Double-eagle	1850	516.	2.58	513.42	50 years
Eagle	1795	258.	1.29	256.71	35 "
Half-eagle	1795	129.	.64	128.36	20 "
Three dollars	1854	77.4	.38	77.02	.....
Quarter-eagle	1796	64.5	.32	64.18	15 years
Dollars	1849	25.8	.13	25.67	.....

504. SILVER COINS OF THE UNITED STATES.

Denominations	Coinage Commenced	Coinage Ceased	Standard Weight Grains	Amount for which a Legal Tender
Dollars (orig.)	1794	1873	412.5	Unlimited
Dollars (stand.)	1878	....	412.5	"
Trade Dollars	1873	1878	420.	Not a legal tender
Half-dollars	1794	....	192.9	Ten dollars
Columbian ½-dols	1892	1893	192.9	Ten dollars
Quarter-dollars	1796	....	96.45	Ten dollars
Columbian ¼-dols	1893	1893	96.45	Ten dollars
Twenty cents	1875	1878	77.16	Five dollars
Dimes	1796	....	38.58	Ten dollars
Half-dimes	1795	1873	19.29	Five dollars
Three cents	1851	1873	11.52	Five dollars

505. MINOR COINS OF THE UNITED STATES.

Denominations	Coinage Commenced	Coinage Ceased	Standard Weight Grains	No allowance for abrasion	Amount for which a Legal Tender
Five cents	1866	....	77.16		Twenty-five cents
Three cents	1865	1889	30.		Twenty-five cents
Two cents	1864	1872	96.		Twenty-five cents
Cents	1793	....	48.		Twenty-five cents
Half-cents	1793	1857	....		Not a legal tender

REMARK. The five-cent and three-cent coins are nickel; the cent is copper, nickel and bronze.

506. The intrinsic value, in U. S. money, of the units of currency of the principal nations of the world, as proclaimed by the Secretary of the Treasury on Oct. 1, 1906, is shown by the following tables:

TABLE I.

(Gold Standard.)

Country	Monetary Unit	Value in U.S. Money	Coins
Argentine Republic	Peso	$0.965	Gold: argentine ($4.824) and ½ argentine. Silver: peso and divisions
Austria-Hungary	Crown	.203	Gold: 10 and 20 crowns. Silver 1 and 5 Crowns
Belgium	Franc	.193	Gold: 10 and 20 francs. Silver: 5 francs
Brazil	Milreis	.546	Gold: 5, 10, and 20 milreis. Silver: ½, 1, and 2 milreis
British Pos'sions, N.A.*	Dollar	1.000	
Central Amer. States—			
Costa Rica	Colon	.465	Gold: 2, 5, 10, and 20 colons ($9.307). Silver: 5, 10, 25, and 50 centimos
British Honduras	Dollar	1.000	
Chile	Peso	.365	Gold: escudo ($1.825), doubloon ($3.65), and condor ($7.30). Silver: peso and divisions
Colombia	Dollar	1.000	Gold: condor ($9.647), and double-condor. Silver: peso
Denmark	Crown	.268	Gold: 10 and 20 crowns
Ecuador	Sucre	.487	Gold: 10 sucres ($4.8665). Silver: sucre and divisions
Egypt	Pound	4.943	Gold: pound (100 piasters), 5, 10, 20, and 50 piasters. Silver: 1, 2, 5, 10, and 20 piasters
Finland	Mark	.193	Gold: 20 marks ($3.859), 10 marks ($1.93)
France	Franc	.193	Gold: 5, 10, 20, 50, and 100 francs. Silver: 5 francs
German Empire	Mark	.238	Gold: 5, 10, and 20 marks
Great Britain	Pd. Sterling	4.8665	Gold: sovereign (pound sterling) and ½ sov.
Greece	Drachma	.193	Gold: 5, 10, 20, 50, and 100 drachmas. Silver: 5 drachma
Haiti	Gourde	.965	Gold: 1, 2, 5, and 10 gourdes. Silver: gourde and divisions
India	Pd. Sterling	4.8665	Gold: sovereign (pound sterling). Silver: rupee and divisions
Italy	Lira	.193	Gold: 5, 10, 20, 50, and 100 lire. Silver: 5 lire
Japan	Yen	.498	Gold: 5, 10, and 20 yen. Silver: 10, 20, and 50 sen
Liberia	Dollar	1.000	
Mexico	Peso	.498	Gold: 5 and 10 pesos. Silver: dollar or (peso) and divisions
Netherlands	Florin†	.402	Gold: 10 florins. Silver: ½, 1, and 2½ florins
Newfoundland	Dollar	1.014	Gold: 2 dollars ($2.027)
Norway	Crown	.268	Gold: 10 and 20 crowns
Panama	Balboa	1.000	Gold: 1, 2½, 5, 10, and 20 balboas. Silver: peso and divisions
Peru	Libra	4.8665	Gold: ½ and 1 libra. Silver: sol and divisions
Philippine Islands	Peso	.500	Silver peso: 10, 20, and 50 centavos
Portugal	Milreis	1.080	Gold: 1, 2, 5, and 10 milreis
Russia	Ruble	.515	Gold: 5, 7½, 10 and 15 rubles. Silver: 5, 10, 15, 20, 25, 50, and 100 copeks
Spain	Peseta	.193	Gold: 25 pesetas. Silver: 5 pesetas
Sweden	Crown	.268	Gold: 10 and 20 crowns
Switzerland	Franc	.193	Gold: 5, 10, 20, 50, and 100 francs. Silver: 5 francs
Turkey	Piaster	.044	Gold: 25, 50, 100, 250, and 500 piasters
Uruguay	Peso	1.034	Gold: peso. Silver: peso and divisions
Venezuela	Bolivar	.193	Gold: 5, 10, 20, 50, and 100 bolivars. Silver: 5 bolivars

*Except Newfoundland.

†Another name for this coin is Guilder.

TABLE II.

(Silver Standard.)

Country	Monetary Unit		Value in U. S. Money	Coins
Bolivia	Boliviano		$0.485	Silver: boliviano and divisions
Central Amer. States—Guatemala, Honduras, Nicaragua, Salvador	Peso		.485	Silver: peso and divisions
China	Tael	Amoy	.730	
		Canton	.792	
		Cheefoo	.760	
		Chin Kiang	.776	
		Fuchau	.735	
		Haikwan	.808	
		Hankow	.743	
		Kiaschow	.770	
		Nankin	.786	
		Niuchwang	.745	
		Ningpo	.764	
		Peking	.775	
		Shanghai	.726	
		Swatow	.734	
		Takau	.799	
		Tientsin	.770	
	Dollar	Hongkong	.523	
		British	.523	
		Mexican	.526	
Persia	Kran		.089	Gold: ½, 1 and 2 tomans ($3.409) Silver: ¼, ½, 1, 2 and 5 krans
Straits Settlements	Dollar		.523	

REMARK 1. The rate at which bills of exchange are bought and sold is fixed by the commercial par of exchange (501, Rem. 2). When foreign coin can be bought and transportation paid at a less cost than is fixed by the commercial par of exchange, gold coin can be *exported* at a profit; and when the commercial par of exchange is less than the intrinsic value increased by cost of transportation, gold coin can be *imported* at a profit; hence the commercial relations existing between countries have much to do with the exportation and importation of gold.

REM. 2. Exchange with most foreign countries is quoted by giving the value of the monetary unit in denominations of U. S. money; thus, Great Britain, 4.85. Exchange with France, and other countries in which the unit is the franc, is quoted by expressing the value of $1 in francs; thus, Belgium, 5.20. Exchange with Germany is quoted by expressing the value of 4 marks.

507. A **Letter of Credit** is a circular letter issued by a banking house to a person traveling in a foreign country, *and addressed*, in most cases, to bankers located at *different places*, requesting them to furnish the traveler money

in sums to suit his convenience, not to exceed the amount that may be due on presentation.

508. To find the cost of a foreign bill of exchange.

1. What will be the cost of a bill of exchange on Liverpool, of £260 3s. 6d., exchange being quoted at 4.85?

Operation.

£260, 3s. 6d.=£260.175
$4.85 \times 260.175 = \$1261.85$

EXPLANATION. We first reduce the 3s. 6d. to the decimal part of a pound, then the value of £1 ($4.85) multiplied by the number of pounds gives the entire cost, which is $1261.85.

2. What is the cost of a bill of exchange on Paris of 850 fr. at a quotation of 5.18?

850 fr. (value of bill)÷5.18 fr. (value of $1)=164.09 (number of dollars equal in value to 850 fr.).

EXAMPLES FOR PRACTICE.

Find the cost of the following bills of exchange:

	Face.	Quotation.		Face.	Quotation.
3.	£720	4.85½.	*9.*	795 guilders	41¾.
4.	£315	4.86.	*10.*	2560 "	40½.
5.	£84 2s. 6d.	4.85.	*11.*	1842 "	39¾.
6.	2564.5 fr.	5.19½.	*12.*	465 milreis	55¼.
7.	5716.75 fr.	5.18⅞.	*13.*	1420 crowns	27½.
8.	£123 5s.	4.86¾.	*14.*	525 florins	32¾.

15. What will I be required to pay for a bill of exchange on Frankfort of 1250 marks, at a quotation of 93½?

$.935 (val. of 4 marks) × 1250 (number of marks)=$1168.75 (4 times the value of 1250 marks).

$1168.75÷4=$292.19 (value of 1250 marks).

16. I am owing a merchant in Berlin, Germany, 7460 marks. What will a bill of exchange cost with which to pay the debt, if German exchange is quoted at 94¼?

REMARK. When a bill of exchange is bought through a broker, the gross cost of 1 unit is the market value plus the brokerage, and when a bill of exchange *is sold, the prime* cost of a unit is the market value minus the *brokerage.*

17. I purchased, through a broker whose rate of brokerage was ¼%, a bill of exchange on London of £640 5s. at 4.86. What was the total cost?

18. The following bills of exchange were sold through a broker, whose rate was ⅛%: 1500 fr. on Paris at 5.20, 940 marks on Hamburg at 95½. What were the net proceeds?

19. A hardware merchant bought an invoice of cutlery from a wholesale house in Sheffield on 60 days' credit. Wishing to balance the account at once, he purchased and remitted a 60-day bill of exchange. The commercial par of exchange was 4.84½, and the amount of the bill was £140 15s. What was the cost of the bill of exchange, money being worth 4% per annum?

509. To find the face of a foreign bill of exchange.

1. I placed $920 in the hands of a broker, with instructions to purchase a bill of exchange on Lyons, France. If the quotation was 5.20 and brokerage ¼%, what was the face of the bill?

$920 (gross cost)÷1.0025 (rate of gross cost)=$917.71 (prime cost).
5.20 fr. (what $1 will buy) × 917.71=4772 fr. 1 dc. (what $917.71 will buy).

2. I sold through a broker a bill of exchange on Berne, Switzerland, and realized $583.27 as net proceeds. What was the face of the bill, exchange being 5.19½ and brokerage ⅛%?

$583.27÷(100%−⅛%) (rate of net proceeds)=$584 (gross proceeds).
5.19½ fr. × 584=3033 fr. 9 dc. (face of bill of exchange).

3. The gross cost of a bill of exchange on Oldenburg, Germany, including brokerage ¼%, was $2273.67. What was the face of the bill, the quotation being 94½?

$2273.67÷(100%+¼%) (rate of gross cost)=$2268 (prime cost).
$2268÷$.945 (value of 4 marks)=2400 (¼ the number of marks).
2400 marks × 4=9600 marks (face of the bill).

EXAMPLES FOR PRACTICE.

Find the face of each bill of exchange in the following examples:

	Market Value.	Course of Exchange.
4.	$5420	$4.86 to £1.
5.	$1375	$4.85½ to £1.
6.	$600	5.19 fr. to $1.
7.	$396.50	5.20¼ fr. to $1.
8.	$572	5.18½ fr. to $1.
9.	$1760.80	38½¢ to 1 guilder.
10.	$920.50	37¢ to 1 guilder.
11.	$780	94½¢ to 4 marks.
12.	$1230	95¼¢ to 4 marks.
13.	$218	95¢ to 4 marks.

Find the face of each bill of exchange in the following list:

	Gross Cost.	Quotation.	Brokerage.
14.	$6450	$4.85 to £1	⅛%.
15.	$720	$4.85½ to £1	¼%.
16.	$1350	$4.85¼ to £1	¼%.
17.	$900	$4.86 to £1	⅛%.
18.	$2500	$4.84¾ to £1	⅛%.
	Net Proceeds.	**Quotation.**	**Brokerage.**
19.	$1500.30	$4.86 to £1	¼%.
20.	$700	5.19 fr. to $1	¼%.
21.	$327.45	5.18½ fr. to $1	⅛%.
22.	$480	95¢ to 4 marks	¼%.
23.	$528.48	95½¢ to 4 marks	½%.

24. I sold through a broker a bill of exchange on Paris. If the net proceeds were $251.37, brokerage ¼%, and the quotation at which the sale was made 5.18, what was the face of the bill?

25. A broker in Chicago sold for a London capitalist, 40 shares of N. Y. Central R. R. stock at 148 and charged ⅛% brokerage. He at once notified his principal of the

sale, and soon after received instructions to remit a bill of exchange of 48000 francs to DeVoe & Co., of Paris, and to remit the remainder due him in a bill of exchange on London. If the course of exchange on Paris was 5.19 fr. to $1, and the course of exchange on London was $4.86 to £1, what was the face of the London bill, brokerage of $\frac{1}{8}\%$ being allowed on the Paris bill only?

LONDON STOCK QUOTATIONS.

510. American securities are quoted in London on the basis of $5 to the £, regardless of the commercial par of exchange; hence, it is evident that London and American quotations, regarding American securities, will differ in all cases except when the course of exchange with England is $5. Whatever the relation may be between the assumed value of £1 ($5), and the value as expressed by the quotation on exchange, it must be the same as the relation between London and American quotations of American securities. When the course of exchange is 4.85, the London quotations of American securities will be $\frac{5.00}{4.85}$ of the American quotations. Hence, multiplying the American quotation by 5, and dividing the result by the commercial par of exchange, will give the London quotation.

1. When Michigan Central R. R. stock is quoted in America at 130, what should be the corresponding London quotation, exchange being quoted at 4.86?

$130×5=$650, and $650÷4.86=133$\frac{3}{4}$—.

The quotations given in the following examples are American. Find the corresponding London quotations.

	Quotations.	Course of Exch.		Quotations.	Course of Exch.
2.	95½	4.85½.	*6.*	210	4.84¾.
3.	83	4.86.	*7.*	60	4.85¼.
4.	138	4.85¼.	8.	140	4.86½.
5.	160	4.86¼.	9.	104	4.85.

10. When U. S. 3% bonds are quoted in London at 107½, what should be the American quotation, if the course of exchange is 4.86¾?

11. When Union Pacific R. R. stock is quoted in London at 112, what should be the American quotation, if the course of exchange is 4.87?

ARBITRATION OF EXCHANGE.

511. Arbitration of Exchange is the process of computing exchange between two places by means of one or more intermediate exchanges.

1. A merchant in Baltimore paid a demand in Paris of 4200 fr., by remitting to Amsterdam at the rate of \$.40½ per guilder, and thence to Paris at the rate of 2.15 fr. per guilder. How much Federal money was required?

Operation.

.43 ~~2.15~~ francs = 1 guilder

1 guilder = \$.~~405~~ .081

\$ ——— = 4200 fr.

(\$.081 × 4200) ÷ .43 = \$791.16

EXPLANATION. We write the existing equalities expressed by the conditions of the example; and, since the three elements on the left of the signs of equality are equal respectively to the corresponding elements on the right, it is evident that the products of these elements must be equal. Rejecting equal factors by cancellation, and dividing the product by the product of the given factors, we obtain \$791.16, the correct result.

2. An agent in New York collected \$2500 for a firm in Paris, and charged ½% commission for collection; he remitted the net proceeds to the Paris firm through their branch house in Liverpool, charging ¼% brokerage on the draft. If the course of exchange on Liverpool was 4.85, and the rate between Liverpool and Paris was 26½ francs to the pound sterling, what sum did the firm in Paris receive?

Operation.

.97 $~~$4.85~~ = £1

£1 = ~~26.5~~ fr. 5.3

$1 = $.995 *

.0401 $~~$1.0025~~ = $1 †

fr. = $~~$2500~~ 100

$$\frac{5.3\times.995\times100}{.97\times.0401}=13557.6 \text{ fr.}$$

EXPLANATION. We make the statement as in the former example. Then, since the agent is to deduct ½% as commission on the collection, we place $1 on the left, and the net proceeds of $1 ($.995), on the right, as there exists an equality in mathematical effect; for a similar reason we place the gross cost of $1 ($1.0025) on the left and $1 on the right, and obtain by the processes of cancellation, multiplication, and division, 13557.6 fr. as the proceeds of the exchange.

* The true meaning of the equation is, $1 *without a collection charge* is equal, *in effect*, to $.995 *with a collection charge*.

† $1.0025 *with brokerage* is equal, *in effect*, to $1 *without brokerage*.

RULE I. *Arrange the given rates of exchange so that, in in any two consecutive equations, the same unit of currency shall stand on opposite sides.*

II. *When there is commission for drawing, place 1 minus the rate on the left if the cost of exchange is required, and on the right if proceeds are required; and when there is commission for remitting, place 1 plus the rate on the right if cost is required, and on the left if proceeds are required.*

III. *Represent the required sum by a dash, with the name of the proper unit of currency affixed, and place it equal to the given sum on the right.*

IV. *Cancel equal factors from the two sides and divide the product of the remaining factors on the right by the product of the remaining factors on the left; the result will be the required answer.*

EXAMPLES FOR PRACTICE.

3. A speculator in Albany, having purchased 58 shares of railroad stock in St. Louis, at 95, remits to his agent in Baltimore, a draft purchased at 2% premium, with orders *for the agent* to remit the sum due in St. Louis. If exchange on St. Louis is at ½% discount in Baltimore, and

the agent's commission for remitting is $\frac{1}{4}\%$, how much does the stock cost in Albany?

4. If at Boston exchange on London is 4.89½, and at London on Paris 24 francs 96½ centimes per £1, what is the arbitrated course of exchange between Boston and Paris, through London?

5. An American resident of Amsterdam, wishing to obtain funds from the United States to the amount of $6400, directs his agent in London to draw on the U. S. and remit the proceeds to him in a bill of exchange on Amsterdam, exchange on the U. S. being at 4.85 in London, and the course between London and Amsterdam being 18d. per guilder. If the agent charges commission at $\frac{1}{2}\%$ both for drawing and remitting, how much better is this arbitration than to draw directly on the U. S. at 41 cents per guilder?

6. A banker in New York remits to his correspondent bank in London $25000 by arbitration as follows: First to Paris at 5.16 fr. per $1; thence to Hamburg at 125 fr. per 100 marks; thence to Amsterdam at 1.71½ marks to the guilder; thence to London at 11.82 guilders per £1 sterling. What is the gain over direct exchange at 4.91½?

7. If in London exchange on Paris is 25¾, and in New York on Paris it is 5.20, what is the arbitrated course of exchange between New York and London?

STOCKS AND BONDS.

512. A **Company** is an association of individuals engaged in the prosecution of some enterprise. Companies may be incorporated or unincorporated.

REMARK. An unincorporated company is called a *Co-partnership* or *Firm.*

513. A **Corporation** is an artificial person created by *law and endowed with* power to act as a single individual.

514. A **Charter** is the legal act of incorporation and specifies the powers and obligations of the corporate body.

515. The **Capital Stock** of a corporation is the money or property contributed by its members for the purpose of carrying on the business of the company.

516. Certificates of Stock are the documents issued to members giving them their respective titles to the capital.

517. A **Share** is one of the equal parts into which the capital is divided. The value of a share is not a fixed legal sum, and is different in different corporations, varying from 10 cents to $1000. The usual value of a share is $100.

518. A **Stockholder** is one who owns some portion of the capital stock, either as an original contributor, or as a subsequent purchaser.

519. The **Par Value** of a share is the sum specified in the certificate, and represents the cost to the original owner.

520. The **Market Value** of a share is the price at which it can be sold in the stock market.

REMARK. When the market value of a share is greater than the par value, it is *above par*, or at a *premium;* and when the market value is less than the par value, it is *below par*, or at a *discount.*

521. The **Gross Earnings** of a corporation are the total receipts from the business.

522. The **Net Earnings** of a corporation are what remain after all expenses have been paid.

523. A **Dividend** is that portion of the net earnings which is divided among the stockholders.

524. An **Assessment** is a sum levied upon the stockholders of a corporation to pay its losses or meet its deficiencies.

REMARK. Dividends and assessments are a certain rate per cent. of the par value of the stock. A 5% dividend is frequently mentioned as $5 per *share.*

525. Preferred Stock is stock having advantage over he ordinary or common stock.

REMARK. A corporation may become financially embarrassed, and to rotect it from loss or disaster, additional funds must be obtained at once. hese necessary funds are frequently secured by placing on the market dditional stock, made desirable in consequence of certain guaranties that o not apply to the stock in common; as, a pledge to pay some specified ate of dividend annually, or a promise to pay some stipulated rate per cent ut of the net earnings before any dividend can be declared on the common tock. Such an issue would be called *preferred stock*.

526. Watered Stock is stock which has been reduced in alue by inflation.

REMARK 1. Stock becomes inflated by increasing the number of hares without increasing the amount of capital. If the original number f shares of a corporation is 500, representing a capital of $50000, and 100 dditional shares are distributed among the stockholders gratuitously, the 00 shares will represent the same amount of capital as the 500 did before he distribution. The stock, under such conditions, is said to have been *vatered*.

REM. 2. The objects for watering the stock of a corporation are vari- us; sometimes the directors wish to keep the public in ignorance concern- ng their prosperity, and resort to this means to make the dividends seem o be at a less rate than they really are. Sometimes the object is to secure dditional credit by making the capital stock seem greater than it is. nother object is to place them in a position to declare larger dividends han their charter will allow. If the charter of a corporation restricts the nnual dividend to 10%, and the directors wish to issue a 15% dividend, hey can water the stock to the amount of 50%, and a 10% dividend will hen be 15% of the actual capital.

527. The **Franchise** of a corporation is the privilege, ight, or power, granted by its charter.

528. A **Bond** is a written obligation to pay a sum of noney at some future time, with interest payable accord- ng to its terms.

REMARK 1. Bonds are issued by public and by private corporations, nd by individuals.

REM. 2. Provision for payment of interest is usually made by coupons ttached. (See coupon note, 462.)

REM. 3. Government and state bonds usually derive their names from the rate of interest which they bear; as U. S. 4's, N. Y. 6's, etc.

REM. 4. Quotations in stocks and bonds usually denote the market price of $100 par. The method of quoting a rate per cent. of premium or of discount is in vogue in some cities. The quotation of stocks worth $25 per share par, is the value of 4 shares. The quotation of half-share stocks ($50 per share), is the value of 2 shares.

REM. 5. Mining stock is usually quoted at the market price per share regardless of par value.

529. There are two methods of computation regarding stocks and bonds, the *percentage* method, and the *unit* method. The unit method will generally be found the more convenient.

530. Calculations in stocks and bonds are treated, in this work, under two separate heads, *Stock Investments*, and *Stock Speculations*.

STOCK INVESTMENTS.

531. Stock Investments have relation to cost, brokerage, income, and liability.

REMARK. The cost of stocks and bonds is the market value, which depends upon premium or discount.

532. The **income** to the owner of *stocks* is the amount received from dividends; and the liability is the sum paid in assessments.

533. The **income** to the owner of *bonds* is the sum collected from interest.

534. A **Stock Broker** is a person who buys and sells stocks and bonds, either for himself or for others.

535. Brokerage is a fee or compensation charged by a broker for negotiating a purchase or a sale for another.

REMARK. Brokerage is usually computed at a certain rate per cent. of the par value.

536. Whenever the percentage method is employed in *stock investments*, the par value is the base, and premium,

scount, market value, brokerage, dividends, and assess-ents are *percentages*, since each represents a certain num-er of hundredths of the base.

REMARK. The number of units expressing a rate in the percentage ethod, corresponds to the number of dollars expressed by the unit of the nit method. Thus 15% premium is \$15 per share; ¼% brokerage is ¼ of dollar, or 25 cents, per share, etc.

From the elements given in the following examples, find ne market value:

1. \$14200 par of bonds, at a premium of $5\frac{3}{4}\%$.
2. \$23500 par of stocks, at a discount of $12\frac{1}{2}\%$.
3. 135 shares of stock, at a premium of 13%.
4. 147 shares of stock, at a premium of $3\frac{3}{16}\%$.
5. \$11400 par of stocks, quoted at $85\frac{3}{4}$.
6. 113 shares of stock, quoted at $106\frac{3}{4}$.
7. What is the cost of \$7200 par of stocks, if the pre-mium is 15% and brokerage $\frac{1}{8}\%$.

\$7200 (par value)×1.15⅛ (rate of gross cost)=\$8289 (gross cost);
or \$115.125 (cost per share)×72 (number of shares)=\$8289 (cost).

In the following examples, find gross cost, or net pro-ceeds as may be required.

8. \$7800 par of bonds, bought at $6\frac{1}{4}\%$ premium, bro-kerage $\frac{1}{8}\%$.
9. 56 shares of stock bought at 8% discount, brokerage ⅛%.
10. \$5200 par of bonds sold at $101\frac{3}{4}$, brokerage $\frac{3}{8}\%$.
11. 95 shares of stock bought at $97\frac{1}{2}$, brokerage $\frac{1}{8}\%$.
12. \$2700 par of stocks sold at 16% premium, broker-age $\frac{1}{8}\%$.
13. If the par value of stocks is \$5700, and the divi-dend \$342, what is the rate of dividend?

\$342 (dividend)÷\$5700 (par value)=6% (rate of dividend);
or \$342 (dividend)÷57 (number of shares)=\$6 (rate per share)=6%.

From *the elements* given in the following examples,

find the rate per cent. and name the element to which belongs, in each case.

14. The par value is $4500, and the dividend is $202.5

15. The par value is $3800, and the brokerage is $9.5

16. The par value is $6300, and the assessment is $18

17. The par value is $5600, and the market value i $5824.

18. The par value is $2700, and the discount is $67.5

19. The par value is $3400, and the premium is $30

20. If the par value is $9800, and the market value i $5634, what is the rate of discount?

21. If the par value is $8800, and the market value i $9416, what is the rate of premium?

Find and name the wanting element in each of the fol lowing examples:

22. Premium, $810; rate of premium, $4\frac{1}{2}\%$.

23. Dividend, $115; rate of dividend, 5%.

24. Discount, $242.25; rate of discount, $7\frac{1}{8}\%$.

25. Brokerage, $3.50; rate of brokerage, $\frac{1}{8}\%$.

26. Assessment, $87.50; rate of assessment, $3\frac{1}{2}\%$.

27. Market value, $4837.50; quotation, $112\frac{1}{2}$.

28. Market value, $7410; rate of discount, 22%.

29. Market value, $8399; rate of premium, $13\frac{1}{2}\%$.

30. Gross cost, $1478; quotation, 92; brokerage, $\frac{3}{8}\%$.

31. Net proceeds, $8799; quotation, 105; brokerage, $\frac{1}{4}\%$.

REMARK. The gross cost of 1 share is the quotation + brokerage. The net proceeds of one share is the quotation—brokerage.

32. Gross cost, $4396; quotation $137\frac{1}{4}$; brokerage, $\frac{1}{8}\%$.

33. Net proceeds, $1207.50; quotation, $86\frac{3}{8}$; brokerage, $\frac{1}{8}\%$.

34. If the cost of 120 shares of stock, including brokerage at $\frac{1}{8}\%$, was $11625, at what quotation were they bought?

Gross cost ÷ No. of shares = gross cost per share.
Gross cost per share — brokerage per share = quotation.

35. If the net proceeds from the sale of 60 shares of ock was \$6217.50, at what quotation were they sold, lowing $\frac{1}{8}\%$ brokerage?

Net proceeds÷No. of shares=Net proceeds of 1 share.
Net proceeds of 1 share+brokerage per share=quotation.

36. If the proceeds of 62 shares of stock, sold through broker whose rate is $\frac{1}{4}\%$ were \$6804.50, at what rate of emium were they sold?

37. A man invested \$7010 in manufacturing stock, inuding brokerage at $\frac{1}{8}\%$. What was his income during e first year if the quotation at which the stock was ought was $87\frac{1}{2}$, and two dividends were declared, the first %, and the other 5%?

38. If I sell 50 shares of stock at $12\frac{1}{2}\%$ discount and ay $\frac{1}{4}\%$ brokerage, what are the net proceeds of the sale?

39. On Jan. 1, 1906, a merchant invested \$7370 in raiload stock at 8% discount, paying brokerage $\frac{1}{8}\%$; 4 months ater, the stock was watered by issuing a stock dividend of $12\frac{1}{2}\%$; on July 1, of the same year, a semi-annual dividend of 6% was declared. What sum did the merchant receive?

40. If the stock mentioned in the foregoing example vas quoted at 96 immediately before inflation, what should t have been quoted at immediately thereafter?

41. The directors of a corporation having a capital of \$150000, ordered that 75% of the net earnings be divided among the stockholders. A, who owned 40 shares, received \$220. What were the net earnings?

42. A retired merchant invested \$25100 in stocks as follows: 96 shares of manufacturing stock quoted at $82\frac{3}{8}$, 112 shares of bank stock quoted at 105, and the remainder in railroad stock quoted at $112\frac{1}{2}$, brokerage in each case $\frac{1}{8}\%$. During the year he received the following dividends: 5% on the manufacturing stock, 7% on the bank stock, and 8% on *the railroad stock*. What was his annual income?

STOCK SPECULATIONS.

537. Stock Speculations are the dealings in stocks and bonds which result in losses or gains.

REMARK 1. In a mathematical sense, stock speculations are neither more nor less than a subject of loss and gain in which stocks and bonds are the commodities bought and sold; therefore market value, when there is no brokerage, is the *base*, since it represents gross cost. When brokerage is an element, market value plus brokerage is the *base*.

REM. 2. When the unit method of computation is employed, a $100 share of stock, or a $100 bond is the unit.

1. If I buy 6% stocks at 25% discount, what rate per cent. do I make on my investment?

$6 (income from 1 share)÷$75 (gross cost of 1 share)=8% (rate on investment).

2. If I buy stocks at 19¾% premium and pay ¼% brokerage, and the dividend is such that my income is 5% on the investment, what is the rate per cent. of dividend?

($119¾+$¼) (gross cost)×.05 (rate of income)=$6 (income on 1 share)=6%.

3. At what quotation must 8% bonds be bought that 10% may be realized on the investment, the charge for brokerage being ¼%.

$8 (income from 1 unit)÷.10 (rate of income)=$80 (gross cost per unit.)
$80−$¼ (brokerage)=$79¾ (quotation).

In the following examples, find the rate per cent. on the investment:

4. Quotation, 75; dividend, 5%.

5. Quotation, 109⅜; brokerage ¼%; dividend, 5½%.

6. Quotation, 119⅜; brokerage, ¼%; assessment, 3%.

7. Quotation, 233; brokerage, ⅓%; quarterly dividend, 3½%.

8. Premium, 19⅞%; brokerage, ⅛%; semi-annual dividend, 4%.

9. Discount, 20½%; brokerage, ¼%; dividend, 4%.

In the following examples, find rate of dividend:

10. Quotation, 59⅞; brokerage, ⅛%; rate on investment, 10%.

11. Quotation, $119\frac{3}{4}$; brokerage, $\frac{1}{4}\%$; rate on investment, 5%.

12. Quotation, $84\frac{7}{8}$; brokerage, $\frac{1}{8}\%$; rate on investment, $7\frac{1}{17}\%$.

13. Premium, $23\frac{7}{8}\%$; brokerage, $\frac{1}{8}\%$; rate on investment, $6\frac{14}{31}\%$.

14. Discount, $4\frac{1}{4}\%$; brokerage, $\frac{1}{4}\%$; rate on investment, $8\frac{1}{3}\%$.

In the following examples, find quotations at which stocks must be bought to realize the expressed conditions:

15. Rate on investment, 10%; rate of dividend, 7%.

16. Rate on investment, 9%; rate of dividend, 6%; brokerage, $\frac{1}{4}\%$.

17. Rate on investment, $12\frac{1}{2}\%$; rate of dividend, 7%; brokerage, $\frac{1}{8}\%$.

18. At what quotation must 4% bonds be bought, allowing brokerage $\frac{1}{8}\%$, to enable the purchaser to realize an income of 5% on his investment?

19. Which will be the better investment of \$7980, and by what difference: $5\frac{1}{2}\%$ bonds at $104\frac{3}{4}$, or 5% stocks at $94\frac{3}{4}$, brokerage in each case being $\frac{1}{4}\%$?

20. I own 120 shares of 5% stocks, which are paying me 6% on my investment; if I sell the same at a profit of 15%, how much will I receive?

21. A retired grain dealer bought 80 shares of 6% bank stock at 90, and exchanged the same for 70 shares of railroad stock quoted at 98. At the expiration of the year, he sold his railroad stock at 95, the bank stock in the meantime having advanced to par. How much did the grain dealer lose by the exchange?

22. An investor bought 72 shares of steamboat stock at 110 and realized 5% on his investment. What was the annual rate of dividend declared?

23. *A speculator* purchased city bonds at 20% discount

and realized 5% on his investment. He also purchased school district bonds bearing the same rate of interest at 5% premium, and received therefrom an annual income of $1200. He afterward sold the school district bonds at a price 8% in advance of cost. What were the proceeds of the sale?

24. An individual bought stock at $16\frac{1}{8}$% discount, and after receiving two dividends of 4% each, sold the stock at $20\frac{7}{8}$% discount, netting a gain of $522. If he paid a broker $\frac{1}{8}$% for buying, and the same for selling, what was the par value of the stock?

25. What rate per cent. is gained by buying stocks at 10% discount and selling the same at 10% premium?

26. At what price must 4% bonds, maturing in 5 years, be purchased to enable the purchaser to realize 10% on his investment, no allowance being made for interest on the annual interest payments?

$100+($4×5)=$120, principal and interest of a $100 bond at maturity.
$1+($.10×5)=$1.50, amount of $1 for given time at 10%.
$120 (amount)÷$1.50=$80, principal, or sum to be paid to realize 10% per annum on the investment.

27. If 3% bonds, maturing in 20 years from date of investment, are purchased at $94\frac{3}{8}$, through a broker whose rate is $\frac{1}{8}$%, what rate per cent. will be realized on the sum invested, no allowance being made for interest on annual interest payments?

$100+($3×20)=$160, amount of a $100 bond at maturity.
$160—($94$\frac{3}{8}$+$\frac{1}{8}$)=$65.50, total income from investment in 20 years.
$94.50×.01×20=$18.90, interest on sum invested at 1%.
$65.50 (income)÷$18.90 (income at 1%) = $3\frac{8}{189}$ (number of times 1%)=$3\frac{8}{189}$%.

28. At what quotation must I buy 5% bonds, maturing in 10 years, so as to make 6% on my investment, allowing $\frac{1}{4}$% brokerage?

29. What rate per cent. per annum will be realized on *the investment*, if 6% bonds, maturing in 6 years, are *bought at 104*?

CAPITALIZATION.

538. Capitalization is the act of determining the income resulting from a given capital, or of determining what capital will produce a given income.

NOTE. Capitalization is not properly a subject belonging to stocks and bonds, but as it is so nearly identical in mathematical relations, we treat it under that head.

1. If a house is worth \$3600, and is rented at a capitalization of 7%, what rent does the tenant pay monthly?

2. What is the value of a store building, rented at \$60 per month, at a capitalization of 5%?

3. The annual income from a ground rent worth \$3000 is \$165. At what rate per cent. does the property capitalize?

NOTE. A piece of real estate, when made a subject of rental, is frequently termed a *ground rent.*

4. A man bought a \$220 ground rent at a capitalization of 5%, and sold the same at a capitalization of $5\frac{1}{2}$%. Did he gain or lose, and what amount?

5. A block of stores, yielding an annual income of \$4200 from rentals, was sold at a capitalization of 7%. What were the net proceeds of the sale if sold by a real estate agent who charged a commission of $\frac{3}{4}$%?

6. By buying a ground rent at a capitalization of 5% and selling the same at a capitalization of $4\frac{4}{5}$%, I gain \$375, which is $4\frac{1}{6}$% of the investment. What is the annual income from rentals?

7. A speculator bought a block containing 4 stores with 6 office rooms above, for \$51000; he paid \$1000 for repairs, and paid taxes and insurance to the amount of \$800. The stores rented at \$40 per month each, and the offices at \$10 each per month. What was the net rate of capitalization?

539. ADDITIONAL EXAMPLES FOR REVIEW.

1. At what quotation must $5\frac{1}{2}\%$ bonds be bought to enable the purchaser to realize $6\frac{1}{4}\%$ on his investment, allowing $\frac{1}{4}\%$ for brokerage?

2. If canal stock bought at 112 yielded the purchaser semi-annual dividends of 5%, what per cent. of income per annum did he receive on his investment?

3. A speculator invested $21600 in manufacturing stock quoted at $119\frac{1}{2}$, and sold 110 shares of the same at $121\frac{1}{4}$ and the remainder at $124\frac{3}{8}$. What was his net gain, allowing $\frac{1}{8}\%$ brokerage on each transaction?

4. The directors of a corporation watered the stock 25%, and then declared a dividend of 8%. What would have been the rate had the dividend been declared before inflation?

5. A banker bought 45 shares of 8% stock, and during the first year his income was 10% on his investment. During the second year the stock declined to such an extent that his income was 12% of the market value. What was the amount of his loss?

6. A merchant sold 38 shares of 8% stock at $115\frac{1}{8}$, and with the proceeds of the sale bought 7% stock at $5\frac{1}{8}\%$ discount. If he paid brokerage $\frac{1}{8}\%$ for each transaction, what sum did he gain or lose?

7. A speculator bought insurance stock at $124\frac{1}{2}$, and sold the same at $128\frac{3}{4}$, netting a profit of $459. How many shares were involved in the transaction, the par value of each share being $25?

8. A lady inherited $15264, which she placed in the hands of a stock broker for investment. With the funds he purchased 83 shares of preferred gas stock at $107\frac{1}{2}$, with an annual dividend of 6% guaranteed. The remainder was invested in equal sums in Indiana 5's at $4\frac{3}{4}\%$ *premium*, and Virginia 6's at $89\frac{3}{4}$. If the broker charged

$\frac{1}{4}\%$ brokerage and $\frac{1}{10}\%$ for collecting and remitting interest and dividends, what annual income did the lady receive?

9. What greater rate per cent. on the investment will be realized by buying 6% railroad stock at 5% discount than by buying 5% Tennessee bonds at 10% discount?

10. How much better is an investment of $6000 in a ground rent that will capitalize at 8%, than in 6% stocks at 20% discount?

11. Which is the better investment and what is the difference in rate: $4\frac{1}{2}\%$ bonds at 115, and having 15 years to run, or 6% bonds at 108, and having 5 years to run?

12. If I buy bonds running 12 years, at 85, and realize 5% per annum on the investment, what rate of interest do the bonds draw?

13. The charter of a new railroad company limits the stock to $750000 and the indebtedness to $250000. Three installments of the stock, 10%, 20% and 30%, respectively, have been paid in. The expenditures in the construction of the road have already reached the sum of $720000, and the estimated cost of completion is $300000. If the company call in the final installment of its stock, and issue bonds to its full limitation, and provide for the remaining outlay by levying an assessment upon the stockholders, what will an individual be obliged to pay who is the owner of 75 shares?

14. A street railway company having a stock capital of $48000, paid, during the last year, 2 semi-annual 6% dividends. If the yearly expenditures amounted to $3850, what were the gross earnings?

CUSTOM HOUSE BUSINESS.

540. Duties or **Customs** are taxes levied on imported *goods by the general government*, to protect home indus-

tries and to create a revenue for the support of the government.

541. A **Custom House** is an office established by the government for the transaction of business relating to duties.

REMARK. Ports at which custom houses are established are called "ports of entry;" "ports of delivery" sometimes exist in the same collection district, at which vessels may discharge their cargoes if they have previously entered and cleared at a port of entry.

542. An **Invoice** (sometimes called a "Shipper's Manifest,") is an itemized list of goods subject to duty, showing the quantity and price of each kind.

REMARK. Invoices are required to be made out according to the weights, measures, and currency of the country from which they are exported. Invoices from countries of which the value of the monetary unit has not been proclaimed by the Secretary of the Treasury, must be accompanied by a consular certificate denoting values in U. S. money.

543. Duties are of two kinds—*Ad Valorem* and *Specific*.

544. An **Ad Valorem Duty** is a duty levied at a certain rate per cent. of the market cash value of the goods at the time and place of shipment.

545. A **Specific Duty** is a duty levied at a certain price per unit on the weight or measure of the goods, regardless of their cost.

REMARK 1. In all computations of ad valorem duties, the fractional part of a dollar equaling or exceeding $\frac{1}{2}$ is regarded as $1; and fractional parts of a dollar less than $\frac{1}{2}$ are rejected.

REM. 2. In computing specific duties, the long ton of 2240 lb., and its corresponding divisions, are employed in cases of weight.

REM. 3. Some goods are subject to both an ad valorem and a specific duty.

546. **Allowances** are deductions made in estimating specific duties, and are distinguished as *Tare*, *Breakage* and *Leakage*.

REMARK 1. *Tare* is a deduction allowed for the weight of whatever contains the goods.

REM. 2. *Breakage* is a deduction allowed for the waste of liquids imported in bottles.

REM. 3. *Leakage* is a deduction allowed for the waste of liquids imported in casks or barrels.

547. Computations of ad valorem duties are made according to the principles of percentage, the market value at the place of export being the *base*, and the ad valorem duty the *percentage*.

1. What is the duty on 600 yards of carpeting 27 inches wide, bought at Lyons, France, and invoiced at 9.80 fr. per yard, the rate of duty being 30 cents per square yard and 25% ad valorem?

Operation.

9.80 fr. × 600 = 5880 fr.
$.193 × 5880 = $1134.84
$1135 × .25 = $283.75

27 in. = $\frac{2}{3}\frac{7}{6} = \frac{3}{4}$ yd., width
$\frac{3}{4} \times 600 = 450$ sq. yd.
$.30 × 450 = $135

$283.75 + $135 = $418.75

EXPLANATION. Multiplying the price of 1 yard by the number of yards gives 5880 francs, the market cash value in France. Multiplying the value of 1 fr. ($.193), as shown in the table (506), by the number of francs, gives $1134.84, the value in U. S. money; then multiplying $1135 (545, REM. 1.), by the rate of duty, gives $283.75, the ad valorem duty. We next find the number of square yards by multiplying length by width, and $.30 (specific duty per sq. yd.) multiplied by 450, the number of square yards, gives $135, the specific duty, and combining the specific duty with the ad valorem duty gives $418.75, the total amount of duty.

RULE. *Ad Valorem Duty.—Reduce the net invoice price to U. S. money (506); if there are allowances, deduct them and multiply the remainder by the rate of ad valorem duty.*

Specific Duty.—Deduct all allowances from the quantity specified in the invoice, and multiply the rate per unit of specific duty by the number of units subject to duty.

NOTE. The net invoice price is what remains of the bill after attending charges (boxing, cartage, etc.) and all discounts are deducted.

EXAMPLES FOR PRACTICE.

Find the ad valorem duty upon each of the following importations.

2. *£280 15s.*, allowing 5% for leakage, rate of duty, 40%.

3. £116 12s. 6d., allowing 8% for breakage, rate of duty, 25%.

4. 1240 fr., allowing 1½% for tare, rate of duty, 30%.

5. 4520 marks, rate of duty, 50%.

6. 8720 guilders, allowing 4% for breakage, rate of duty, 60%.

7. $1600 (Mexico), allowing 3% for tare, rate of duty, 15%.

Find the specific duty upon each of the following importations:

8. 3260 yd. of dress goods, 27 in. wide, rate of duty, 32¢ per sq. yd.

9. 60 doz. bottles wine, breakage 10%, duty $1.80 per doz.

10. 5 tons of sugar, tare 1 cwt. 40 lb., duty 1½¢ per lb.

11. 720 gal. brandy, leakage 3%, duty $1.20 per gal.

12. 80 tons of medicinal herbs, rate of duty ¾¢ per lb.

13. 120 gross bottles, breakage 8%, rate of duty 15¢ per doz.

14. 10 great gross buttons, rate of duty 5¢ per doz.

15. At 40% ad valorem, what is the duty on 2400 yd. silk, invoiced at 9.6 fr. per yd., cartage, 7.5 fr.

16. A merchant in Philadelphia received an importation of English watches, bought on 60 days' credit, and subject to a discount of 3% if paid in 10 days. The invoice amounted to £2750 5s., with additional charges amounting to £1 2s. 6d. for boxing and cartage. What was the duty at 25% ad valorem?

17. At 50% ad valorem, what is the duty on 120 oriental rugs, invoiced at £9 2s. 6d. each?

18. A merchant in Cincinnati bought of a house in London, 2400 yards of carpeting, billed at 2s. 6d. per yd., *insurance* and other charges were added, amounting to £5 5s. If the duty was 40% ad valorem, and the bill was

paid by a bill of exchange bought at 4.90, what was the gross price per yard of the carpeting?

19. If the duty, at 60%, on 71 casks of wine, each containing 30 gallons net, amounted to $2556, at what price per gallon was the wine invoiced?

20. If the duty on an importation of 20 gross bottles of porter, at 60¢ per dozen, amounted to $136.80, how many bottles were broken?

21. A liquor dealer imported 40 casks of port wine, each containing originally 36 gallons, invoiced at 13 fr. per gallon. He paid the bill by remitting a bill of exchange bought at 5.20, and also paid transportation at $1.20 per cask, and a bill for cartage amounting to $6. If the duty was 30%, and 1½% was allowed for leakage, at what price per gallon must the wine be sold to net a profit of 20%?

INSURANCE.

548. Insurance is a contract whereby one party, for a consideration, becomes bound to indemnify another party, or his representatives, against loss by certain risks.

REMARK. Insurance is classified as *property insurance* and *personal insurance.*

549. Property Insurance is that which indemnifies the owner of property for loss by fire, wind, or other casualty.

REMARK. The leading functions pertaining to property insurance are *Fire Insurance*, which is an indemnity for loss by fire; *Marine Insurance*, which is an indemnity for loss occasioned by perils of navigation; *Tornado Insurance*, which is an indemnity for loss by destructive winds; *Transit Insurance*, which is an indemnity for loss of, or damage to, merchandise during transportation; *Title Insurance*, which is an indemnity for loss by defective titles; and *Accident Insurance*, which is an indemnity for loss by breakage or other accidents.

550. Personal Insurance is the insurance of persons.

REMARK. *The leading functions of personal insurance are, Life Insurance, which secures the* payment of a specified sum to the beneficiary

at the death of the insured; *Health Insurance*, which secures the payment of a weekly sum during illness; and *Accident Insurance*, which secures the payment of some definite sum in case of accident.

551. The **Insurer** or **Underwriter** is the party taking the risk.

REMARK. Insurance is usually effected through corporations called *Insurance Companies*.

552. Insurance Companies are classified as *Stock Companies*, *Mutual Companies*, and *Mixed Companies*.

REMARK 1. A *Stock Insurance Company* is one of which the capital is owned by the stockholders, who alone share the profits and losses.

REM 2. A *Mutual Insurance Company* is one in which the parties insured share the profits and losses.

REM. 3. A *Mixed Insurance Company* is one in which the profits, above a stipulated sum paid to the stockholders as a dividend, are divided among the insured.

553. The **Insured** is the party protected by the insurance.

554. Premium is the sum paid for insurance. It is a certain per cent. of the sum insured, and varies according to the nature of the risk assumed.

555. The **Term of Insurance** is the time for which the property or person is insured.

REMARK 1. The term of fire insurance is usually from 1 to 5 years. The term of marine insurance is usually for a voyage; sometimes, however, the vessel is insured until its arrival at a certain port. The term of life insurance is usually the life of the insured.

REM. 2. The contract between the company and the insured is termed the *Policy*.

FIRE AND MARINE INSURANCE.

556. Fire Insurance is a contract whereby the insurers agree, for a stipulated sum, to indemnify the insured for any loss occasioned by fire to the property described in the policy, for the time therein specified.

REMARK 1. The policy is also a protection to the insured against loss *occasioned* by the means employed to extinguish a fire, or to prevent it *from spreading*.

564. The rates of premium for life insurance are based on the expectations of life, as shown by the foregoing table, and the probable rates of interest at which money can be loaned.

REMARK. The premium contains three elements: 1st, *Reserve*, or that portion of each premium which, compounded semi-annually at 3% for a term equal to the expectation of life, will amount to the face of the policy. 2d. An estimated amount of each man's equitable share of annual losses, (payments of policies becoming claims before the holders reach the expectation of life). 3d. *Loading*, which is a certain per cent. added to meet current expenses.

565. Most companies in this country are mutual, and divide the profits among the policy holders. The dividends operate to decrease subsequent rates or to increase the amount of insurance, as the policy holders may direct.

REMARK 1. Profits result from the following conditions: A realization of more than 3% interest on the reserve fund; a less number of losses than was estimated; and a less amount in expenditures than was anticipated.

REM. 2. Stock companies do not declare dividends, but usually make their premiums from 20 to 30 per cent. less than mutual companies. The rates given in the following tables are those of mutual companies.

566. Rates of Life Insurance are expressed at a given sum on each $1000 of the face of the policy.

NOTE 1. Tables of rates found in most commercial arithmetics are based upon a 4% reserve and are below the rates of most companies at the present time. Most banks have changed their rate of interest on time deposits from 4% to 3%, thus making a revision of life insurance rates a necessity. The following table is correct and up to date.

TABLE OF INSURANCE RATES,

SHOWING PREMIUMS FOR A POLICY OF $1000.

Age.	Life Policies. Premiums payable annually for: Life.	20 years.	15 years.	10 years.	Endowment Policies. Premiums payable annually during life of policy: 25 years.	20 years.	15 years.	10 years.	Age.
21	$19.62	$29.84	$36.00	$48.56	$39.38	$50.07	$68.40	$105.84	21
22	20.06	30.31	36.55	49.30	39.50	50.17	68.50	105.92	22
23	20.51	30.80	37.13	50.06	39.63	50.28	68.60	106.02	23
24	20.99	31.31	37.73	50.85	39.76	50.40	68.70	106.11	24
25	21.49	31.83	38.35	51.67	39.90	50.53	68.82	106 22	25
26	22.01	32.37	38.98	52.51	40.06	50.66	68.94	106.33	26
27	22.56	32.94	39.65	53.38	40.23	50.81	69.07	106.44	27
28	23.14	33.52	40.33	54.28	40.41	50.97	69.21	106.56	28
29	23.74	34.13	41.05	55.21	40.61	51.13	69.35	106.70	29
30	24.38	34.76	41.78	56.18	40.82	51.31	69.51	106.84	30
31	25.05	35.42	42.55	57.18	41.05	51.51	69.68	106.98	31
32	25.75	36.11	43.34	58.21	41.31	51.72	69.86	107.15	32
33	26.50	36.82	44.16	59.28	41.59	51.95	70.06	107.32	33
34	27.28	37.56	45.02	60.38	41.89	52.20	70.27	107.50	34
35	28.11	38.34	45.91	61.53	42.23	52.47	70.50	107.70	35
36	28.98	39.15	46.83	62.71	42.60	52.78	70.75	107.91	36
37	29.90	40.00	47.79	63.94	43.00	53.10	71.02	108.14	37
38	30.88	40.89	48.79	65.21	43.45	53.47	71.32	108.39	38
39	31.91	41.81	49.83	66.53	43.94	53.87	71.64	108.66	39
40	33.01	42.79	50.92	67.90	44.49	54.31	72.00	108.96	40
41	34.16	43.82	52.06	69.32	45.09	54.80	72.40	109.28	41
42	35.39	44.90	53.24	70.79	45.75	55.33	72.83	109.63	42
43	36.70	46.04	54.49	72.32	46.49	55.93	73.32	110.03	43
44	38.08	47.25	55.79	73.91	47.30	56.59	73.85	110.46	44
45	39.55	48.52	57.16	75.57	48.20	57.32	74.44	110.94	45
46	41.12	49.87	58.60	77.30	49.19	58.14	75.10	111.47	46
47	42.79	51.31	60.11	79.10	50.29	59.03	75.83	112.46	47
48	44.57	52.83	61.71	80.98	51.49	60.03	76.64	112.72	48
49	46.46	54.45	63.39	82.95	52.82	61.13	77.54	113.44	49
50	48.48	56.17	65.16	84.99	54.29	62.34	78.53	114.24	50
51	50.62	58.01	67.03	87.12		63.67	79.62	115.13	51
52	52.91	59.97	69.01	89.35		65.15	80.82	116.10	52
53	55.35	62.06	71.10	91.68		66.76	82.15	117.17	53
54	57.95	64.29	73.31	94.11		68.55	83.61	118.35	54
55	60.72	66.69	75.66	96.66		70.51	85.21	119.64	55

NOTE 2. To find the semi-annual rate, add 4% to the annual rate and divide by 2. To find quarterly rate add 6% and divide by 4.

EXAMPLES FOR PRACTICE.

1. What sum must a man aged 35 years pay annually to secure a 20-year life policy of $4500?

$38.34 (rate per $1000) × 4½ (number of thousands) = $172.53 (annual cost).

2. A man aged 31 years secured an ordinary life policy of $3000, and died after making his 30th semi-annual payment. What was the total cost of his insurance?

$\frac{\$25.05 \times 1.04}{2}$ (566, note 2) = $13.03 (semi-annual rate per $1000).

$13.03 × 30 (number of payments) = $390.90 (cost per $1000).

$390.90 × 3 (number of thousands) = $1172.70 (total cost).

3. What annual premium will a man aged 27 years pay to secure an endowment policy for $2500, payable to himself in 20 years, or to his heirs if death occur prior?

4. What quarterly payment will secure to a person aged 25 years, a life policy of $3500?

5. A man aged 33 years obtained a 10-year life policy for $2000. What annual premium did he pay?

6. What semi-annual payment for life will be required of a person 34 years of age to secure a policy of $15000?

7. What quarterly payment for 15 years will secure to a person 38 years of age, a life policy of $8000?

8. How much more will a 10-year endowment policy of $5000 cost a person aged 21 years, than a 15-year endowment policy for the same amount?

9. A man 24 years of age took a policy in a mutual company for $4000, the premium continuing annually until death, which occurred after making 23 payments. If the dividends increased the policy $1350, how much more did the beneficiary receive than the insured paid the company?

10. At the age of 25 years, each of two brothers, Charles and Henry, who were twins, came into possession of *the first installment* of a 20-year annuity of a sufficient

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23	20.51	30.80	37.13	50.06	39.63	50.28	68.60	106.02	23
24	20.99	31.31	37.73	50.85	39.76	50.40	68.70	106.11	24
25	21.49	31.83	38.35	51.67	39.90	50.53	68.82	106 22	25
26	22.01	32.37	38.98	52.51	40.06	50.66	68.94	106.33	26
27	22.56	32.94	39.65	53.38	40.23	50.81	69.07	106.44	27
28	23.14	33.52	40.33	54.28	40.41	50.97	69.21	106.56	28
29	23.74	34.13	41.05	55.21	40.61	51.13	69.35	106.70	29
30	24.38	34.76	41.78	56.18	40.82	51.31	69.51	106.84	30
31	25.05	35.42	42.55	57.18	41.05	51.51	69.68	106.98	31
32	25.75	36.11	43.34	58.21	41.31	51.72	69.86	107.15	32
33	26.50	36.82	44.16	59.28	41.59	51.95	70.06	107.32	33
34	27.28	37.56	45.02	60.38	41.89	52.20	70.27	107.50	34
35	28.11	38.34	45.91	61.53	42.23	52.47	70.50	107.70	35
36	28.98	39.15	46.83	62.71	42.60	52.78	70.75	107.91	36
37	29.90	40.00	47.79	63.94	43.00	53.10	71.02	108.14	37
38	30.88	40.89	48.79	65.21	43.45	53.47	71.32	108.39	38
39	31.91	41.81	49.83	66.53	43.94	53.87	71.64	108.66	39
40	33.01	42.79	50.92	67.90	44.49	54.31	72.00	108.96	40
41	34.16	43.82	52.06	69.32	45.09	54.80	72.40	109.28	41
42	35.39	44.90	53.24	70.79	45.75	55.33	72.83	109.63	42
43	36.70	46.04	54.49	72.32	46.49	55.93	73.32	110.03	43
44	38.08	47.25	55.79	73.91	47.30	56.59	73.85	110.46	44
45	39.55	48.52	57.16	75.57	48.20	57.32	74.44	110.94	45
46	41.12	49.87	58.60	77.30	49.19	58.14	75.10	111.47	46
47	42.79	51.31	60.11	79.10	50.29	59.03	75.83	112.46	47
48	44.57	52.83	61.71	80.98	51.49	60.03	76.64	112.72	48
49	46.46	54.45	63.39	82.95	52.82	61.13	77.54	113.44	49
50	48.48	56.17	65.16	84.99	54.29	62.34	78.53	114.24	50
51	50.62	58.01	67.03	87.12		63.67	79.62	115.13	51
52	52.91	59.97	69.01	89.35		65.15	80.82	116.10	52
53	55.35	62.06	71.10	91.68		66.76	82.15	117.17	53
54	57.95	64.29	73.31	94.11		68.55	83.61	118.35	54
55	60.72	66.69	75.66	96.66		70.51	85.21	119.64	55

NOTE 2. To find the semi-annual rate, add 4% to the annual rate and divide by 2. To find quarterly rate add 6% and divide by 4.

564. The rates of premium for life insurance are based on the expectations of life, as shown by the foregoing table, and the probable rates of interest at which money can be loaned.

REMARK. The premium contains three elements: 1st, *Reserve*, or that portion of each premium which, compounded semi-annually at 3% for a term equal to the expectation of life, will amount to the face of the policy. 2d. An estimated amount of each man's equitable share of annual losses, (payments of policies becoming claims before the holders reach the expectation of life). 3d. *Loading*, which is a certain per cent. added to meet current expenses.

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NOTE 1. Tables of rates found in most commercial arithmetics are based upon a 4% reserve and are below the rates of most companies at the present time. Most banks have changed their rate of interest on time deposits from 4% to 3%, thus making a revision of life insurance rates a necessity. The following table is correct and up to date.

TABLE OF INSURANCE RATES,

SHOWING PREMIUMS FOR A POLICY OF $1000.

Age.	Life Policies. Premiums payable annually for: Life.	20 years.	15 years.	10 years.	Endowment Policies. Premiums payable annually during life of policy: 25 years.	20 years.	15 years.	10 years.	Age.
21	$19.62	$29.84	$36.00	$48.56	$39.38	$50.07	$68.40	$105.84	21
22	20.06	30.31	36.55	49.30	39.50	50.17	68.50	105.92	22
23	20.51	30.80	37.13	50.06	39.63	50.28	68.60	106.02	23
24	20.99	31.31	37.73	50.85	39.76	50.40	68.70	106.11	24
25	21.49	31.83	38.35	51.67	39.90	50.53	68.82	106 22	25
26	22.01	32.37	38.98	52.51	40.06	50.66	68.94	106.33	26
27	22.56	32.94	39.65	53.38	40.23	50.81	69.07	106.44	27
28	23.14	33.52	40.33	54.28	40.41	50.97	69.21	106.56	28
29	23.74	34.13	41.05	55.21	40.61	51.13	69.35	106.70	29
30	24.38	34.76	41.78	56.18	40.82	51.31	69.51	106.84	30
31	25.05	35.42	42.55	57.18	41.05	51.51	69.68	106.98	31
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43	36.70	46.04	54.49	72.32	46.49	55.93	73.32	110.03	43
44	38.08	47.25	55.79	73.91	47.30	56.59	73.85	110.46	44
45	39.55	48.52	57.16	75.57	48.20	57.32	74.44	110.94	45
46	41.12	49.87	58.60	77.30	49.19	58.14	75.10	111.47	46
47	42.79	51.31	60.11	79.10	50.29	59.03	75.83	112.46	47
48	44.57	52.83	61.71	80.98	51.49	60.03	76.64	112.72	48
49	46.46	54.45	63.39	82.95	52.82	61.13	77.54	113.44	49
50	48.48	56.17	65.16	84.99	54.29	62.34	78.53	114.24	50
51	50.62	58.01	67.03	87.12		63.67	79.62	115.13	51
52	52.91	59.97	69.01	89.35		65.15	80.82	116.10	52
53	55.35	62.06	71.10	91.68		66.76	82.15	117.17	53
54	57.95	64.29	73.31	94.11		68.55	83.61	118.35	54
55	60.72	66.69	75.66	96.66		70.51	85.21	119.64	55

NOTE 2. To find the semi-annual rate, add 4% to the annual rate and divide by 2. To find quarterly rate add 6% and divide by 4.

EXAMPLES FOR PRACTICE.

1. What sum must a man aged 35 years pay annually to secure a 20-year life policy of $4500?

$38.34 (rate per $1000) × 4½ (number of thousands) = $172.53 (annual cost).

2. A man aged 31 years secured an ordinary life policy of $3000, and died after making his 30th semi-annual payment. What was the total cost of his insurance?

$\frac{\$25.05 \times 1.04}{2}$ (566, note 2) = $13.03 (semi-annual rate per $1000).

$13.03 × 30 (number of payments) = $390.90 (cost per $1000).

$390.90 × 3 (number of thousands) = $1172.70 (total cost).

3. What annual premium will a man aged 27 years pay to secure an endowment policy for $2500, payable to himself in 20 years, or to his heirs if death occur prior?

4. What quarterly payment will secure to a person aged 25 years, a life policy of $3500?

5. A man aged 33 years obtained a 10-year life policy for $2000. What annual premium did he pay?

6. What semi-annual payment for life will be required of a person 34 years of age to secure a policy of $15000?

7. What quarterly payment for 15 years will secure to a person 38 years of age, a life policy of $8000?

8. How much more will a 10-year endowment policy of $5000 cost a person aged 21 years, than a 15-year endowment policy for the same amount?

9. A man 24 years of age took a policy in a mutual company for $4000, the premium continuing annually until death, which occurred after making 23 payments. If the dividends increased the policy $1350, how much more did the beneficiary receive than the insured paid the company?

10. At the age of 25 years, each of two brothers, Charles and Henry, who were twins, came into possession of the first installment of a 20-year annuity of a suffic

amount to pay the annual premium on a 20-year endowment policy of $5000. Charles availed himself of the benefits of the insurance, but Henry saw fit to loan each installment of his annuity, as received, at 6% simple interest. What sum did each possess, as the result of his annuity, at the end of 20 years, provided that the policy held by Charles increased in value $2625 by virtue of dividends?

11. A clergyman, aged 42 years, takes an endowment policy for $2500, payable to himself in 15 years, or to his wife at death, and dies after making 9 payments. How much money would he have saved had he insured for the same amount on the annual life plan?

12. A man, aged 46 years, obtained a 15-year endowment policy of $6000, but he died after making his 5th annual payment of premium. What sum did his beneficiary receive above the cost of insurance?

13. A gentleman at the age of 37 years secured an ordinary life policy of $3500, and died after making his 9th semi-annual payment of premium. How much greater was the sum received by the heirs than the total amount paid in premiums?

14. A man at the age of 38 years obtained from a stock company, whose rates of premium are 25% below those of mutual companies, a 15-year life policy of $2000, and died after making his 10th payment of premium. How much less would his heirs have received had the sums paid in premiums been loaned at 6% simple interest?

TAXES.

567. A **Tax** is a sum of money levied on the person, property, or income of an individual for public purposes.

568. A **Poll-Tax** is a sum levied on the person of each *male citizen* above a certain age (usually 21 years), who *is not* exempt by law.

569. A **Property Tax** is a sum levied on property at a certain rate per thousand of its assessed valuation.

570. An **Assessor** is a public officer whose duty it is to estimate the value of each piece of property within his assessment district.

571. An **Assessment Roll** is a list or schedule containing the names of all persons liable to taxation in a given assessment district, and the valuation of each person's taxable property.

572. An **Equalization Board** is an official body of men to whom the assessment rolls are delivered for inspection, and for correction in cases where the assessed valuations are regarded, by the members thereof, too high or too low.

REMARK. Methods of equalizing valuations are not uniform in all states. According to the legal requirement of many states, each assessor, after completing his work, delivers the assessment roll to a local board of his district, who, after making needful corrections, causes the same to be delivered to the county equalization board, whose duty it is to compare the assessment rolls received from the different assessment districts of the county, and make such changes as may be necessary to render valuations of the same class of property equal throughout the county. The state equalization board, after receiving an abstract of values fixed by the county boards, will then make such changes as may be necessary to equalize valuations throughout the state.

573. The principles of percentage will apply in tax computations, the valuation of the property being the *base*, and the property tax corresponding to the *percentage*.

REMARK 1. The *unit rate*, in tax computations, is *1 mill*, or .001 of a dollar; therefore a tax is reckoned at a certain *rate per thousand*, and not properly at a *rate per cent.*

REM. 2. In determining tax rates, the gradation, in most states, is by tenths of a mill; thus, if the exact rate is a complex decimal having a value between $17\frac{3}{10}$ mills and $17\frac{4}{10}$ mills, the rate to be employed is $17\frac{4}{10}$ mills. A few states require a gradation by fourths of a mill.

1. At a district school meeting, it was voted that the sum of *$1500 be raised* by taxation. If the valuation of

the real estate belonging to the district was $58740, and of the personal property $36328, how much was A's school tax, provided his real estate was assessed at $2240, and his personal property at $974?

Operation.

$58740+$36328=$95068
$1500÷$95068=.0158—(rate)
($2240+$974)×.0158=$50.78

EXPLANATION. The value of the personal property belonging to the district added to the value of the real estate gives $95068, which is the total assessed valuation of the property to be taxed. Then dividing the sum to be raised by the amount of taxable property, will evidently give the amount of tax on each dollar, or the rate, which is .0158. A's total assessed valuation multiplied by the rate gives $50.78, the amount of his tax.

2. What would have been the rate in the above example if 83 persons of the district had been required to pay a poll tax of 75 cents each?

$.75×83=$62.25, amount raised from poll tax.
$1500—$62.25=$1437.75, amount of property tax.
$1437.75÷$95068=.0151+, therefore the rate is .0152.

NOTE. In determining rates, the gradations in the above, and in all following examples, are by tenths of a mill.

RULE I. *To find rate. Divide the property tax by the assessed valuation, and carry the division to the 4th decimal place, and if a remainder occur, increase the terminal decimal by 1; the quotient thus obtained will be the number of mills and tenths on each dollar.*

II. *To find each individual tax. Multiply each individual's valuation by the rate.*

NOTE. If a poll tax is included, it must be deducted from the total tax to find the property tax.

EXAMPLES FOR PRACTICE.

3. In a certain township, $3500 is to be raised by direct taxation on the property of the township. The assessed valuation of $123800 was raised 10% by the county board of equalization, and the state board has made another

raise of 5%. What is C's tax, whose valuation, as returned by the assessor, is $3600?

4. In school district No. 24, the valuation of real estate is $62340, and of personal property, $39718; the valuation of all other real estate in the same township is $189420, and of all other personal property, $116343. The township has voted to raise by taxation, $2500 for town purposes, and school district No. 24 has voted to raise $850 for school purposes. The following levies have been ordered: for state purposes, 4 mills; for county purposes, 5 mills; for support of poor, $\frac{7}{10}$ of a mill; for general school purposes 1 mill. What will be A's total tax if he resides in school district No. 24, and his real estate is valued $2630, and his personal property $1487?

5. The following tax list page contains $\frac{1}{10}$ the valuations of all the property belonging to each school district. The districts enumerated have voted the following sums: District 15, $450; district 16, $1200; district 17, $675; district 18, $1600. The rate of general tax, .0143, includes all items except special school tax, and applies to each valuation. Compute changes in valuations made by county and state equalizing boards, recording, in each case, the nearest unit; find the rate of school tax for each district; make all required computations and extensions; add the columns, and see that the sum of totals equals grand total.

rate premium" for the expired part of the term. Short rates are rates charged for periods less than a year, and are higher in proportion to time than annual rates.

561. Computations in fire and marine insurance accord with the principles of percentage; the sum insured being the base, and the premium the percentage.

Find the premium that should be paid in each of the following examples:

Sum insured.	Rate of premium.	Sum insured.	Rate of premium.
1. \$4500	$1\frac{1}{2}\%$.	*5.* \$960	$1\frac{3}{8}\%$.
2. \$8750	$2\frac{1}{8}\%$.	*6.* \$3250	$2\frac{3}{4}\%$.
3. \$6300	$1\frac{3}{4}\%$.	*7.* \$5600	$2\frac{1}{4}\%$.
4. \$12600	$\frac{7}{8}\%$.	*8.* \$7400	$\frac{1}{2}\%$.

Find the sum insured in each of the following examples:

Premium.	Rate of premium.	Premium.	Rate of premium.
9. \$23.70	$1\frac{1}{2}\%$.	*12.* \$18.45	$2\frac{1}{4}\%$.
10. \$77	$1\frac{3}{8}\%$.	*13.* \$21.70	$\frac{7}{8}\%$.
11. \$94.20	$\frac{3}{4}\%$.	*14.* \$107.75	$2\frac{1}{8}\%$.

Find the rate of premium in each of the following examples:

Sum insured.	Premium.	Sum insured.	Premium.
15. \$3850	\$57.75.	*18.* \$2440	\$21.35.
16. \$2140	\$16.05.	*19.* \$27800	\$312.75.
17. \$23120	\$115.60.	*20.* \$620	\$13.95.

21. What will it cost to insure a house worth \$4200 if the company assumes a risk of $\frac{3}{4}$ the value and charges a premium of $1\frac{3}{4}\%$?

22. I insured a warehouse worth \$7200 for $\frac{3}{4}$ its value. In case of total destruction by fire, what will be my loss, if the company charged a premium of $1\frac{1}{8}\%$?

23. The Phœnix Insurance Co. took a risk of \$4500 on *a country* store building at $1\frac{1}{2}\%$ premium; and, the same *day, insured* $\frac{2}{3}$ the risk in the American Insurance Co. at

$1\frac{1}{4}\%$ premium. What rate per cent. did the Phœnix Co. receive for assuming the remainder of the risk?

24. I insured my house for $\frac{4}{5}$ its value and paid a premium of \$58.95, the rate being \$1.50 per \$100. What was the value of the house?

25. A man owning a warehouse has it insured for \$3200, which is $\frac{2}{3}$ its value. If the rate of premium is 2%, and the property burns before the expiration of the insurance term, what will be the owner's loss per cent.?

26. A house, which was insured for $\frac{3}{5}$ its value at a $1\frac{1}{2}\%$ rate of premium, was totally destroyed by fire. If the owner's total loss was \$1287.50, what was the face of the policy?

27. A steamboat was insured for $\frac{3}{4}$ of its value at $1\frac{1}{4}\%$; A, who was the owner of a $\frac{3}{5}$ interest, paid \$236.25 of the premium. What was the value of the boat? If, during the life of the policy, the boat was damaged by fire, and A received, as his part due from the insurance company, \$2808, what was the total damage?

28. A ship, valued at \$61860, is insured for enough more than its value to cover the cost of insurance at $1\frac{1}{2}\%$, and \$4 for the policy and survey fee. What is the sum insured?

29. The owners of an ocean vessel insure it for a sum equal to its value, \$72000, and the cost of insurance, which is at the rate of 2%. What is the face of the policy?

30. My house is insured in 3 companies; one-half the insurance is carried by the Home at $1\frac{1}{8}\%$; three-eighths is carried by the Germania at $1\frac{1}{4}\%$, and the remainder is carried by the Hartford at $1\frac{1}{2}\%$. If the total cost of the insurance in the three companies is \$68.25, what is the sum insured?

LIFE INSURANCE.

562. Life Insurance is a contract between an insurance company and the insured by virtue of which the company pledges to pay a certain sum of money to the heirs of the insured, at his death, or to himself if he survive a certain number of years, in consideration of payments made, according to the terms of the policy.

REMARK 1. Policies issued by life insurance companies are of two kinds, *Life Policies* and *Endowment Policies*. *Life policies* are payable to the beneficiaries at the death of the insured. *Endowment policies* are payable to the insured at the expiration of a certain time, or, in the event of death prior to the expiration of the stated time, to the beneficiary or beneficiaries named in the policy.

REM. 2. The premium may be paid in one sum, on the receipt of the policy, or annually, semi-annually, or quarterly during life, or during a term of years; usually 10, 15, or 20 years.

563. The **Expectation of Life** is the average number of years which persons of different ages have been found to live.

REMARK. The duration of life of any single individual cannot be determined with accuracy, but it has been found, by carefully preserved records, that for every person of any given age who lives longer than the expectation, some other person will die correspondingly earlier. The following is the

AMERICAN TABLE OF MORTALITY.

EXPECTATION OF LIFE IN YEARS.

Age	Expectation of life	Age	Expectation of life	Age	Expectation of life	Age	Expectation of life	Age	Expectation of life	Age	Expectation of life
		28	36.73	37	30.35	46	23.80	55	17.40	64	11.68
20	42.20	29	36.03	38	29.62	47	23.08	56	16.72	65	11.10
21	41.53	30	35.33	39	28.90	48	22.36	57	16.05	66	10 54
22	40.85	31	34.62	40	28.18	49	21.63	58	15.39	67	10.00
23	40.17	32	33.92	41	27.45	50	20.91	59	14.74	68	9.48
24	39.49	33	33.21	42	26.72	51	20.20	60	14.09	69	8.98
25	38.81	34	32.50	43	25.99	52	19.49	61	13.47	70	8.48
26	*38.11*	35	31.78	44	25.27	53	18.79	62	12.86	71	8.00
27	*37.43*	*36*	31.07	45	24.54	54	18.09	63	12.26	72	.7 54

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23	20.51	30.80	37.13	50.06	39.63	50.28	68.60	106.02	23
24	20.99	31.31	37.73	50.85	39.76	50.40	68.70	106.11	24
25	21.49	31.83	38.35	51.67	39.90	50.53	68.82	106 22	25
26	22.01	32.37	38.98	52.51	40.06	50.66	68.94	106.33	26
27	22.56	32.94	39.65	53.38	40.23	50.81	69.07	106.44	27
28	23.14	33.52	40.33	54.28	40.41	50.97	69.21	106.56	28
29	23.74	34.13	41.05	55.21	40.61	51.13	69.35	106.70	29
30	24.38	34.76	41.78	56.18	40.82	51.31	69.51	106.84	30
31	25.05	35.42	42.55	57.18	41.05	51.51	69.68	106.98	31
32	25.75	36.11	43.34	58.21	41.31	51.72	69.86	107.15	32
33	26.50	36.82	44.16	59.28	41.59	51.95	70.06	107.32	33
34	27.28	37.56	45.02	60.38	41.89	52.20	70.27	107.50	34
35	28.11	38.34	45.91	61.53	42.23	52.47	70.50	107.70	35
36	28.98	39.15	46.83	62.71	42.60	52.78	70.75	107.91	36
37	29.90	40.00	47.79	63.94	43.00	53.10	71.02	108.14	37
38	30.88	40.89	48.79	65.21	43.45	53.47	71.32	108.39	38
39	31.91	41.81	49.83	66.53	43.94	53.87	71.64	108.66	39
40	33.01	42.79	50.92	67.90	44.49	54.31	72.00	108.96	40
41	34.16	43.82	52.06	69.32	45.09	54.80	72.40	109.28	41
42	35.39	44.90	53.24	70.79	45.75	55.33	72.83	109.63	42
43	36.70	46.04	54.49	72.32	46.49	55.93	73.32	110.03	43
44	38.08	47.25	55.79	73.91	47.30	56.59	73.85	110.46	44
45	39.55	48.52	57.16	75.57	48.20	57.32	74.44	110.94	45
46	41.12	49.87	58.60	77.30	49.19	58.14	75.10	111.47	46
47	42.79	51.31	60.11	79.10	50.29	59.03	75.83	112.46	47
48	44.57	52.83	61.71	80.98	51.49	60.03	76.64	112.72	48
49	46.46	54.45	63.39	82.95	52.82	61.13	77.54	113.44	49
50	48.48	56.17	65.16	84.99	54.29	62.34	78.53	114.24	50
51	50.62	58.01	67.03	87.12		63.67	79.62	115.13	51
52	52.91	59.97	69.01	89.35		65.15	80.82	116.10	52
53	55.35	62.06	71.10	91.68		66.76	82.15	117.17	53
54	57.95	64.29	73.31	94.11		68.55	83.61	118.35	54
55	60.72	66.69	75.66	96.66		70.51	85.21	119.64	55

NOTE 2. To find the semi-annual rate, add 4% to the annual rate and divide by 2. To find quarterly rate add 6% and divide by 4.

EXAMPLES FOR PRACTICE.

1. What sum must a man aged 35 years pay annually to secure a 20-year life policy of $4500?

$38.34 (rate per $1000) × 4½ (number of thousands) = $172.53 (annual cost).

2. A man aged 31 years secured an ordinary life policy of $3000, and died after making his 30th semi-annual payment. What was the total cost of his insurance?

$\frac{\$25.05 \times 1.04}{2}$ (566, note 2) = $13.03 (semi-annual rate per $1000).

$13.03 × 30 (number of payments) = $390.90 (cost per $1000).

$390.90 × 3 (number of thousands) = $1172.70 (total cost).

3. What annual premium will a man aged 27 years pay to secure an endowment policy for $2500, payable to himself in 20 years, or to his heirs if death occur prior?

4. What quarterly payment will secure to a person aged 25 years, a life policy of $3500?

5. A man aged 33 years obtained a 10-year life policy for $2000. What annual premium did he pay?

6. What semi-annual payment for life will be required of a person 34 years of age to secure a policy of $15000?

7. What quarterly payment for 15 years will secure to a person 38 years of age, a life policy of $8000?

8. How much more will a 10-year endowment policy of $5000 cost a person aged 21 years, than a 15-year endowment policy for the same amount?

9. A man 24 years of age took a policy in a mutual company for $4000, the premium continuing annually until death, which occurred after making 23 payments. If the dividends increased the policy $1350, how much more did the beneficiary receive than the insured paid the company?

10. At the age of 25 years, each of two brothers, Charles and Henry, who were twins, came into possession of *the first installment* of a 20-year annuity of a sufficient

amount to pay the annual premium on a 20-year endowment policy of $5000. Charles availed himself of the benefits of the insurance, but Henry saw fit to loan each installment of his annuity, as received, at 6% simple interest. What sum did each possess, as the result of his annuity, at the end of 20 years, provided that the policy held by Charles increased in value $2625 by virtue of dividends?

11. A clergyman, aged 42 years, takes an endowment policy for $2500, payable to himself in 15 years, or to his wife at death, and dies after making 9 payments. How much money would he have saved had he insured for the same amount on the annual life plan?

12. A man, aged 46 years, obtained a 15-year endowment policy of $6000, but he died after making his 5th annual payment of premium. What sum did his beneficiary receive above the cost of insurance?

13. A gentleman at the age of 37 years secured an ordinary life policy of $3500, and died after making his 9th semi-annual payment of premium. How much greater was the sum received by the heirs than the total amount paid in premiums?

14. A man at the age of 38 years obtained from a stock company, whose rates of premium are 25% below those of mutual companies, a 15-year life policy of $2000, and died after making his 10th payment of premium. How much less would his heirs have received had the sums paid in premiums been loaned at 6% simple interest?

TAXES.

567. A **Tax** is a sum of money levied on the person, property, or income of an individual for public purposes.

568. A **Poll-Tax** is a sum levied on the person of each *male citizen* above a certain age (usually 21 years), who *is not exempt* by law.

569. A **Property Tax** is a sum levied on property at a certain rate per thousand of its assessed valuation.

570. An **Assessor** is a public officer whose duty it is to estimate the value of each piece of property within his assessment district.

571. An **Assessment Roll** is a list or schedule containing the names of all persons liable to taxation in a given assessment district, and the valuation of each person's taxable property.

572. An **Equalization Board** is an official body of men to whom the assessment rolls are delivered for inspection, and for correction in cases where the assessed valuations are regarded, by the members thereof, too high or too low.

REMARK. Methods of equalizing valuations are not uniform in all states. According to the legal requirement of many states, each assessor, after completing his work, delivers the assessment roll to a local board of his district, who, after making needful corrections, causes the same to be delivered to the county equalization board, whose duty it is to compare the assessment rolls received from the different assessment districts of the county, and make such changes as may be necessary to render valuations of the same class of property equal throughout the county. The state equalization board, after receiving an abstract of values fixed by the county boards, will then make such changes as may be necessary to equalize valuations throughout the state.

573. The principles of percentage will apply in tax computations, the valuation of the property being the *base*, and the property tax corresponding to the *percentage*.

REMARK 1. The *unit rate*, in tax computations, is *1 mill*, or .001 of a dollar; therefore a tax is reckoned at a certain *rate per thousand*, and not properly at a *rate per cent.*

REM. 2. In determining tax rates, the gradation, in most states, is by tenths of a mill; thus, if the exact rate is a complex decimal having a value between $17\frac{3}{10}$ mills and $17\frac{4}{10}$ mills, the rate to be employed is $17\frac{4}{10}$ mills. A few states require a gradation by fourths of a mill.

1. At a district school meeting, it was voted that the sum of *$1500 be* raised by taxation. If the valuation of

the real estate belonging to the district was $58740, and of the personal property $36328, how much was A's school tax, provided his real estate was assessed at $2240, and his personal property at $974?

Operation.

$58740+$36328=$95068
$1500÷$95068=.0158—(rate)
($2240+$974)×.0158=$50.78

EXPLANATION. The value of the personal property belonging to the district added to the value of the real estate gives $95068, which is the total assessed valuation of the property to be taxed. Then dividing the sum to be raised by the amount of taxable property, will evidently give the amount of tax on each dollar, or the rate, which is .0158. A's total assessed valuation multiplied by the rate gives $50.78, the amount of his tax.

2. What would have been the rate in the above example if 83 persons of the district had been required to pay a poll tax of 75 cents each?

$.75×83=$62.25, amount raised from poll tax.
$1500—$62.25=$1437.75, amount of property tax.
$1437.75÷$95068=.0151+, therefore the rate is .0152.

NOTE. In determining rates, the gradations in the above, and in all following examples, are by tenths of a mill.

RULE I. *To find rate. Divide the property tax by the assessed valuation, and carry the division to the 4th decimal place, and if a remainder occur, increase the terminal decimal by 1; the quotient thus obtained will be the number of mills and tenths on each dollar.*

II. *To find each individual tax. Multiply each individual's valuation by the rate.*

NOTE. If a poll tax is included, it must be deducted from the total tax to find the property tax.

EXAMPLES FOR PRACTICE.

3. In a certain township, $3500 is to be raised by direct taxation on the property of the township. The assessed valuation of $123800 was raised 10% by the county board of equalization, and the state board has made another

raise of 5%. What is C's tax, whose valuation, as returned by the assessor, is $3600?

4. In school district No. 24, the valuation of real estate is $62340, and of personal property, $39718; the valuation of all other real estate in the same township is $189420, and of all other personal property, $116343. The township has voted to raise by taxation, $2500 for town purposes, and school district No. 24 has voted to raise $850 for school purposes. The following levies have been ordered: for state purposes, 4 mills; for county purposes, 5 mills; for support of poor, $\frac{7}{10}$ of a mill; for general school purposes 1 mill. What will be A's total tax if he resides in school district No. 24, and his real estate is valued $2630, and his personal property $1487?

5. The following tax list page contains $\frac{1}{10}$ the valuations of all the property belonging to each school district. The districts enumerated have voted the following sums: District 15, $450; district 16, $1200; district 17, $675; district 18, $1600. The rate of general tax, .0143, includes all items except special school tax, and applies to each valuation. Compute changes in valuations made by county and state equalizing boards, recording, in each case, the nearest unit; find the rate of school tax for each district; make all required computations and extensions; add the columns, and see that the sum of totals equals grand total.

Names	School dist.	Assessed Value	County val.* + 15%	State val.† − 5%	School dist. No. 15 Rate.....	School dist. No. 16 Rate.....	School dist. No. 17 Rate.....	School dist. No. 18 Rate.....	General tax $14\frac{3}{10}$ m.	Total tax
A	17	$872	.....	....			....		.... ..	.. ..
B	17	1480	.....	.....			.. ...		.... ..	.... ..
C	15	728	.....	.....	....				.... ..	.... ..
D	18	365	.....	.....				....	.... ..	.... ..
E	15	2412	.....	.....	....				.... ..	.... ..
F	16	1886	.....	.....		....			.... ..	.... ..
G	16	932	.....	.....		.. ...			.... ..	.... ..
H	17	2148	.....	.....			....		.... ..	.... ..
I	18	3620	.....	.....				....	.... ..	.... ..
J	15	184	.....	.....	.. ...				.... ..	.... ..
K	17	615	....	....			....		.... ..	.... ..
L	16	4260	.....	.....		....			.... ..	.... ..
M	18	1542	.....	.....				....	.... ..	.... ..
N	18	718	.....	....				....	.... ..	....
O	17	668	.....	.....			....		.... ..	.. ..
P	16	1756	.....	.....		...			.... ..	.... ..
Q	16	240	.....	.....		....			.... ..	.. .
R	18	384	.....	.....				....	.... ..	.... ..
Totals			.....	.....	....	....	....	....	.... ..	.... ..

*The county board of equalization raised valuations 15%.
†The state board reduced values fixed by county board 5%.

6. What rate of taxation will be necessary to raise $3200, above collector's commission, which is $1\frac{1}{2}\%$, the assessed valuation, upon which the tax is to be levied, being *$287520*?

7. In a certain county the assessed valuation of prop-

erty, unchanged by equalizing boards, is $7327500. The estimate for county expenses is $8000 for salaries to county officials, $6500 for incidentals, $12000 for roads and bridges, and $3000 for aid to the poor. The interest on bonded indebtedness is $4000, and it is decided to create a sinking fund of $5000 each year with which to pay the principal of county bonds at maturity. There is now in the county treasury $7500 more than is necessary to meet current expenses; and it is estimated that the revenue resulting from fines and licenses during the coming year will amount to $4500. What rate of taxation will be necessary for county purposes?

RATIO AND PROPORTION.

RATIO.

574. Ratio is the relation existing between two like numbers with respect to comparative value.

REMARK. There are two methods of comparing numbers with respect to value; first by a system of differences, as, 8 is 3 more than 5; second, by a system of quotients, thus, 4 is $\frac{1}{3}$ of 12. The relation expressed by a difference is sometimes designated *Arithmetical Ratio*, and the relation expressed by a quotient, *Geometrical Ratio*. The ordinary conception of ratio is by a system of quotients.

575. The **Unit** or **Standard of Ratio** is the number to which the other number is compared; thus, if 15 is compared to 5, the number 5 is the unit of comparison, and the quotient obtained by dividing 15 by this unit gives 3 as the ratio, which signifies that 15 represents 3 units, if the number 5, to which it is compared, is regarded as 1 unit.

576. The **Sign of Ratio** is a colon (:), and differs from the sign of division by the omission of the horizontal line.

REMARK 1. The sign of ratio has a double signification; it is a sign of operation and also of relation. As a sign of operation it indicates a division of the number before it by the number placed after it; as a sign of relation it expresses a comparison in value of the number before it to the number after it.

REM. 2. Ratio is sometimes indicated fractionally by writing the number compared as the numerator, and the unit of comparison as the denominator; thus, 18 : 6 and $\frac{18}{6}$ have the same signification.

REM. 3. The two numbers compared, when taken separately, are called the *terms of the ratio;* when taken together they constitute a *couplet*. The term on the left of the sign, in an indicated ratio, is called the *antecedent*, and the term on the right is called the *consequent*.

REM. 4. When the quantity compared is made the antecedent, and the standard of comparison the consequent, the ratio is said to be *direct; when this* order of arrangement is reversed, the ratio is termed *inverse, indirect,* or *reciprocal.*

577. A Simple Ratio is one containing but a single couplet; as 16 :12.

578. A Compound Ratio is the product of two or more simple ratios; as $\left\{\begin{matrix}6:3\\5:2\end{matrix}\right\}$. It is read, 6×5 is to 3×2.

REMARK 1. As a conventionalism, such an expression as 8:4 is sometimes spoken of as a ratio, but, according to the true meaning of the term, it is not a ratio; it is only an *indication of a ratio.* The ratio of 8:4 is 2.

REM. 2. In the application of ratio to business affairs, each of the terms is *concrete.* The terms are also like numbers, and, when necessary, must be reduced to the same denomination. The ratio is always *abstract.*

579. Since the antecedent is a dividend, the consequent a divisor, and the ratio a quotient, it is evident that the principles of division will apply to ratio.

GENERAL PRINCIPLES.—1. *Any change in the antecedent by multiplication or division produces a like change in the ratio.*

2. *Any change in the consequent by multiplication or division produces an opposite change in the ratio.*

3. *Any like change in both antecedent and consequent by multiplication or division produces no change in the ratio.*

FORMULAS.

1. Quantity compared÷unit of comparison=direct ratio.
2. Unit of comparison÷quantity compared=inverse ratio.
3. Antecedent÷consequent=ratio.
4. Antecedent÷ratio=consequent.
5. Consequent×ratio=antecedent.

EXAMPLES FOR PRACTICE.

1. If 36 yards is compared to 9 yards, what is the direct ratio?

2. If 120 pounds is compared to 15 pounds, what is the reciprocal ratio?

3. If 14 acres is compared to 70 acres, what is the inverse ratio?

4. What is the ratio of $\$\frac{2}{3}:\$\frac{3}{4}$?

5. *What is the* ratio of $17\frac{1}{2}$ to $\frac{4}{5}$?

6. If 14.4 tons is compared to 1.2 tons, what is the direct ratio?

7. 252 bushels is compared to 18 bushels; indicate the inverse ratio.

8. If the direct ratio between two numbers is $1\frac{1}{7}$, what is the inverse ratio?

9. What is the ratio of $5\frac{1}{4}$ miles : 23.8 miles?

10. What is the ratio of 216 rods : 4 miles?

11. What is the ratio of 5 lb. 8 oz. 10 pwt. : 5 oz. 16 pwt. 12 gr.?

12. What is the ratio of a tract of land 24 ch. 75 l. long and 18 ch. 20 l. wide to another tract whose length is 60 rd. 8 ft. 6 in. and width 48 rd. 12 ft. 9 in.?

13. What is the difference between the direct ratio of 5 hr. 24 min. : 1 hr. 15 min. and the inverse ratio of £18 : £2 6s.?

14. If the antecedent is $16.50 and the direct ratio is 11, what is the consequent?

15. If the consequent is $4\frac{1}{2}$ bushels and the inverse ratio is 9, what is the antecedent?

16. If the antecedent is 5 rd. 4 yd. 6 in. and the reciprocal ratio is $\frac{1}{4}$, what is the consequent?

PROPORTION.

580. Proportion is an expression of equality between two ratios.

REMARK 1. The sign of proportion in ordinary use is the double colon (::), and when placed between two indicated ratios it implies that the ratios of the two couplets are equal. Thus, 6 : 3 :: 14 : 7 signifies that the ratio of 6 compared to 3 is equal to the ratio of 14 compared to 7. The expression is read, 6 is to 3 as 14 is to 7. The sign of equality is sometimes substituted for the ordinary sign. Proportion may also be expressed fractionally; thus, $\frac{6}{3}=\frac{14}{7}$.

REM. 2. *Simple proportion* is an expression of equality between

vo simple ratios; thus, 6 bu. : 18 bu. :: $5 : $15. It should be read 6 bu. :e to 18 bu. as $5 are to $15.

REM. 3. *Compound Proportion* is an expression of equality beveen ratios, one or both of which are compound; thus, $\left\{\begin{matrix}12:5\\5:4\end{matrix}\right\}::24:8$. ; should be read, 12×5 is to 5×4 as 24 is to 8.

581. The **Terms of a proportion** are the quantities used ı the comparison.

REMARK. Since a simple ratio contains two terms, the fewest terms ıat a proportion can have is four.

582. The **Extremes** of a proportion are the first and ɔurth terms; that is, those placed at the extremes.

583. The **Means** of a proportion are the second and ıird terms; that is, those placed between the extremes.

584. Three quantities are proportional when the first ı to the second as the second is to the third. The quantiıes 4, 6, and 9 are proportional, since 4 : 6 : : 6 : 9.

REMARK. When three quantities are proportional, the second quantity a *mean proportional* between the other two.

585. Theorem. *The product of the extremes of a proporɔn is equal to the product of the means.*

PROOF. The proportion 6:9::8:12 may be written $\frac{6}{9}=\frac{8}{12}$ (580, Rem. 1). Multiplying each of these equal fractions by their common denominator,)8, gives $6\times12=9\times8$. Now the first of these products is that of the exemes, and the second is that of the means; and these products must be ıual, since they are results obtained by multiplying equals by equals. Axiom.)

586. From the foregoing theorem and proof we deduce he following additional principles:

1. *The quotient obtained by dividing the product of the exemes of a proportion by one of the means is equal to the other ıean.*

2. *The quotient obtained by dividing the product of the means f a proportion by one of the extremes is equal to the other extreme.*

587. To find an omitted term of a proportion when the other terms are known.

1. Find the term omitted from the proportion, 45 bu. : 52 bu. : : $36 : ($?).

Operation.

52×36=1872, product of means
1872÷45=41.60, other extreme

EXPLANATION. We first obtain the product of the means, which is also the product of the extremes (Theo.). Dividing this product by the given extreme, 45, gives the other, $41.60, which was omitted from the proportion (Prin. 2).

EXAMPLES FOR PRACTICE.

Find the term omitted from each of the following proportions:

2. 5 rd. : 13 rd. : : 21¢ : (?).

3. 120 lb. : 15 lb. : : (?) : $2.50.

4. 13 T. : (?) : : $78 : $90.

5. 175 yd. : 112 yd. : : (?) : $14.56.

6. $4\frac{1}{2} : 98\frac{1}{3} : : 6\frac{3}{4} : (?)$.

7. (?) : 12 : : 2 : $16\frac{2}{3}$.

8. { 24 yd. : 15 yd. / 18 in. : (?) } : : $14.40 : $12.

9. { 20 men : 16 men / 5 da. : 8 da. } : : $640 : (?).

10. { 150 lb. : 420 lb. / (?) : 25 mi. } : : $.30 : $1.05.

11. { 9 men : 7 men / 5 da. : (?) / 8 hr. : 10 hr. } : : { 16 ft. : $74\frac{2}{3}$ ft. / 4 ft. : 3 ft. / 3 ft. : 2 ft. }

12. The ages of James and Charles are in the same ratio as the respective numbers of marbles which they own. James is 10 years old, and owns 15 marbles. How old is Charles, who owns 18 marbles?

13. A's money is to B's, inversely, as the number of days belonging to February, 1906, is to the number of *days* belonging to June. If B has $420, how much has A?

14. A certain triangle has a base of 24 inches, and an altitude of 30 inches. Another triangle of the same form has a base of 18 inches; what is its altitude?

NOTE 1. Similar surfaces (those of the same form) have their corresponding dimensions proportional.

15. If the circumference of a circle, whose radius is 2 inches, is 12.5664 inches, what is the circumference of a circle whose radius is 5 inches?

16. The area of a triangle, whose longest side is 10 inches, is 24 square inches. What is the area of a similar triangle whose longest side is 15 inches?

NOTE 2. The areas of similar surfaces are to each other as the squares of their corresponding dimensions.

NOTE 3. The volumes of similar solids are to each other as the cubes of their corresponding dimensions.

17. If a globe 2 inches in diameter weighs 12 ounces, avoirdupois, what is the weight of a globe of the same material which is 4 inches in diameter?

18. The altitude of the greater of two similar pyramids is 3 feet, and of the smaller $1\frac{1}{2}$ feet; what is the ratio of their volumes?

588. The difficulty experienced by many students in computations pertaining to proportion results from a failure to understand the existing relations between expressed facts, and, in consequence of such failure, the proportion, as stated by the student, is often faulty. In each of the foregoing examples, the proportion is either stated, or the language is such as to direct the statement, hence we see little occasion for difficulty. In the examples which follow, the student will be thrown upon his own resources, but, by a due exercise of the powers of thought, all obstacles may be overcome.

Most questions in proportion may be considered as a comparison of two related causes and the effects resulting

therefrom. Thus, if $5 as a cause will buy 8 bushels, as an effect, $10 as a cause will buy 16 bushels as an effect. It is evident that the relation between effects is the same as the relation between the causes that produce the effects.

We believe that a careful study of *Cause and Effect*, as here presented, will have a tendency to remove much of the difficulty experienced by the student in stating questions in proportion.

589. Like causes, under the same conditions will produce like effects. Related causes, under the same conditions, will produce effects having the same relations as their respective causes. Since the foregoing statements are axiomatic, argument is unnecessary.

590. A **Simple Cause or Effect** is one containing but a single element, as price, quantity, cost, time, or any single factor used as a term in proportion.

591. A **Compound Cause or Effect** is the product of two or more elements. In the statement, 5 men, in 3 days, can dig a ditch 12 rods long, 3 feet deep, and 2 feet wide, both cause and effect are compound.

592. When cause and effect are considered in stating a proportion, the following formulas should be observed:

1st Cause : 2d Cause :: 1st Effect : 2d Effect
or 1st Effect : 2d Effect :: 1st Cause : 2d Cause

Since each term of a proportion is considered abstract in computation, some prefer the following formula:

1st Cause : 1st Effect :: 2d Cause : 2d Effect

But as ratio is the result of comparing two numbers or things of the *same kind*, the first form is regarded as the more logical.

1. If 80 bushels of wheat will pay a debt of $56, how many bushels at the same price will be required to pay a debt of $44.10?

1st Cause	:	2d Cause	::	1st Effect	:	2d Effect
80 bu.	:	(?)	::	$56	:	$44.10

$$\frac{44.10 \times 80}{56} = 63. \quad Ans.$$

2. If 5 men, in 12 days, can do a certain job of work, how many men will be required to perform 3 times as much work in 30 days?

Operation.

1st C.	2d C.	1st E.	2d E.
5	(?)		
2	6	1	3
12	30		

$$\frac{2 \times 3}{1} = 6. \quad Ans.$$

EXPLANATION. It is evident that the cause which produces the work is 5 men taken in connection with the number of days they labor. Since the character of the work is not stated, we represent its value by 1, since a collective unit is the whole of anything. Arranging the elements of the 1st cause in the 1st position, and the 1st effect, 1, in the 3d position, according to previous instruction, and the elements of the 2d cause (representing the one unknown by (?) in the 2d position, and the 2d effect, which is 3 times the first effect, in the 4th position, we have each term in the position shown in the operation. Then cancelling like factors from means and extremes, and dividing the product of the remaining factors of the extremes by the product of the remaining factors of the means (the factor 1 in this example) gives 6, the number of men required.

EXAMPLES FOR PRACTICE.

3. If 14 barrels of apples cost $49, what will 9 barrels cost, purchased at the same rate?

4. If 175 pounds of sugar cost $10.50, what will 213 pounds cost at the same rate?

5. If 42 acres of land will produce 693 bushels of wheat, what number of acres will be required to produce 957 bushels, at the same rate of yield?

6. A money loaner received $121 interest on a loan of $2420; how much would he have received had the loan been $3500 for the same time and rate?

7. If 125 oranges are worth 150 lemons, how many lemons are 315 oranges worth?

8. If the shrinkage on a barrel of sugar, invoiced at 297 pounds, is 3 lb. 7 oz., what will be the quantity sold from an invoice of 1 T. 268 lb.?

9. If a principal will accrue $\frac{1}{20}$ part of itself as interest, in 300 days, in what time will it accrue interest equal to $\frac{3}{8}$ of itself, if loaned at the same rate?

10. If \$12 will yield 18 cents of interest when loaned for 90 days, what should be received for the use of \$360 for 200 days, loaned at the same rate?

SUGGESTION. \$12 and 90 days are elements of 1st cause, which produce 18 cents as an effect. The elements of the 2d cause are \$360 and 200 days, the effect of which is to be found.

11. If a laborer's services are worth \$22.40 for 14 days, how many days should he work to earn \$36.80?

12. If 8 men can chop 60 cords of wood in 4 days, how many days will be required for 5 men to chop 75 cords?

13. If $1\frac{1}{3}$ yds. of cloth cost $\frac{7}{8}$ of a dollar, what part of a dollar will $\frac{4}{7}$ of a yard cost?

14. A corporation declared a dividend of 6%, whereby A, who owned 36 shares, received \$216. How much would he have received had he owned 42 shares and the dividend had been 8%?

15. The capital of a firm is \$20800, of which A furnished \$8200; if the gains amount to \$2392, how much should A receive?

16. If 5 men can chop 80 cords of wood in 8 days, how many men will be required to chop 168 cords in 12 days?

17. If 5 men can saw 5 cords of wood in 5 hours, how many men will be required to saw 20 cords in 20 hours?

18. If a tower 150 feet high casts a shadow 90 feet long, what must be the height of a steeple that will cast a shadow, at the same time, 72 feet long?

19. If 4 men can mow a field 80 rods long by 60 rods

wide in $3\frac{1}{2}$ days, how many days will be required for 5 men to mow a field 25 chains long by 23 chains 10 links wide?

20. If \$500 will yield \$6 of interest in 72 days, when loaned at 6%, what will \$1450 produce if loaned for 9 mo. 12 da. at 8%?

21. At what rate per cent. must \$960 be loaned for 3 mo. 18 da. to produce \$14.40 interest?

NOTE. Make the statement from the known fact that \$1 will produce 1 cent in 60 days, at 6%.

22. What principal loaned for 17 days at 9% will produce \$5.10 interest?

23. A is given 24 feet the start of B in a race. If A's rate of speed is 495 feet per minute, and B's is 500 feet per minute, how far will A run before he is overtaken by B?

24. At 4 o'clock, the hour hand of a watch is 20 minutes in advance of the minute hand. At what time before 5 o'clock will the hands be together?

25. By selling an organ for \$90, I gain 25%; what will be my rate of gain if I sell it for \$93.60?

593. ADDITIONAL EXAMPLES FOR REVIEW.

1. If a man can saw a cord of basswood in 3 hours, how long a time will be required to saw 7 cords of maple, provided the sawing of 3 cords of maple will require the same time as the sawing of 5 cords of basswood?

2. If $\frac{2}{3}$ as much time is required to saw a cord of basswood as to saw a cord of maple, how many cords of maple can 7 men saw in 3 days, if 3 men can saw $3\frac{1}{2}$ cords of basswood in $\frac{1}{2}$ a day?

3. A merchant's sales during the months of January and February were the same, but during January he sold at 20% profit and gained \$240. If his gains for February amounted to \$300, at what rate of profit did he sell?

4. At what times between 5 and 6 o'clock will the hands of a watch form a right angle?

5. If 8 men, in one day of 8 hours, can dig a field of potatoes, how many days will be required for 5 men, working 10 hours each day, to dig the potatoes from three times as large a field?

6. A man walking by the side of a railroad track at the rate of 4 miles per hour is overtaken by a train moving at the rate of 40 miles per hour. If the train is 15 seconds passing him, what is the length of the train?

7. If 15 pounds of butter will pay for a barrel of apples at present prices, how many pounds of butter will be required to pay for a barrel of apples after butter advances in price 25% and apples decline in price 10%?

8. A farmer gave in exchange 16 bushels of potatoes for a barrel of flour. A month later, after flour had declined in price $12\frac{1}{2}\%$, 12 bushels of potatoes would pay for a barrel of flour. At what rate per cent. had potatoes advanced in price during the month?

9. Two vessels sailed from a given point at the same time, the course of one being north and the other east. At the end of the third hour they were 75 miles apart. If the ratio of speed was as 3 to 4, how far apart were they at the end of the seventh hour, the surface of the earth being regarded as a plane?

10. A ship's crew of 20 men set sail with provisions sufficient for 6 weeks, but on the eve of the 14th day from port, two men were drowned. How long a time from the date of the fatality will the provisions maintain the remainder of the crew?

BANKRUPTCY.

594. A **Bankrupt** is a person who fails in business to such an extent as to be unable to meet his financial obligations.

REMARK. Legal proceedings in bankruptcy, under the United States bankrupt law, may be instituted at the request of the bankrupt himself, by satisfying a court of proper jurisdiction that he is unable to pay his debts; or at the instance of one or more of his creditors on formal application to have him adjudged a bankrupt. Firms and corporations may be adjudged bankrupt the same as single individuals.

595. The **Assets** of a bankrupt are his total resources, including his property and all debts due him from others.

REMARK. Assets are of two kinds, *Available*, those having a marketable cash value, and *Nominal*, including such as probably cannot be converted into cash except at a discount from estimated value.

596. A **Schedule** is an itemized list of the bankrupt's assets and liabilities.

597. An **Assignee** or **Receiver** is a person appointed by the court to take charge of the assets of a bankrupt for the purpose of converting the same into cash, and to pay such a proportion of the liabilities as may be possible, after first deducting all expenses attending the adjustment.

598. A **Dividend** is the proportion of the available assets paid by the receiver to the creditors.

REMARK. Under the bankrupt law, claims for labor, to a certain limit, may be paid in full before a division is made to the other claimants. Those holding claims of this character are termed *preferred creditors*.

599. A **Discharge** is a decree issued by a court of bankruptcy, relieving the bankrupt from further obligations to his creditors.

600. Since the available assets are a certain number of hundredths of the liabilities, it is evident that the principles of percentage will apply to computations in bankruptcy. The principles of proportion will also apply, since the total indebtedness sustains the same relation to any individual claim as the available assets sustain to the amount the individual will receive.

1. A merchant failed in business with liabilities as follows: Notes outstanding, \$2800; due D. C. Brayton on account, \$1640; due Wm. B. Wright & Co., \$2150; due John D. White, \$810; due his clerk for last month's salary, \$50. His available assets were: cash, \$290; merchandise, \$2480; personal accounts, \$927. If the expenses of adjustment were \$243, how much should each creditor receive?

First Operation.

Assets.		Liabilities.	
Cash	\$290	Notes outstanding	\$2800
Merchandise	2480	D. C. Brayton	1640
Personal accounts	927	Wm. B. Wright & Co.	2150
Total assets	\$3697	John D. White	810
Clerk's salary	\$50		
Expenses	243		
Net assets	\$3404	Total liabilities	\$7400

\$3404 ÷ \$7400 = 46%, rate of dividend.

\$1640 × .46 = \$754.40, dividend to Brayton.
2150 × .46 = 989.00, " " Wright & Co.
810 × .46 = 372.60, " " White.

Second Operation.

After first finding net assets and total liabilities as above, we proceed by proportion; thus

\$7400 : \$1640 :: \$3404 : (Brayton's dividend).
\$7400 : \$2150 :: \$3404 : (Wright & Co.'s dividend).
\$7400 : \$810 :: \$3404 : (White's dividend).

RULE I. *From the total assets deduct expenses and claims hat are to be paid in full and divide the remainder by the total abilities. The quotient expressed in hundredths will be the ate per cent. of dividend.*

II. *Multiply each creditor's claim by the rate per cent. of ividend.*

NOTE. The results may also be obtained by proportion, in which the atement in each case should be as follows: Total liabilities : An individual claim :: Total assets : The individual dividend.

EXAMPLES FOR PRACTICE.

2. A mercantile firm failed with liabilities amounting o $8400, and assets amounting to $3520. If the expense f settlement was $118, what amount should a creditor reeive who held a claim of $1854.50?

3. A manufacturing company failed in business with he following liabilities: Notes outstanding, $8720; accrued interest on same, $423.80; personal accounts, 4785.40. The schedule revealed the following assets: eal estate, $3000; cash, $756.75; available personal accounts, $1812; machinery, $2500; wares on hand, $1500. f the cost of settlement was $508.92, how much should axter & Co. receive, who hold the company's note for 1200, with interest accrued to date amounting to $82.30, nd an account of $972.25, which is 4 months past due, nd on which they are entitled to interest at the rate of % per annum?

4. If I hold a claim amounting to $560.40 against a ankrupt, and there remains $4820 of the assets after paying expenses of adjustment, how much should I receive, if he total liabilities are $15062.20?

5. The liabilities of a bankrupt firm amounted to 47480. The available assets were: Cash, $2124.50; a noninterest bearing note of $2400 due 84 days after date of adjustment; personal accounts upon which $3250 was col-

lected; fixtures which were sold for $700; and real estate appraised at $6000. The receiver discounted the note at a bank at 6% per annum and declared a first dividend to creditors before selling the real estate. Three months later the real estate was sold for $6300, and a second dividend declared If the expense attending the first adjustment amounted to $350, and that of the second, $75, how much should a creditor receive from each dividend if his claim was for $1345?

6. The liabilities of a bankrupt were $7835; the assets were $4180, and the expenses attending the settlement were $184. How much was paid on a dollar?

7. The liabilities of a merchant who failed in business were $15230, including taxes amounting to $254.60, which were paid in full. The assets were: Merchandise, $9450; available personal accounts, $1800; and cash, $74.35. If the expenses attending settlement were $408, how much should a creditor have received whose claim was $1320?

STORAGE.

601. **Storage** is the act of placing merchandise or other movable property in a warehouse or storeroom for safe keeping.

REMARK. The term is also frequently employed to designate the compensation charged for storing and caring for goods belonging to another.

602. The **Rate of Storage** is the price charged per unit for each storage term.

603. A **Storage Term** is the time for which the rate is charged. It does not usually exceed one month, and is often less.

REMARK. In some cities it is customary, in the computation of storage bills, to regard a fractional part of a term a full term, while, in other cities, a fraction of a term, less than one-half, is regarded one-half; and a fraction exceeding one-half, is counted a full term. In the following examples, a fractional part of a term is to be regarded as a full term.

604. To find the average storage when no deliveries occur.

1. A warehouseman received the following quantities of wheat for storage: Aug. 12, 300 bu.; Aug. 30, 500 bu.; Sept. 14, 250 bu.; Oct. 3, 200 bu. If the rate was 1 cent a bushel per month, what was the amount of storage due Oct. 13?

Operation.

Aug. 12 to Oct. 13=62 da.×300=1 bu. for 18600 da.
" 30 " " 13=44 " ×500=1 " " 22000 "
Sept. 14 " " 13=29 " ×250=1 " " 7250 "
Oct. 3 " " 13=10 " ×200=1 " " 2000 "
The total storage is equivalent to 1 bu. for 49850 da.

49850÷30=1662— mo., and \$.01×1662=\$16.62, the storage bill.

RULE I. *Multiply the number of days between the date of each receipt and date of settlement by the number of units belonging to such receipt.*

II. *Divide the sum of the products thus obtained by the number of days belonging to the storage term; and multiply the rate of storage by the number of terms.*

EXAMPLES FOR PRACTICE.

2. At 1 cent a bushel per month, what is the storage due Oct. 28, on 180 bu. received Aug. 16; 200 bu. received Aug. 30; 350 bu. received Sept. 10; and 150 bu. received Sept. 15?

3. A warehouseman received for storage, 150 bbl. pork Sept. 22; 280 bbl. pork Oct. 30; and 80 bbl. beef Nov. 8. At 2 cents a barrel per month, what was the storage bill due Dec. 5?

4. At 2¾ cents a barrel per month, what is the storage due March 15, on the following receipts: Dec. 20, 90 bbl.; Jan. 6, 130 bbl.; Jan. 18, 40 bbl.; Jan. 31, 85 bbl.; Feb. 10, 170 bbl.; and Feb. 24, 60 bbl.?

To find the average storage when there are deliveries.

1. A warehouseman's dealings with Geo. Duncan, during the months of January, February, and March, were as shown by the following statement:

Received.		Delivered.	
Jan. 26	300 bbl.	Feb. 5	180 bbl.
Feb. 13	150 "	" 9	50 "
Mar. 10	240 "	Mar. 12	280 "
		" 30	180 "

What was the storage bill March 30, at 3 cents a barrel per month?

Operation.

63 da. (Jan. 26 to Mar. 30) ×300=18900 da.
45 " (Feb. 13 " " ")×150= 6750 "
20 " (Mar. 10 " " ")×240= 4800 "
1 barrel for 30450 da.

53 da. (Feb. 5 to Mar. 30)×180= 9540 da.
49 " (" 9 " " ")× 50= 2450 "
18 " (Mar. 12 " " ")×280= 5040 "
1 barrel for 17030 da.

30450 da.—17030 da.=13420 da.=448 mo.
$.03×448=$13.44, storage bill.

EXPLANATION. We first compute the storage on each receipt for the full time, as in the former case, and obtain 30450 days, which would be correct had no deliveries been made; but since 180 bbl. were delivered 53 days, 50 bbl. 49 days, and 280 bbl. 18 days before the date of settlement, it is evident that this excess of time, which is 17030 days, must be deducted as shown in the operation. Hence storage should be computed on 1 bbl. for 448 mo., which, at a 3-cent rate, amounts to $13.44.

RULE I. *Multiply the number of days between the date of each transaction and date of settlement by the number of units belonging to such transaction.*

II. *Subtract the sum of the products relating to deliveries from the sum of the products relating to receipts.*

III. *Divide the difference between the sums of the products thus obtained by the number of days belonging to the storage term, and multiply the rate of storage by the number of terms.*

2. Harris & Kemp received from Geo. Vroman, for storage, produce as follows: Sept. 24, 120 sks. potatoes and 40 sks. onions; Oct. 5, 75 sks. potatoes; Oct. 20, 35 sks. potatoes and 15 sks. onions. The following deliveries were made upon orders received from Vroman: Oct. 2, 75 sks. potatoes and 40 sks. onions; Oct. 16, 50 sks. potatoes from receipts of Oct. 5, and 30 sks. from receipts of Sept. 24; Nov. 8, all produce remaining from receipts of Sept. 24 and Oct. 5, and 20 sks. potatoes and 15 sks. onions from receipts of Oct. 20; Nov. 17, 15 sks. potatoes. What was

the storage bill Nov. 17, at the rate of 4 cents a sack per month?

605. In a storage warehouse, computations are usually made from a receiving book, which is an auxiliary book containing a record of receipts and deliveries. Computations are generally made upon each delivery at the time of removal, and charged to the customer's account, if not paid. Different methods of storage accounting are in use. The following plan for recording receipts and deliveries, as illustrated by its application to the foregoing example, will be found very convenient:

GEO. VROMAN *in acct. with* HARRIS & KEMP.

Date		Recd.	Deliv.	Goods	Memoranda	Amt.	
Sept.	24	120 sks.		Potatoes			
"	"	40 "		Onions			
Oct.	2		75 sks.	Potatoes	8 da. × 75 = 600 da.		
"	"		40 "	Onions	8 " × 40 = 320 "		
"	16		30 "	Potatoes	22 " × 30 = 660 "		
Nov.	8		15 "	"	45 " × 15 = 675 "		
Oct.	5	75 sks.		Potatoes			
"	16		50 sks.	"	11 da. × 50 = 550 da.		
Nov.	8		25 "	"	34 " × 25 = 850 "		
Oct.	20	35 sks.		Potatoes			
"	"	15 "		Onions			
Nov.	8		20 sks.	Potatoes	19 da. × 20 = 380 da.		
"	"		15 "	Onions	19 " × 15 = 285 "		
"	17		15 "	Potatoes	28 " × 15 = 420 "		
					30)4740 da.		
Total storage is equivalent to storage of 1 sk. for 158 mo.							
					$.04 × 158 =	6	32
					Chd. in acct.		

NOTE 1. We regard the above statement and computation self-explanatory.

NOTE 2. When goods are received from a customer on which there are different rates of storage, make separate computations for each class and *combine the results.*

EXAMPLES FOR PRACTICE.

3. What is the storage bill due on the following account at the date of the last delivery, computed at $2\frac{1}{4}$ cents a bushel per month?

Received.				Delivered.			
Aug.	20,	180 bu.	potatoes.	Aug.	29,	120 bu.	potatoes.
"	31,	65 "	turnips.	"	30,	40 "	"
Sept.	12,	320 "	potatoes.	Sept.	16,	200 "	"
"	19,	75 "	onions.	"	16,	50 "	turnips.
				"	25,	90 "	potatoes.
				Oct.	4,	15 "	turnips.
				"	4,	40 "	onions.
				"	13,	50 "	potatoes.
				"	15,	35 "	onions.

4. Compute the storage on the following account to March 5, 1904, at 2 cents a case per month?

1904.	Received.		1904.	Delivered.	
Jan.	21,	160 cases eggs.	Feb.	10,	200 cases eggs.
Feb.	7,	190 " "	"	17,	70 " "
Mar.	1,	80 " "	Mar.	2,	95 " "
"	3,	120 " "	"	5,	185 " "

5. What amount of storage was due on the following account at the date of the last delivery, the rate on the cheese being 4 cents a hundred pounds per month, and the rate on the apples $2\frac{3}{4}$ cents a barrel per month? (See note 2.)

1906.	Received.		1906.	Delivered.	
Sept.	18,	400 bbl. apples.	Sept.	24,	280 bbl. apples.
"	27,	350 lb. cheese.	Oct.	6,	100 " "
Oct.	9,	200 bbl. apples.	"	6,	225 lb. cheese.
			"	23,	125 " "
			"	30,	220 bbl. apples.

CASH STORAGE.

606. Cash Storage is storage computed on each delivery.

NOTE. In computing storage *on account*, as illustrated by the foregoing cases, the deliveries combine to form an *average time;* hence storage on account is sometimes termed *average storage.* In *cash storage*, as shown by the definition, the deliveries are not combined in computation, but each is computed *separately.*

1. The receipts and deliveries at a warehouse were as follows:

Received.	Delivered.
Oct. 4, 300 bu. corn.	Oct. 24, 350 bu. wheat.
" 18, 500 " wheat.	" 30, 200 " corn.
" 27, 200 " "	Nov. 8, 100 " wheat.
Nov. 12, 150 " barley.	" 15, 220 " "
	" 15, 100 " corn.
	" 22, 30 " wheat.
	" 30, 150 " barley.

What was the storage bill on the above at the rate of 3 cents per bushel for the first 10 days or part thereof, and 2 cents per bushel for each subsequent 10 days or part thereof?

STATEMENT.

Date		Rec'd	Deliv.	Mdse.	Memoranda	Amt.	
Oct.	4	300 bu.		corn			
"	30		200 bu.	"	26 da. (3 terms) 7¢ × 200 =	14	
Nov.	15		100 bu.	"	42 " (5 ") 11¢ × 100 =	11	
Oct.	18	500 bu.		wheat			
"	24		350 bu.	"	6 da. (1 term) 3¢ × 350 =	10	50
Nov.	8		100 bu.	"	21 " (3 ") 7¢ × 100 =	7	
"	15		50 bu.	"	28 " (3 ") 7¢ × 50 =	3	50
Oct.	27	200 bu.		wheat			
Nov.	15		170 bu.	"	19 da. (2 terms) 5¢ × 170 =	8	50
"	22		30 bu.	"	26 " (3 ") 7¢ × 30 =	2	10
Nov.	12	150 bu.		barley			
"	30		150 bu.	"	18 da. (2 terms) 5¢ × 150 =	7	50
					Total storage....	64	10

EXPLANATION. The first delivery of 350 bu. of wheat was made Oct. 24, from the receipts of Oct. 18, the first wheat received, hence the time in store was 6 days, which is counted as one term. The storage of 350 bu. at 3 cents a bushel amounts to $10.50, as shown in the statement. The second delivery of 200 bu. of corn was necessarily made from the corn received Oct. 4. The time in store was from Oct. 4 to Oct. 30, (26 days or 3 terms). The rate is 3¢+2¢+2¢=7¢; and the storage on 200 bu. at that rate amounts to $14. The third delivery was wheat, and was necessarily taken from what was received Oct. 18. As the time in store was 21 days, (Oct. 18 to Nov. 8), the charge was for 3 terms, and amounted to $7. The fourth delivery of 220 bu. of wheat comprised the remaining 50 bu. of what was received Oct. 18, and 170 bu. of what was received Oct. 27. The 50 bu. had been in store 28 days, which counted as 3 storage terms, and the 170 bu. had been in store 19 days, or 2 terms. The fifth delivery of 100 bu. of corn was in store 42 days, or 5 terms, (Oct. 4 to Nov. 15); the rate per bushel being 3+2+2+2+2=11 cents. The 30 bu. of wheat delivered Oct. 22 had been in store 26 days, for which the storage charge was $2.10. The last delivery was received Nov. 12, and delivered Nov. 30, after being in store 18 days, or 2 terms. The total storage is the sum of the storage on all the deliveries, $64.10, as shown by the statement.

2. Compute the storage on the following account at 5 cents per barrel for the first 10 days, or part thereof, and 3 cents per barrel for each subsequent 10 days, or part thereof.

Received.		Delivered.	
May 28	325 bbl.	June 7	280 bbl.
June 15	420 "	" 29	350 "
July 6	200 "	July 10	175 "
		Aug. 2	140 "

3. Find the cash storage on the following account, the term being 15 days, and the rate 8 cents for the first term or part thereof, and 5½ cents for each subsequent term or part.

Received.		Delivered.	
June 17	500 bbl.	June 30	600 bbl.
" 26	475 "	July 15	420 "
July 13	200 "	Aug. 4	250 "
Aug. 3	360 "	" 8	200 "
" 5	240 "	" 12	150 "
" 10	150 "	" 15	80 "
		" 20	225 "

EQUATION OF PAYMENTS.

607. Equation of Payments is the process of finding the *mean* or *equitable* time at which one payment can be made in settlement of two or more debts due at different dates, without loss to either debtor or creditor.

608. The **Term of Credit** is the time given by the creditor for the payment of a debt.

609. The **Average Term of Credit** is the time to elapse before several debts, due at different dates, may be paid in one sum without loss to either debtor or creditor.

610. The **Equated Time** is the date which limits the average term of credit.

611. The **Focal Date** is any assumed date taken as a standard to which other dates are compared.

REMARK. Any date may be taken as a standard of comparison, as the result of the computation will be the same; but, as a matter of convenience, we would suggest that the *first* or *last* of the given dates, or the day preceding the first day of the month in which the earliest item falls due, be taken as the focal date.

612. To find the equated time, or average date, when all the terms of credit begin at the same time.

1. If I owe a creditor \$400 due in 60 days, \$500 due in 30 days, \$200 due in 20 days, and \$300 due in 10 days, what term of credit should be given if I pay the whole indebtedness in one sum?

Operation.

60 days × 400 = 24000 da.
30 " × 500 = 15000 "
20 " × 200 = 4000 "
10 " × 300 = 3000 "
(?) " × 1400 = 46000 "

46000 ÷ 1400 = 33— days.

EXPLANATION. By the terms of credit as expressed in the example, I am to have the use of \$400 60 days, which is equivalent to the use of \$1 for 400 times 60 days, or 24000 days. I am also entitled to the use of \$500 for 30 days, which is equivalent to the use of \$1 for 15000 days. Regarding the third and fourth items, I am entitled to the use of

$1 for 4000 days and for 3000 days, respectively. The credit to which I am entitled, regarding the entire indebtedness, is equivalent to the use of $1 for 46000 days, which is equivalent to the use of what I owe, $1400, or $\frac{46000}{1400}$ days, or 33 days. Hence the credit that should be given, for payment in one sum, is 33 days.

RULE I. *Multiply each term of credit by the item to which it belongs, and divide the sum of the products by the sum of the items; the result is the average term of credit.*

II. *Add the average term of credit to the date at which all credits begin, to find the equated time of payment.*

NOTE 1. A fraction of a day less than $\frac{1}{2}$ should be disregarded; a fraction of a day equal to, or exceeding $\frac{1}{2}$, should be counted as 1 day.

NOTE 2. If an item contains a fractional part of $1, the result will be practically correct if the nearest unit is employed in the computation.

EXAMPLES FOR PRACTICE.

2. A retail dealer bought of a wholesale house a bill of goods amounting to $4500, agreeing to pay for the same on the instalment plan, as follows: $2000 in 10 days, $500 in 30 days, $1000 in 60 days, and the remainder in 90 days. How many days' credit should be given for payment in one sum, in equity to both debtor and creditor?

3. An invoice of goods amounting to $1800 was bought June 12, on the following terms: $500 payable in 20 days, $700 in 30 days, and $600 in 60 days. What is the equated date for payment in one sum?

4. A man purchased a farm March 18, upon the following terms: cash, $5000; $2500 in 6 months; $2000 in 8 months, and the remainder, $1500, in one year. What is the equated date for payment of the entire sum?

5. A commission merchant received a consignment of 500 barrels of apples on Oct. 5. On Oct. 12, he sold 125 barrels at $3.25; on Oct. 29, 200 barrels at $3.00; on Nov. 14, he sold the remainder at $3.10. What is the equated date of total sales?

Find the average term of credit of each of the following bills:

6.

Items.	Terms of credit.
$ 80	5 months
75	3 "
120	6 "
50	2 "

7.

Items.	Terms of credit.
$150	15 days
130	40 "
220	30 "
90	60 "

8.

Items.	Terms of credit.
$ 800	5 days
1200	8 "
300	20 "
500	60 "

Find the equated date for the payment of each of the following:

9.

Apr. 4, $130 on 20 days' credit.
" 280 " 10 " "
" 60 " 40 " "
" 175 " 60 " "

10.

Aug. 15, $720 on 30 days' credit.
" 600 " 15 " "
" 450 " 20 " "
" 900 " 30 " "

613. To find the average date for the payment of two or more debts contracted at different times.

NOTE. This case will apply to ledger accounts having only a debit or a credit side.

1. A retail merchant's purchases from T. B. Arnold & Co., during the months of May and June, were as follows: May 3, $750; May 12, $500; May 27, $800; June 10, $600; June 23, $900. What is the equated date for the payment of the several bills in one sum?

FIRST OPERATION—PRODUCT METHOD.

Focal date, May 0 (= Apr. 30).

Dates.	Days from focal date.		Items.		Products.
May 3....	3	×	750	=	2250
" 12 ...	12	×	500	=	6000
" 27....	27	×	800	=	21600
June 10....	41	×	600	=	24600
" 23....	54	×	900	=	48600
			3550		103050

103050÷3550=29 days from focal date (May 29).

EXPLANATION. We assume that the entire sum should have been paid May 0 (=Apr. 30); but as each item was due on the day of purchase, it is evident that, had the first item, $750, been paid on our assumed date, (May 0), the debtor would have been deprived of its use 3 days, which is equivalent to the use of $1 for 750 times 3 days, or 2250 days. Had each of the other items been paid on the assumed date, the debtor would have been deprived of the use of $500 for 12 days, or of $1 for 6000 days; of $800 for 27 days; or of $1 for 21600 days; of $600 for 41 days, or of $1 for 24600 days; and of $900 for 54 days, or of $1 for 48600 days. Had the whole indebtedness been paid May 0, the debtor would have been deprived of the use of $1 for 103050 days, or of the amount of the debt, $3550, for 103050÷3550=29 days; therefore the equated day is May 0+29, or May 29.

Second Operation. Interest Method.

Focal date, May 0. Rate of interest, 12% per annum.

Dates.	Items.	Interest.					
May 3....	$750....	$.75	int. of	$750	for	3	days.
" 12....	500....	2.00	"	500	"	12	"
" 27....	800....	7.20	"	800	"	27	"
June 10....	600....	8.20	"	600	"	41	"
" 23....	900....	16.20	"	900	"	54	"
	$3550	$34.35					
		3					

Int. of $3550 for 3 da.=3.55) 103.05(29+ days. Hence, the average date is May 0+29 days=May 29.

EXPLANATION. We assume May 0 as the date at which the entire indebtedness should be paid, as in the first operation; then, computing the interest on each item for the time intervening between the focal date and the due date, we find that, if payment were made on May 0, the debtor would be deprived of interest amounting to $34.35; hence, the date for payment, *without loss of* interest to either party, should be as many days *after May 0 as will enable* the whole sum, $3550, to produce $34.35 interest

The interest of $3550 at 12% for 3 days (unit of time) is $3.55. Now it is evident that the interest, $34.35, divided by the interest for one day, ($3.55÷3), will give the number of days; but it is more convenient to divide three times the interest by the interest for 3 days, as shown in the operation. The quotient obtained shows that the debtor should have the use of the amount he owes until May 29.

NOTE. Any rate of interest may be employed in the computation, as the result will be the same by one rate as by another. 12% will generally be found the most convenient.

614. From the foregoing explanations we deduce the following rules:

PRODUCT METHOD. I. *Take as the focal date any convenient date, not later than the earliest item falls due.*

II. *Find the difference in days between the focal date and the date at which each item falls due, and multiply each difference by the item to which it respectively applies.*

III. *Divide the sum of the products by the sum of the items; the quotient will denote the number of days to count forward from the focal date to find the equated date.*

NOTE 1. If the date at which the last item falls due, or any date thereafter, be taken as the focal date, it is evident that the equated date will be found by counting *backward* from the focal date.

NOTE 2. The most convenient focal date is the day preceding the first day of the month on which the earliest item falls due; it is usually recorded as 0 of the said month, and is equivalent to the last day of the preceding month.

INTEREST METHOD. I. *Select the focal date as directed in the preceding rule.*

II. *Compute the interest on each item for the time between the focal date and the date at which such item falls due; and divide three times the sum of the interest items by .001 of the total indebtedness.*

III. *The quotient will denote the number of days to count forward from the focal date to determine the equated date.*

NOTE 3. The statement contained in Note 1, will apply to the interest method the same as to the product method.

EXAMPLES FOR PRACTICE.

Find the equated date for the payment of each of the ollowing bills:

2.

ec. 5, 1903, $400, no cr.
" 22, " 350, "
an. 7, 1904, 520, "
eb. 10, " 180, "

3.

Dec. 18, 1903, $175, no cr.
Jan. 3, 1904, $300, "
Feb. 21, " 150, "
Mar. 10, " 80, "

4.

pr. 16, 1905, $80, no cr.
" 30, " 75, "
lay 13, " 90, "
une 7, " 65, "
" 29, " 50, "

5.

Sept. 16, 1906, $130, no cr.
Oct. 1, " 145, "
" 26, " 30, "
Nov. 4, " 72, "
" 20, " 65, "

Find the equated date for the settlement of each of the ollowing accounts:

6. WM. D. SPENCER.

906											
ug.	9	To mdse.		1200	00						
"	23	"	"	842	50						
ept.	11	"	"	95	80						
ct.	14	"	"	750	00						

7. JORDAN BROS. & SMITH.

						1906					
						Jan.	1	Bal. from old %		124	20
						"	"	Interest to date		4	50
						Feb.	18	By mdse.		340	00
						Mar.	24	"	"	200	00
						Apr.	3	"	"	170	00

8. CARL HEMINGWAY.

1906											
May	15	To mdse.		250	00						
"	30	"	"	180	00						
June	7	"	"	300	00						
"	28	"	"	160	00						
July	12	"	"	420	00						
"	20	"	"	500	00						

9. A dealer purchased of a jobbing firm the following invoices: June 12, 1905, 10 bbl. sugar, 2980 lb., at 5 cents per pound; July 17, 650 lb. coffee at 18 cents per pound; Aug. 4, 12 bbl. flour at $5.20 per barrel; and 480 lb. corn meal at $2.75 per hundredweight. What is the total amount of indebtedness, and the equated date of payment?

10. A manufacturing firm sold to a customer wares which were charged in account as shown by the following entries: Nov. 18, 1906, $450; Dec. 16, 1906, $360; Jan. 14, 1907, $500. Jan. 22, 1907, the firm received in settlement the entire sum shown by the account, with interest at the rate of 6% per annum from the equated date. What sum balanced the account?

615. To find the average date for the payment of two or more debts contracted at different dates when credit is given.

1. The following sales were made by a mercantile house to C. E. Motley: May 19, 1906, merchandise amounting to $380 on 30 days' credit. June 10, 1906, merchandise amounting to $250 on 10 days' credit. June 29, 1906, merchandise amounting to $400 on 60 days' credit. July 17, 1906, merchandise amounting to $320, no credit.

Operation. Product Method.

Focal date, May 0, 1906.

Days.		Items.		Products.
49	×	380	=	18620.
51	×	250	=	12750.
120	×	400	=	48000.
78	×	320	=	24960.
		1350		)104330(77+

May 0 + 77 days = July 14.

EXPLANATION. The first item was due 30 days from the date of sale, or 49 days later than the focal date; the other items were due 51 days, 120 days, and 78 days, respectively, later than the focal date. Multiplying each of these periods of time by the item to which it applies, as explained in the former case, we have the several products shown in the operation. Dividing the sum of the products by the sum of the items, as previously explained, gives 77 days, the time to count forward from the focal date to find the equated date.

REMARK. When the term of credit is the same respecting each item, *apply 614*, and add the uniform term of credit to the result.

EXAMPLES FOR PRACTICE.

Find the equated date for the payment of each of the following accounts:

2. S. J. LARSON.

Aug.	15	To merchandise per inv. at 30 days	600	00		
"	30	" " " " " 60 "	720	00		
Sept.	18	" " " " " 15 "	400	00		
Oct.	2	" " " " " 30 "	250	00	1970	00

3. WM. GARDNER & SON.

Feb.	13	To merchandise per inv. at 60 days	124	50		
"	28	" " " " " " "	87	40		
Mar.	8	" " " " " " "	215	75		
"	14	" " " " " " "	350	00		
Apr.	1	" " " " " " "	200	00	977	65

4. HANSON & MEAD.

Oct.	7	To merchandise per inv. at 2 months	1500	00		
"	24	" " " " " 2 "	850	00		
"	30	" " " " " 4 "	730	00		
Nov.	18	" " " " " 3 "	1100	00		
Dec.	24	" " " " " 3 "	1320	00		
"	31	" " " " " 1 "	460	00	5960	00

5. JEROME COOPER.

Apr.	4	To merchandise per inv. at 30 days	75	00		
"	22	" " " " " 10 "	120	40		
May	8	" " " " " 10 "	95	20		
"	17	" " " " " 15 "	100	00		
"	31	" " " " " 30 "	64	30		
June	14	" " " " " no credit	85	00		
"	30	" " " " " " "	90	60		
July	6	" " " " " " "	20	75		
"	20	" " " " " " "	210	00	861	25

6. Geo. Peabody purchased the following invoices of a wholesale firm: March 10, 1906, 160 bbl. flour at $5.50 per bbl., on 60 days' credit; April 12, 1906, 480 lb. dairy butter at 21½¢ per pound, on 30 days' credit; April 30, 1906,

12 bbl. sugar, averaging 260 lb. per bbl., at $4\frac{7}{8}¢$ per pound, on 2 months' credit; May 19, 24 cases eggs, 720 doz., at 15¢ per doz. At what date should the entire account be paid by average?

AVERAGING ACCOUNTS.

616. The **Average of an Account** is the average or equated time for the payment of the balance of an account having both a debit and a credit side.

REMARK. Averaging accounts is only an additional case belonging to Equation of Payments. In the former cases presented, all the items were either debits or credits. In averaging accounts, we are required to deal with both debit and credit items in each example, and to find an equitable date for the payment of the balance.

617. To find the equated date for payment of the balance of an account containing both debit and credit items.

1. Find the equated date for payment of the balance of the following account:

JOHN G. RADCLIFF.

1906					1906				
Aug.	4	To mdse. (30 da.)	300	00	Sept.	5	By cash.....	200	00
"	22	" " (60 da.)	500	00	"	20	" "	400	00
Sept.	8	" "	100	00	Oct.	10	" "	150	00
"	25	" " (20 ")	250	00					

OPERATION—PRODUCT METHOD.

Focal date, Aug. 0.

Days.		Items.		Products.	Days.		Items.		Products.
34	×	300	=	10200	36	×	200	=	7200
82	×	500	=	41000	51	×	400	=	20400
39	×	100	=	3900	71	×	150	=	10650
76	×	250	=	19000			750		38250
		1150		74100					
		750		38250					
		400		)35850(90—					

Aug. 0+90 da.=Sept. 29.

EXPLANATION. Since the first item, $300, was sold on 30 days' credit, *it falls due* 4+30=34 days later than the focal date; the second item falls *due 60 days after* the date of sale, or 82 days later than the focal date; the *third item falls* due (since no credit was given) on the date of sale, which is

39 days later than the focal date; the fourth item was due 20 days after date of sale, or 76 days later than the focal date. Considering the debit side an independent account, as if there were no credits, we find that, if the whole sum of the debits was paid Aug. 0, Radcliff would be deprived of the use of $1 for 74100 days. Considering the credit side as a separate account against the merchant, and assuming that Radcliff should receive the whole sum, $750, on Aug. 0, the merchant would be deprived of the use of $1 for 38250 days. By combining these relations we find that, if Radcliff had paid the balance Aug. 0, he would have been deprived of the use of $1 for 74100—38250=35850 days, or of the use of the balance of his indebtedness, $400, for $\frac{35850}{400}$=90— days. Therefore the balance should be paid 90 days later than the focal date, or Sept. 29.

NOTE. The preceding rule, modified according to the foregoing explanation, will be found applicable to this case.

EXAMPLES FOR PRACTICE.

Find the equated date for payment of the balance of each of the following accounts:

2. H. B. FOSTER.

1907					1907				
Jan.	25	To mdse	840	00	Feb.	2	By cash	500	00

3. SIMON B. GATES.

1907					1907				
Apr.	10	To mdse.....	700	00	Apr.	20	By cash.....	600	00
"	24	" "	400	00	June	29	" "	350	00
May	16	" "	300	00					

REMARK 1. If the balance of the account and the product balance are on the same side of the account, the quotient obtained by dividing the difference of products by the difference of items should be added to the focal date to find the equated date; if the item balance is in favor of one side, and the product balance is in favor of the other side, the quotient should be subtracted from the focal date. The reason for this instruction is obvious.

REM. 2. If the date at which the last item falls due, or any date thereafter, be taken as the focal date, the operations directed in Rem. 1 should be reversed.

4. WHITNEY & BATES.

1905					1905				
Feb.	3	*To mdse. (10 da.)*	465	00	Mar.	14	By cash.....	600	00
Mar.	*9*	" " *(30 ")*	850	00					

5. FRANK S. NEWMAN.

1906					1906				
July	17	To mdse.....	354	00	Aug.	1	By cash.....	250	00
Aug.	6	" " (30 da.)	120	00	Sept.	3	" "	200	00

6. THEODORE PARKER.

1906					1907				
Oct.	12	To mdse.(4 mo.)	3450	00	Jan.	5	By cash	1200	00
					"	24	" "	1500	00
					Feb.	15	" "	350	00

7. LATTIMER BROS.

1904					1905				
Dec.	16	To mdse.(60 da.)	2450	00	Feb.	12	By cash	2000	00
"	31	" " (30 ")	372	50					

8. FREEMAN, BLAKE & CO.

1905					1905				
June	22	To mdse.(30 da.)	1720	00	July	3	By cash	500	00
July	11	" " (30 ")	1475	00	Aug.	10	" Accept. 30 da.	1200	00
"	31	" "	800	00					

9. ADAMS & MUNSON.

1907					1907				
Mar.	5	To mdse.....	574	80	Mar.	15	By cash	400	00
"	29	" "	635	40	"	21	" "	100	00
Apr.	4	" "	217	25	Apr.	10	" "	550	00
"	26	" " (60 da.).	720	00	May	1	" "	80	00
					"	"	" accept. 10 da.	1017	45

10. KINSLEY & WISE.

1904					1904				
Jan.	13	To mdse. (3 mo.)	175	00	Mar.	1	By cash.....	150	00
Feb.	7	" " (3 ")	212	45	Apr.	7	" "	200	00
Mar.	8	" " (3 ")	375	25	May	9	" "	175	00

11. D. G. WAGNER.

1903					1903				
Apr.	18	To mdse. (60 da.)	540	00	May	7	By cash.....	800	00
May	3	" "	378	20	June	2	" "	300	00
"	29	" "	416	30					

12. KENDALL & RICHARDS.

1906					1906				
Aug.	18	To mdse.....	825	15	Aug.	25	By cash.....	700	00
"	30	" "	364	60	Sept.	13	" "	400	00
Oct.	11	" "	512	25	Nov.	3	" "	500	00

13. JOHN V. HANISCH.

1906						1906				
Nov.	4	To mdse. (30 da.)	428	40		Dec.	16	By cash	1200	00
"	28	" " (30 da.)	723	35		"	30	" "	150	00
Dec.	9	" " (30 da.)	268	40						
"	31	" " (30 da.)	514	10						

14. OSMOND & KENT.

1907						1907				
Feb.	23	To mdse. (90 da.)	1200	00		May	6	By cash	1000	00
Mar.	10	" " (90 ")	850	00		"	19	" "	300	00
"	30	" " (90 ")	975	50		"	24	" "	200	00
Apr.	12	" " (90 ")	643	75						

15. BRIGGS & KNOWLTON.

1905						1905				
June	17	To mdse.	520	00		June	24	By cash	500	00
"	30	" "	630	00		July	26	" accept. (30 da.)	1065	00
July	21	" "	415	00		Sept.	15	" cash	600	00
Aug.	8	" " (30 da.)	728	40		"	20	" note with int.	128	40
"	23	" " (30 ")	156	20		Oct.	1	" 30-day note without int.	200	00
"	31	" " (30 ")	395	90		"	10	" cash	52	10

16. Walter P. Chase bought of Mead & Hartland, the following invoices, on 60 days' credit, and subject to a discount of 2%, in each case, if paid in 10 days: Sept. 4, 1906, $1600; Sept. 24, 1906, $950; Oct. 18, 1906, $1150; Nov. 3, 1906, $520; Nov. 20, 1906, $1340. Sept. 12, 1906, he paid $1176, and was allowed the discount as per terms. Dec. 1, 1906, he paid cash $1800, and on Dec. 15, 1906, he paid cash $1000 and gave his note for the balance. What was the face of the note, and from what date should it draw interest?

AVERAGING ACCOUNT SALES.

618. When a commission agent receives goods to be sold on commission, he pays transportation charges, unless prepaid by the consignor, and all other necessary items of expense, and charges the same to the consignment. It is also a custom, under certain conditions, to make cash ad-

vances, or to accept time drafts, which gives the consignee the right to retain the net proceeds a sufficient length of time to offset interest on such advances.

REMARK. The averaging of an account sales does not differ from the method employed in averaging any other account, except as to finding equated date for commission.

1. Find the equated date for paying the net proceeds of the following account sales of 450 bbl. flour, received from Geo. P. Wellman, Chicago, to be sold on his account and risk:

1906					
Oct. 9	Sold 120 bbl. for cash at $5.10	$ 612	00		
" 25	" 200 " " " " 5.15	1030	00		
Nov. 14	" 130 " " " " 5.20	676	00	$2318	00
	Charges.				
Oct. 2	To freight and cartage	196	80		
" 3	" advertising	12	00		
" 3	" insurance	5	00		
" 26	" cash advanced consignor	500	00		
	" commission @ 3%	69	54	783	34
	Net proceeds	.....	...	$1534	66

OPERATION—PRODUCT METHOD.

Focal date, Oct. 0.

Days.	Items.	Products.	Days.	Items.	Products.
2	197	394	9	612	5508
3	17	51	25	1030	25750
26	500	13000	45	676	30420
27	70	1890		2318	)61678(27—
	784	15335		784	15335
Net proceeds due per equation, Oct. 30.				1534	)46343(30+

EXPLANATION. We first find the equated date for commission by dividing the sum of the products on the credit side by the sum of the items (615), and find that the equated date is 27 days later than the focal date. The items employed are equal to the true items (the commission), multiplied by $33\frac{1}{3}$; the products are also too large in the same proportion, hence the quotient is the same as would be obtained by dividing the commission products by corresponding items of commission. The date for commission being found, we find equated date as explained in 617.

NOTE. Former rules are deemed sufficient for this case.

EXAMPLES FOR PRACTICE.

2. A commission house sold for W. A. Hamlin the following: 1904, May 5, 520 bu. wheat at 82¢; May 17, 850 bu. corn at 43¢; June 12, 1300 bu. wheat at 83¢; June 29, 400 bu. corn at 44¢ and 720 bu. wheat at 83½¢. The charges were: Apr. 26, freight $215; May 10, cash advanced, $500; June 12, storage, $13; commission 2%. Find equated date for remitting net proceeds.

3. Wilber & McCormick sold on commission for Amos Henderson the following: 1905, Sept. 25, 800 lb. dairy butter at 22¢ and 1800 lb. cheese at 12½¢. Oct. 8, 750 lb. dairy butter at 21¢; Oct. 19, 560 lb. dairy butter at 22½¢; Nov. 7, 220 cases eggs, 30 doz. each, at 18¾¢; Nov. 20, 300 lb. dairy butter at 25¢. The charges were: Sept. 17, freight and drayage, $65, and advertising, $9. Henderson drew on the firm at sight on Oct. 20, for $400, and on Nov. 1, at 30 days, for $200. The after charges were: Storage, $8, and commission 4%. On what date should net proceeds be remitted, per equation?

4. Make all computations necessary to obtain the results omitted from the following account sales:

Account Sales of Flour. MINNEAPOLIS, MINN., June 5, 1906.

Sold for account of GEO. B. DIXON *by* JOHN F. MOTT & SON.

1906						
Apr.	13	120 bbl. to Wm. Jones, on % 30 da....$5.50	.....	...		
"	21	180 " " James Ward, " % 30 " ... 5.60	.....	...		
May	2	250 " " C. D. Briggs, cash..... 5.40	.....	...		
"	15	160 " " John Day, " 5.40	.....	...		
"	18	190 " " Wm. F. Snow on % 30 da.. 5.45	....	...	.....	...
		Payment guaranteed.				
		Charges.				
Apr.	6	Freight and drayage.....	145			
"	7	Insurance.....	9			
"	30	Cash advanced, per draft	500			
May	19	Cooperage.....	15			
		Guaranty, 2% (401, Rem. 3).....	.....	...		
		Commission, 3%.....	.. .	...	.....	...
		Net proceeds, per average.....	.....	...	.....	...

CASH BALANCES.

619. A **Cash Balance** is a sum due on an account at a specified date.

REMARK 1. It is customary among business houses to render a statement to their debit customers at the end of certain periods, which may be monthly, quarterly, semi-annually, or annually. Such a statement is sometimes called an *Account Current*.

REM. 2. Some firms charge interest on periodical balances from the time such balances are equitably due; others, who render accounts current, oftener than once a year, do not charge interest until the end of the year.

REM. 3. The cash balance due on an account is the difference between the debit and credit sides, less the bank discount if settled before the equated date, or plus the interest if settled after the equated date; hence the subject of cash balance is but an extension of equation of payments.

1. Find the cash balance of the following account for Oct. 25, at 6% interest.

AARON A. BIXBY in acct. with GEO. BRADLEY.

1904					1904				
Aug.	8	To mdse.....	500	00	Aug.	18	By cash	300	00
"	20	" "	300	00	"	25	" "	200	00
Sept.	14	" "	600	00	Sept.	10	" "	250	00
					"	28	" "	100	00

EXPLANATION. First find the equated date, by either the product or the interest method, which is Aug. 19. Since the balance, $550, is equitably due on Aug. 19, and is paid Oct. 25, 67 days later, the debtor must pay interest on $550 for 67 days, which is $6.14; hence the sum due Oct. 25 is $550+$6.14=$556.14.

REMARK 1. If the date of settlement be taken as the focal date, the quotient of product differences divided by the difference of items, will give the number of days for which interest or discount is to be computed, as it represents the interval between the equated date and the date of settlement.

NOTE. Previous rules and explanations will be found sufficient for this case.

2. Find the cash balance of the following account for June 21, at 6%.

J. P. NILAN.

1905					1905				
May	10	To mdse	100	00	May	20	By cash.....	250	00
"	15	" " (60 da.)..	200	00					

EXPLANATION. Selecting May 0 for the focal date, and computing according to the explanation given for the former example, we obtain, for the equated date, Dec. 6, which is 168 days later than June 21, the date of settlement; therefore the debtor is entitled to the interest of the balance, $50, for 168 days, which deducted, leaves $48.60, the cash balance.

REM. 2. Selecting June 21 (the date of settlement), for our focal date, a plan recommended by some, will introduce conditions which, without the exercise of care and vigilance, may result in error, as the following operation will show:

OPERATION—WITH JUNE 21 AS FOCAL DATE.

Days.	Items.	Products.	Days.	Items.	Products.
42	× 100 =	4200	32	× 250 =	8000
−23	× 200 =	−4600			
	300	−400			
	250	8000			
	50	−8400			

−8400 ÷ 50 = −168.

EXPLANATION. The first item on the debit side is due May 10, or 42 days *earlier* than the focal date; the next item is due July 14, 60 days after date of sale, or 23 days *later* than the focal date. Now it is evident that the 42 days relating to the first item, and the 23 days relating to the second item, are opposite in their relations to the focal date, since the first reckons backward and the other forward. To distinguish between these relations, we regard time reckoned forward from the focal date as *negative*, and mark numbers used in recording it with the minus sign. Multiplying the negative number, 23, by 200 gives a negative product; and 4200 (positive)+4600 (negative)=400 (negative). This statement may look absurd to the student who has never studied the relations of positive and negative numbers; nevertheless, it is true. The combination (addition), of $4 of capital with $5 of insolvency gives $1 of insolvency as the result. Subtracting the credit item, $250, from the sum of the debit items, $300, gives $50; and subtracting the credit product, $8000, from the sum of the debit products, −400, gives −8400, on the same principle that places 16° north and 12° south, respectively, of the equator have a difference of 28° in latitude. Dividing the difference of products by the difference of items gives 168 days, the interest for which, $1.40, is to be discounted, as indicated by the minus sign.

REM. 3. To avoid complications, we recommend that the equated date

be first obtained, and that interest be added for the number of days that the date of settlement is later than the equated date, and subtracted for the number of days that settlement is made earlier than equated date.

EXAMPLES FOR PRACTICE.

Find the cash balance for each of the following accounts, money in each case being worth 6%:

B. F. ROGERS in acct. with BOSTON DRY GOODS CO.

1905					1905				
Jan.	7	To mdse	1350	00	Jan.	15	By cash	1200	00
"	26	" " (2 mo.)..	1500	00	Feb.	20	" 60-da. draft	1600	00
Feb.	12	" " (2 ")..	840	00	Mar.	21	" cash	500	00
Mar.	6	" " (30 da.)..	720	00	Apr.	10	" "	600	00

(Balance paid and account closed Apr. 18, 1905).

C. L. DANFORTH in acct. with FOOTE & EATON.

1906					1906				
Mar.	13	To mdse. (15 da.)	654	30	Apr.	24	By cash	600	00
Apr.	24	" " (3 mo.)	1874	20	July	28	" "	1500	00
					Aug.	11	" "	100	00
					"	29	" "	100	00

(Balance paid Nov. 5, 1906.)

ROBT. L. BERGSTROM in acct. with J. J. HALL.

1904					1904				
Aug.	27	To mdse.....	520	00	Sept.	8	By cash.....	500	00
Sept.	10	" "	375	00	"	21	" "	300	00
"	16	" "	482	75	Oct.	4	" "	200	00
Oct.	13	" " (3 mo.)	752	00	"	15	" "	150	00
					Nov.	3	" "	200	00

(Balance paid Dec. 4, 1904.)

EZRA D. PARKER in acct. with T. S. YOUNG.

1907					1907				
Apr.	13	To mdse. (30 da.)	120	00	May	12	By cash.....	100	00
"	18	" " (30 ")	85	00	"	21	" "	100	00
"	26	" " (60 ")	90	00	June	1	" "	75	00
May	9	" " (60 ")	150	00	"	7	" "	80	00
"	17	" " (30 ")	210	00	"	20	" "	50	00
"	30	" " (30 ")	175	00	Aug.	3	" accept. (30 da.)	100	00
June	12	" " (30 ")	60	00	Sept.	10	" " (60 da.)	275	00
July	6	" " (60 ")	55	00	Oct.	5	" cash	50	00
"	28	" " (30 ")	140	00	"	8	" "	75	00

(Balance paid Oct. 12, 1907.)

PARTNERSHIP.

620. Partnership is a contract relation between two or more persons who combine their capital, skill, or labor for the purpose of engaging in business; each person sharing in the gains or losses as stipulated in their articles of agreement.

REMARK. A partnership is frequently called a *firm* or a *company*, and each individual of the association is called a *partner* or *member*.

621. The **Capital** of a firm or partnership is the money or property invested in the business.

REMARK 1. The *Resources* or *Assets* of a firm are its entire property, together with all obligations due the firm.

REM. 2. The *Liabilities* of a firm are the debts or obligations due creditors.

REM. 3. The *Net Capital* of a firm is the excess of its resources above its liabilities.

622. The **Insolvency** of a firm is the excess of its liabilities above resources.

REMARK. A firm is insolvent when it lacks the financial ability to pay its debts.

623. The **Investment** of a partner is the aggregate sum contributed by him to the firm, as capital.

REMARK. The sum of a partner's investments in excess of his withdrawals is called his *Net Investment;* and the average sum in use during a given time, when investments and withdrawals have occurred at different times, is called his *Average Investment.*

624. The **Net Gain** is the excess of total gains over total losses, during a given period of time.

625. The **Net Loss** is the excess of total losses over total gains, during a given period of time.

REMARK. Partnership settlements usually occur as often as once a

year, and are made according to the provisions of the contract or articles of agreement. Sometimes the contract provides for a salary to one or more of the partners to be credited at the end of the year, either in the investment account or in a private account, in each case, as the contract may direct. Gains and losses are sometimes divided among the partners in proportion to their investments, and sometimes equally, regardless of the respective investments of the different members of the firm, in which case it is generally provided by contract that the parties' interests shall be equitably adjusted through an interest account at some fixed rate.

626. To divide the gain or loss among partners in proportion to investments, when contributed for equal periods of time.

1. A, B, and C formed a partnership with a capital of $15000, of which A invested $4000, B $5000, and C the remainder. Their net gain, at the end of the year, amounted to $2460. How much should each receive?

REMARK. The above, and the following examples, may be solved by applying principles belonging to *Fractions*, to *Percentage*, or to *Proportion*.

ANALYSIS.

Fractional Method. Since A contributed $\frac{4}{15}$ of the capital, he is entited to $\frac{4}{15}$ of the gain, which is $656. B, for a similar reason, should receive $\frac{5}{15}$, ($\frac{1}{3}$), which is $820, and C should receive $\frac{6}{15}$, ($\frac{2}{5}$), which is $984.

Percentage Method. Since the capital, $15000, produced a net gain of $2460, the rate per cent. of gain is $2460 ÷ $15000 = 16$\frac{2}{3}$%. Multiplying each partner's investment by the rate of gain, gives $656 for A, $820 for B, and $984 for C.

By Proportion. Since capital is a cause and gain the effect, it is evident that the following general formula will apply:

Total Capital	:	Each Partner's Capital	::	Total Gain	:	Each Partner's Gain.
$15000	:	$4000	::	$2460	:	($656) A's gain.
15000	:	5000	::	2460	:	($820) B's "
15000	:	6000	::	2460	:	($984) C's "

RULE. *Divide the total gain or loss by the firm capital to find the rate of gain or loss, and multiply each partner's net capital by it.* Or

State the case by proportion, making the total gain or loss and each partner's gain or loss *the right-hand couplet, and*

the total net investment and each partner's net investment as the left-hand couplet, and find the wanting term as instructed in proportion.

EXAMPLES FOR PRACTICE.

2. Four partners invested respectively $3500, $3900, $5600, and $6500. The net gain at the end of the first year was $2632.50. What share of the profits should each receive?

3. O. D. Walstrom and J. L. Grant formed a partnership and invested respectively $8250 and $6875. By what amount was each partner's investment reduced if the ledger, at the end of the year, showed a net loss of $605?

4. Henry Yeomans and Geo. Durand, of the firm of Henry Yeomans & Co., took in S. J. Wells as a partner on Jan. 1, 1906. Yeomans invested a one-half interest of the former partnership, valued at $6720, and $800 in notes; Durand invested the other one-half interest worth $6720, and a store building and lot valued at $4800. Wells invested notes to the amount of $2400, on which there was accrued interest amounting to $180, and $3000 in cash. The net gain at the end of the year was $2769.75. How much of the same should each receive?

5. G. W. McFarland and C. F. Nesbit formed a partnership Jan. 1, 1905. McFarland invested the following resources; Cash, $5000; a note of $2000 due in 8 months, without interest. Nesbit invested merchandise, $6500, and accounts invoiced at $3120. The personal liabilities of each partner were assumed by the firm. Those of McFarland consisted of one note of $800, due in 6 months, without interest, and a note of $400, given Apr. 1, 1904, with interest at 6%, and due one year from date. Nesbit's liabilities were accounts payable, aggregating $650. It was mutually agreed that all notes should be reckoned worth *their bankable cash* value on the basis of 6% interest.

Their net gain, at the close of the year, amounted to $1910.53, which was divided in proportion to net investments, and credited to their respective accounts. What was the net capital of each partner, Jan. 1, 1906?

NOTE. An understanding of the principles explained in their application to example 1, will enable the student to find any wanting element.

6. If the net capital of a firm was $18400, and the net gain during a given year amounted to $2300, what rate per cent. did each partner realize on his investment?

7. A and B engaged in business with a joint capital of $11500. If A's net gain was $750 and B's $975, what was the net investment of each?

8. A's net investment was $12300 and B's $8000. A's net gain was $984 and C's $720. Find B's gain and C's investment.

9. Two contractors and builders, Carlson & Brown, contracted to build a store for $16500. Carlson furnished the material at a cost of $7875; Brown paid all bills for labor, which amounted to $5500.60. If the net gain was divided in proportion to investments, how much of the $16500 should each have received?

627. To divide the gain or loss in proportion to average investments when investments and withdrawals occur at different times.

1. W. C. Wright and J. F. Boyd formed a partnership Jan. 1, 1906, each investing $8000. Wright's subsequent investments and withdrawals were as follows: May 1, withdrew $2000; Aug. 1, invested $1000; Nov. 1, withdrew $800. Boyd withdrew $3000 on June 1, and $2000 on Oct. 1. If the profits, $1143, were divided in proportion to average investments, what part of the same should *each* receive?

OPERATION.

From Jan. 1 to May 1 = 4 mo.....$8000 × 4 = $32000 for 1 mo.
Withdrew 2000
" May 1 to Aug. 1 = 3 mo.....$6000 × 3 = 18000 for 1 mo.
Invested 1000
" Aug. 1 to Nov. 1 = 3 mo.....$7000 × 3 = 21000 for 1 mo.
Withdrew 800
" Nov. 1 to Jan. 1 = 2 mo.....$6200 × 2 = 12400 for 1 mo.
Wright's investment = $83400 for 1 mo.
$83400 ÷ 12 = $6950, his average for the year.

From Jan. 1 to June 1 = 5 mo.....$8000 × 5 = $40000 for 1 mo.
Withdrew 3000
" June 1 to Oct. 1 = 4 mo.....$5000 × 4 = 20000 for 1 mo.
Withdrew 2000
" Oct. 1 to Jan. 1 = 3 mo.....$3000 × 3 = 9000 for 1 mo.
Boyd's investment = $69000 for 1 mo.

$69000 ÷ 12 = $5750, his average for the year.
$6950 + $5750 = $12700, average capital of the firm.
$1143 ÷ $12700 = 9%, rate of gain on investment.
$6950 × .09 = $625.50, Wright's gain.
$5750 × .09 = $517.50, Boyd's gain.

EXPLANATION. Wright invested $8000 in the business Jan. 1, which represented his capital until May 1, or 4 months, the use of which to the firm was equivalent to the use of 4 times $8000, or $32000, for 1 month. On May 1 he reduced his capital to $6000, which remained unchanged until Aug. 1, a period of 3 months, the use of which being equivalent to the use of 3 times $6000 for 1 month. On Aug. 1, by an additional investment of $1000, he changed his capital to $7000, which remained unchanged until Nov. 1, or 3 months, and was equivalent, in use, to 3 times $7000 for 1 month. On Nov. 1 he withdrew $800, thereby reducing his capital to $6200, which remained unchanged until the end of the year, and was equivalent, in use, to 2 times $6200 for 1 month. Hence, Wright's total investment was the sum of $32000, $18000, $21000 and $12400, or $83400, for 1 month, or an average of $\frac{1}{12}$ of $83400, or $6950, for the year. Proceeding in a similar manner concerning Boyd's investments and withdrawals, we find that his average capital for the year was $5750. Having found each partner's average investment, the division of profits may be made according to either of the methods illustrated in 626.

RULE I. *Multiply each partner's original investment by the time intervening between the date of such investment and the date of the next investment or withdrawal.*

II. *Multiply the increased or decreased investment by the time intervening between such increase or decrease, and the date of the next investment or withdrawal; and so continue to the date of adjusting the gains or losses.*

III. *Divide the sum of the products thus obtained by the time belonging to the period of adjustment to find each partner's average investment.*

IV. *Divide the gains or losses as directed in Rule 626.*

EXAMPLES FOR PRACTICE.

2. A, B and C formed a copartnership with the following investments: A, $7200; B, $10000, and C, $2400. At the end of 3 months A invested $1800 additional, and B $2000, and one month later C withdrew $300. At the end of 6 months from the time of formation the partnership was dissolved, and the net gains, $3531, were divided in proportion to average investment. How much of the same did each receive?

3. Emmerson, Clark and Mott formed a partnership Jan. 1, 1905. Emmerson's account at the end of the year, before closing the ledger, showed a credit on Jan. 1 of $6000, a debit on June 1 of $1000, and a credit on Oct. 1 of $2000. Clark's account exhibited two credits, one on Jan. 1 of $4000 and one on Sept. 1 of $2000. Mott invested $1000 on Jan. 1, and withdrew $800 on March 1 and $200 more on Aug. 1. The net gains, which amounted to $1408.33, were divided in proportion to average investments, and carried to the partners' respective accounts. What was the net credit of each Jan. 1, 1906?

4. Wheeler, Newman, and Lang engaged in the grain business on Jan. 1, 1904, Wheeler investing $12000, Newman $9600, and Lang $7000. On June 1, Wheeler invested $1000 additional; on Aug. 1, he withdrew $3000; and on Sept. 1, he invested $2000. On Apr. 1, Newman and Lang

each withdrew $1800; and on July 1, each invested $1000. On Sept. 1, Newman invested $5000, and Lang withdrew $2500. On Nov. 1, Lang withdrew $2000. At the end of the year, the net gain was found to be $6480, which was divided in proportion to average investments. What amount did each receive?

5. C and D engaged in business as partners, and agreed to share the gains and losses in proportion to their average investments. C furnished $\frac{4}{7}$ and D $\frac{3}{7}$ of the capital. At the end of 6 months each withdrew $\frac{1}{4}$ of his original investment. If the net gain during the year was $7200, what was each partner's share?

6. A, B, and C engaged in the mercantile business as partners, on Jan. 1, 1905, agreeing to divide gains and losses in the ratio of average investments. A's original contribution of capital was $6000, B's $4000, and C's $2000. On Feb. 1, C invested $3000 additional. On March 1, A invested $1200 additional, and B withdrew $600. On Apr. 1, C withdrew from the firm and by mutual consent, received in full settlement the total amount of capital he had contributed, together with interest on his average investment at the rate of 6% per annum for the time he had been a member of the firm, which sum was paid from the partnership fund. Immediately after C's withdrawal, the ledger was closed, when it was found that the net gain was $832. A and B continued the business, each investing, at the time of forming the new partnership, Apr. 1, $3000. On closing the ledger at the end of the year, it was found that the net gain of the new firm amounted to $1731.66, which was equitably divided and credited to the partners' respective accounts. What was each partner's net credit on Jan. 1, 1906?

628. To adjust partnership accounts when the ratio of profit or loss is fixed by contract, and interest is allowed

on the excess and charged on the deficiency of each partner's equitable investment.

1. On Jan. 1, 1905, Z. B. Russell, G. W. Benson, and R. C. Waite entered into a copartnership, agreeing that gains and losses should be divided equally, and that interest at the rate of 10% should be allowed on the excess, and charged on the deficiency of each partner's equitable investment. The net gain, at the end of the year, was $4800, which was credited as additional capital. Find the net capital of each partner on Jan. 1, 1906, from the following statement of the partners' respective accounts:

Z. B. RUSSELL.

1905					1905				
May	7	Withdrawal...	1600	00	Jan.	1	Investment...	10000	00
Sept.	13	" ...	3000	00	July	21	" ...	2000	00
Oct.	31	" ...	500	00					

W. G. BENSON.

1905				1905			
Apr.	6	Withdrawal	600	Jan.	1	Investment.....	8000
June	11	"	400	Aug.	22	"	1500
				Oct.	21	"	4000

R. C. WAITE.

1905				1905			
Feb.	12	Withdrawal	1200	Jan.	1	Investment.....	4000
May	7	"	2500	June	12	"	300
Dec.	9	"	600	Aug.	11	"	200

OPERATION AND EXPLANATION.

We find the average investment of each partner as explained in the former case; thus,

From Jan. 1 to May 7=126 days.....$10000×126=$1260000 for 1 day.
Withdrew..... 1600
" May 7 " July 21= 75 days. ..... 8400× 75= 630000 for 1 day.
Invested..... 2000
" July 21 " Sept. 13= 54 days..... 10400× 54= 561600 for 1 day.
Withdrew..... 3000
" Sept. 13 " Oct. 31= 48 days..... 7400× 48= 355200 for 1 day.
Withdrew..... 500
" Oct. 31 " Jan. 1= 62 days..... 6900× 62= 427800 for 1 day.
Russell's investment=$3234600 for 1 day.

$3234600÷365=$8862, Russell's average investment for the year.

REMARK 1. In computing average investments in partnership, take the nearest unit as the average, as it will be found sufficiently accurate for all practical purposes.

Proceeding with the other partners' accounts in the same manner as with Russell's, we find Benson's average capital to be $8664, and Waite's $1508.

$8862+$8664+$1508=$19034, average capital of firm.
$19034÷3=$6344.67, equitable amount each should have furnished.
$8662—$6344.67=$2517.33, excess furnished by Russell.
$8664—$6344.67=$2319.33, " " " Benson.
$6344.66—$1508=$4836.66, Waite's deficit.

By crediting Russell's account $251.73, the interest to which he is entitled, and $1600, his $\frac{1}{3}$ of the net gain, we find that his net credit is $8751.73.

Crediting Benson with his interest, $231.93, and his $\frac{1}{3}$ of the gain, his account shows a credit balance of $14331.93.

By debiting Waite $483.66, the interest he owes, and crediting him for his $\frac{1}{3}$ gain, his account shows a credit balance of $1316.33.

REM. 2. Partnership accounts, belonging to this case may be adjusted by computing the interest on each investment from its date until the end of the year, or until time of settlement, and from the sum of these interest amounts, subtract the interest on all withdrawals, computed to the same time, and, from such result, deduct the interest of the required investment for the given time.

RULE I. *Find each partner's average investment as directed in Rule 627.*

II. *Credit each partner whose average investment is in excess of the capital he should in equity furnish, with interest at the contract rate on such excess, and debit each partner whose average investment is less than his equitable share of capital, for interest on such deficiency.*

III. *Divide gains or losses as directed by contract.*

EXAMPLES FOR PRACTICE.

2. J. Sanford, Wm. Harlow, and F. W. Ives formed a copartnership on Jan. 1, 1903, it being stipulated in contract that Sanford should furnish $\frac{6}{15}$ of the capital, Harlow $\frac{5}{15}$, and Ives $\frac{4}{15}$, and that gains and losses should be divided in proportion to required investments, and that

interest at the rate of 10% per annum should be allowed on the excess and charged on the deficiency of each partner's required investment. On Jan. 1, Sanford invested $6000; on June 12, $4000; and on Sept. 20, withdrew $3000. On Jan. 1, Harlow invested $5000; on July 18, $2500; and on Sept. 20, withdrew $1000. On Jan. 1, Ives invested $4000; and on April 10, $3000; on June 16, he withdrew $2000; and on Oct. 1, $6000. If the net gain during the year was $6000, what was each partner's interest in the firm on Jan. 1, 1904.

3. E, F, and G engaged as partners in the manufacture of furniture, on Jan. 1, 1902. As per articles of agreement, E furnished $6500, F $8000, and G $5000. It was stipulated in the contract that 12% per annum was to be allowed on the excess, and charged on the deficiency of each partner's equitable investment of $\frac{1}{3}$ the capital, and that gains and losses, after crediting each partner $600 for salary, should be shared equally. On Apr. 1, E and F each invested $1500 additional capital, and on May 1, G withdrew $1200. On Aug. 1, E withdrew $4000, and F and G each invested $2000. On Oct. 1, E and F each withdrew $4000, and G invested $3000. On closing the ledger at the end of the year, it was found that, after crediting partner's salaries, the firm had sustained a net loss of $1050. What was the interest of F, and of G, on Jan. 1, 1903, and how much of E's salary was required to pay his indebtedness to the firm?

4. On July 1, 1905, Louis E. Gifford and Aaron A. Cole entered into partnership relations, each investing $5000 capital, and agreed to share gains and losses equally, and that partners' interests should be adjusted through an interest account on the basis of 10% per annum, before computing gains or losses. On Sept. 1, Martin F. Snow, the former book-keeper, was taken into the firm as a part-

ner, it being agreed that he should invest $2000 as capital and receive a salary of $500 a year for services rendered as office manager. On Oct. 1, Gifford and Cole each invested $1200, and Snow $500. On Dec. 1, each withdrew $800. If the net gain at the end of the year was $3600, what was each partner's interest on Jan. 1, 1906?

5. A and B engaged in trade, agreeing that the average investment of each should be the same, and that gains and losses should be shared equally; also that partners' accounts should be adjusted through an interest account at the end of the year, on the basis of 10% per annum. A invested $6000 at first and 4 months afterward $1500 more. B invested at first $5000, and at the end of 8 months withdrew $2500. At the end of the year their net gain was $1260; how much of the same should each receive after adjustment of interest account?

ADDITIONAL EXAMPLES FOR REVIEW.

629. *1.* Three men hire a pasture for $75. A put in 9 cows 13 weeks, and 6 horses 10 weeks; B put in 4 horses 7 weeks and 15 cows 16 weeks; C put in 14 horses 5 weeks, 8 horses 10 weeks, and 11 cows 15 weeks. Estimating the cost of pasturing 3 cows equal to the cost of pasturing 2 horses for the same time, how much of the $75 should each pay?

2. B commenced business with a capital of $12000, and three months later withdrew $1000. At the end of 6 months from the time of commencing business he withdrew $2500, and at the same time C entered into partnership with him and invested as capital 96 shares of bank stock. The net gain at the end of the year was $6400, which was divided in the ratio of average investments for the year, whereby B received $4000, and C $2400. What was the bank stock worth per share?

3. Three men take an interest in an iron mine, agree-

ing to share in gains and losses in proportion to their average investments. A invests his capital for 4 months, and claims $\frac{1}{10}$ of the profits; B's capital is in 8 months; and C invests \$12000 for 6 months and receives $\frac{3}{5}$ of the profits. How much did A and B invest?

4. Wm. Tracy and Geo. E. Brown bought a saw mill for \$6000, each paying $\frac{1}{2}$ of the purchase price. Tracy stocked the mill with logs, and paid the cost, \$2800, from his private funds. Brown took charge of the mill and paid from his private funds, \$750 for labor, \$450 for repairs, and \$320 for sundry expenses. Tracy sold three bills of lumber, aggregating \$820, and retained the money. Brown received \$1150 for lumber sold, and retained all but \$200, which he gave to Tracy. At the close of the season, the lumber remaining on hand was sold for \$3400 and the money divided equally between the partners; the mill was also sold for \$5750, and the money paid to Tracy. How much should he pay Brown on dissolution and settlement?

5. E. F. Greer, of the firm of Baxter & Greer, offered to buy his partner's interest and to pay therefor 90% of whatever sum the books would reveal as his net credit. Baxter accepted the proposition, and asked Greer, who had served in the capacity of bookkeeper and cashier during the existence of the partnership, for a statement. The following was presented: Baxter's net investment, \$12000; Greer's net investment, \$12000; merchandise on hand per inventory, \$27124; cash on deposit, \$5200; cash in safe, \$218.50; furniture and fixtures per inventory, \$925; personal accounts due firm, \$4456; notes on hand, \$2250; interest collected on notes, \$156; interest paid on outstanding notes, \$118; notes outstanding and due on demand, \$2000; owing on personal accounts, \$850. The total purchases of merchandise amounted to \$22820, and the total sales, to

$13510. The total expenses, shown by the statement, amounted to $6874. Appended to the detailed statement was the following recapitulation: Gains, merchandise, $17814; interest, $38. Loss, expense, $6874. Total gains, $17852, less total loss, $6874, equals net gain, $10978. Baxter's $\frac{1}{2}$ net gain, $5489, added to net investment of $12000, equals $17489, 90% of which is $15740.10. Baxter, doubting the accuracy of the statement, submitted it to an expert, who decided, upon examination, that it was erroneous. Upon examining the books and vouchers, it was found that everything was correct except the expense account. What was the correct amount paid for expenses, and what sum should Greer pay Baxter?

6. E and F are partners. E's capital is to F's as 5 to 8; at the end of 4 months E withdraws $\frac{1}{5}$ of his capital, and F $\frac{3}{4}$ of his; at the end of the year, their net gain is found to be $3600. How much belongs to each?

7. A, B, and C engage in a manufacturing industry. A puts in $2400 for 6 months; B, a sum not specified for 12 months; and C $3600 for a time not specified. A received $2640 for his stock and profits, B, $3600 for his, and C, $4080 for his. If gains were divided in proportion to average investments, what was B's investment and C's time?

8. A, B, and C entered into a copartnership Jan. 1, 1905, and agreed that each should be credited at the end of the year with interest on his average investment, at the rate of 10% per annum, and that gains and losses should be divided equally. A invested $\frac{4}{9}$ of the capital at the time the partnership was formed, B $\frac{3}{9}$, and C $\frac{2}{9}$. On May 1, A withdrew $\frac{1}{8}$ of his investment, and B $\frac{1}{12}$ of his. Aug. 1, C withdrew $\frac{1}{3}$ of his investment. On Sept. 1, B withdrew $\frac{1}{4}$, and C invested a sum equal to $\frac{5}{8}$ of his original investment. On Nov. 1, A withdrew $\frac{1}{4}$ of his original investment. The

net gain, after crediting each partner with interest, was $1200. On closing the ledger at the end of the year, A was credited $700, B $500, and C $400 for interest on their respective investments. What was the net credit of each partner Jan. 1, 1906?

9. Two wood-sawyers agreed to work together for a period of 30 days, and to divide the income from their labor in proportion to working ability. A could saw a cord of maple in the same time that B could saw a cord of soft wood, and he could saw two cords of soft wood in the same time that B could saw a cord of maple. During the 30 days their earnings amounted to $72. How much of this sum belonged to each?

ALLIGATION.

630. Alligation is the process of mixing, or averaging, values of different ingredients.

REMARK. The process of finding the average value of a mixture is called *Alligation Medial*, and of finding the proportional quantities to be used in a mixture of a given average value, *Alligation Alternate*.

631. To find the average price or value of a mixture when the quantity and value of each ingredient are given.

1. A grocer mixed 12 pounds of coffee worth 20¢ a a pound, 24 pounds worth 22¢ a pound, and 48 pounds worth 40¢ a pound. What was the average value per pound of the mixture?

Operation.

$.20 \times 12 = \$2.40$
$.22 \times 24 = 5.28$
$.40 \times 48 = 19.20$
$84 \quad \$26.88$
$\$26.88 \div 84 = \$.32$

EXPLANATION. The value of 12 lb. at 20¢ is $2.40; of 24 lb. at 22¢ is $5.28; and of 48 lb. at 40¢ is $19.20. The whole mixture contains 12 lb. +24 lb.+48 lb.=84 lb., the total value of which is $2.40+$5.28+$19.20=$26.88. One pound of the mixture is therefore worth $\frac{1}{84}$ of $26.88, which is 32 cents.

RULE. *Divide the total value of the mixture by the number of units representing the entire quantity of the different ingredients.*

EXAMPLES FOR PRACTICE.

2. If a mixture is formed consisting of 8 lb. of tea at 60¢, 6 lb. at 70¢, 2 lb. at $1.10, and 4 lb. at $1.20, what is the mixture worth per pound?

3. If 5 gal. water be mixed with 5 gal. wine worth 80¢ per gallon, and 22 gal. worth $1.20 per gallon, what will be the value of the mixture per gallon?

4. A goldsmith mixed 4 oz. gold 12 carats fine, 2 oz.

18 carats fine, and 2 oz. 22 carats fine. How many carats fine was the mixture?

5. If 25% of a mixture is worth 2¢ a pound, 18% 3¢ a pound, 40% 4c a pound, and 17% 5¢ a pound, what is the average value per pound of the mixture?

6. If different grades of coffee, worth respectively 12½, 15, and 28 cents per pound, be mixed in the following proportions: 8 lb. of the first, 8 lb. of the second, and 12½ lb. of the third, what will the mixture be worth per pound?

632. To find the quantity of each ingredient necessary to form a mixture of a given average value.

1. A grocer mixed 3 grades of tea, worth respectively 40¢, 50¢, and 85¢ per pound, in such proportions as to make the mixture worth 60¢ a pound. How many pounds of each grade did the mixture contain.

First Operation. Balance Method.

		A	B	C	D	E	
60	40	$\frac{1}{20}$		5		5	lb. @ 40¢
	50		$\frac{1}{10}$		5	5	" " 50¢
	85	$\frac{1}{25}$	$\frac{1}{25}$	4	2	6	" " 85¢

EXPLANATION. If a pound of tea worth 40¢ be placed in a mixture and sold for 60¢, it is evident that a gain of 20¢ will be the result; hence to produce a gain of 1¢, $\frac{1}{20}$ of a pound must be placed in the mixture. Since each gain must be balanced by a corresponding loss to keep the average price 60¢, we must combine $\frac{1}{25}$ of a pound of the 85¢ tea; these quantities, $\frac{1}{20}$ and $\frac{1}{25}$, we place in column A, opposite the prices to which they are related. Reasoning as before, it will require $\frac{1}{10}$ of a pound of the 50¢ tea to produce a gain of 1¢, and this must be balanced by a corresponding loss of 1¢, which will necessitate the mixing of $\frac{1}{25}$ of a pound of the highest priced tea; these quantities we place in column B, opposite the prices to which they are related. Now, since each of the columns, A and B, contains a gain and an equal loss, it is evident that either, or both, of them may be multiplied or divided by any number whatever without affecting the average price of the mixture. We multiply column A by the least common multiple of the denominators of the fractions and place the results in column C; we also multiply column B by 50, the least common multiple of 10 and 25, and place the results in column D. We combine columns C and D, and place results in column E, and we have the proportional quantities expressed in integral *numbers.*

Principle. A gain in the price of one ingredient must be balanced by an equal loss in the price of some other ingredient, to preserve the average price of the mixture.

	Second Operation.	Linking Method.	
60 { 40, 50, 85 }	25 25 20 + 10 = 30.	25 25 30 } ÷ 5 =	5 lb. @ 40¢ 5 " " 50¢ 6 " " 85¢

Principle. The difference between the price of the mixture and the price of an ingredient of less value, is to the difference between the price of the mixture and the price of an ingredient of greater value, inversely as the quantities required of these ingredients.

EXPLANATION. In order to render the foregoing operation clear, it is necessary to show that it is in harmony with the foregoing principle. Stating the question according to the plan taught in proportion, we have the following:

20¢ : 25¢ :: 20 lb. of 3d ingredient : 25 lb. of 1st ingredient.
10¢ : 25¢ :: 10 " " 3d " : 25 " " 2d "

It is evident that the numbers representing pounds in the second couplet of each proportion may be any whatever having a ratio equal to that of the first couplet of the proportion to which it belongs, hence we repeat the unit values of the first couplet in each case, giving 25 lb. of the first ingredient, 25 lb. of the second, and 20 lb.+10 lb.=30 lb. of the third.

In the operation, we link together the 40 cents (a price which is less than the mixture), with 85 cents, a price which is greater than that of the mixture; another combination is also formed by connecting 50 cents with the 85 cents. We have already explained that 20¢ (the difference between 40¢ and 60¢), is to 25¢ (the difference between 85¢ and 60¢) inversely as the number of pounds required of the 40¢ tea is to the number of pounds required of the 85¢ tea; hence, the difference in each case is placed at the other extremity of the link combining the prices under consideration. Passing to the second grade, we find the difference between it and the price of the mixture to be 10, which we place at the other extremity of the link, thus giving 20 lb. + 10 lb. of the highest priced goods. Notice that two links proceed from 85, one terminating at 40, and the other at 50; hence 25 (the difference between 85 and 60), must have relation to each of the first two grades as shown. The proportionate quantities resulting from the computation are 25 lb. of the first grade, 25 lb. of the second grade, and 30 lb. of the third grade; but, since these proportionate quantities have

a common divisor, 5, we divide each by it and thus express the proportionate relations by the smallest integral numbers.

NOTE 1. The true meaning of *Alligation* is "to link or tie together," and the original plan of presenting alligation alternate was by the linking method. The criticism offered by some, that the linking method, as above explained, teaches that a difference in values equals a certain number of units of commodity, is absurd. In all computations in proportion the numbers employed are abstract in character. A solution of the proportion \$12 : \$4 :: 15 bu. : (?) does not require that \$4 be multiplied by 15 bu., and that the product shall be divided by \$12. The meaning of the statement is that the product of the *abstract numbers* 4 and 15, divided by the *abstract number* 12, will give an abstract quotient containing the same number of units as the concrete number representing the bushels required.

633. Rules for finding the quantity of each ingredient when the price of the mixture is given.

BALANCE METHOD I. *Write the prices of the several ingredients in a column, with the price of the mixture at the left.*

II. *Regard any two prices, one of which is less and the other greater than the price of the mixture, as a couplet; find the difference between each of these prices and the price of the mixture, and write the reciprocal of each difference in a column to the right, and on a line with the given price in the couplet, as one of the proportional terms. In like manner form other couplets until all the prices have been employed, writing each pair of proportional terms in a separate column.*

III. *If the proportional terms thus obtained are fractional, multiply each pair by the least common multiple of their denominators, and carry these integral products to a single column, observing to add any two or more that stand in the same horizontal line; the final results will be the proportional quantities required.*

LINKING METHOD I. *Write the prices of the ingredients and of the mixture as directed in the preceding rule.*

II. *Connect by a link any price less than that of the mixture with any price greater than that of the mixture, until all the prices are linked.*

III. *Write the difference between each price and the price of the mixture on a line with the price at the other extremity of the link, remembering that, in cases where one price is linked with two or more prices, the difference must be placed at the extremity of each link; these respective differences will be the proportional quantities required.*

NOTE 2. If the numbers in any couplet resulting from the computations by either method have a common factor, it may be rejected.

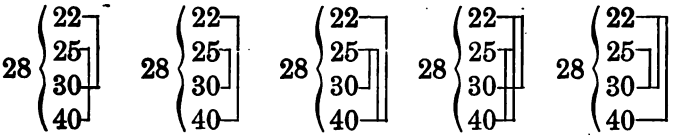
NOTE 3. Since any quantity whose price is less than the price of the mixture may be coupled with *any other* whose price exceeds that of the mixture, it is evident that a problem may have as many different sets of proportional quantities as there are different ways of coupling them. Also the proportionals of any couplet obtained by either method of computation may be multiplied or divided by *any* number, without affecting the accuracy of the results; and, since the number of multipliers and divisors is infinite, it follows that in all cases of more than two ingredients whose price, in each case, differs from that of the mixture, there may be an infinite number of correct answers, unless a limit decides the quantity of one or more of the ingredients.

NOTE 4. If the price of an ingredient is the same as that of the mixture, it should not be coupled, but *any* quantity of such ingredients may be placed in the mixture without affecting the average price.

EXAMPLES FOR PRACTICE.

2. In what proportion can vinegar worth 22, 25, 30, and 40 cents per gallon be mixed to form a compound worth 28 cents per gallon?

The prices named in the foregoing example may be coupled in a variety of ways, as shown by the following illustrations



3. In a certain factory in which men, women and boys are employed, the men receive \$2.50, the women \$1.75, and the boys \$1.25 a day. If the average wages paid is \$1.80

per day, what is the proportionate number of laborers in each class?

4. A merchant has 4 grades of a certain commodity, worth respectively $5\frac{2}{3}$, $7\frac{1}{3}$, $8\frac{3}{4}$, and $11\frac{1}{3}$ cents per pound, which he mixes in such proportions as to form a compound worth $8\frac{3}{4}$ cents per pound. What proportional quantities are used?

5. How many dozen oranges at 25¢, 30¢, 35¢, and 40¢ per dozen must be sold to realize an average price of 28¢ per dozen?

6. A liquor dealer mixed water and three grades of wine in such proportions as to produce a grade worth \$1.10 per gallon. If the respective grades of wine were worth \$.90, \$1.25, and \$1.40, what proportionate quantities of each were used?

634. To find the quantity of each ingredient of a mixture when the quantity of one or more of the ingredients is limited.

1. How many bushels of No. 2 wheat worth \$.80 per bushel, and of No. 3 wheat worth \$.76 per bushel, must be mixed with 12 bushels of No. 1 wheat worth \$.84 a bushel to form a grade worth \$.78 per bushel?

Operation.

	A	B	C	D	E	
76	$\frac{1}{2}$	$\frac{1}{2}$	1	3	4	48
78 80	$\frac{1}{2}$		1		1	$\times 12 =$ 12
84		$\frac{1}{6}$		1	1	12

EXPLANATION. We first find the proportionate quantities by 632; and since there must be 12 times as many bushels of No. 1 wheat as shown by the proportionate results, we multiply each of the proportionate quantities by 12 to preserve the relations.

NOTE. Since column C does not contain the limited quantity, a correct result may be obtained by multiplying column D only, by 12, which would give 37 bu. of No. 3, 1 bu. of No. 2, and 12 bu. of No. 1. These results, although differing from those obtained by the solution, are equally *correct.*

RULE. *Find the proportional quantities by either the balance or linking method, as explained in the former cases. Divide the limited quantity by the obtained proportional of the limited quantity, and multiply each of the proportional quantities by the quotient, or multiply the proportionals of such ingredients only as are coupled with the limited quantity.*

EXAMPLES FOR PRACTICE.

2. A merchant bought 25 hats at $2 each. How many must he buy at $1.25 and $1.60 each to make the average price $1.75?

3. How many gallons of water having a temperature of 95° and 108°, respectively, must be mixed with 12 gallons having a temperature of 120° to make the temperature of the mixture 115°?

4. A dealer mixed 10 pounds of coffee worth 15 cents per pound with other grades worth 13, 20, 27, and 35 cents per pound, making a mixture worth 22 cents per pound. What is the least number of pounds of each grade necessary to form such a mixture?

5. A farmer bought 9 sheep at $5 each and a sufficient number at $3.50 and $4.25 each to make the average price $4.50. How many did he buy at each of the last named prices?

6. How much water will dilute 9 gal. 2 qt. 1 pt. of alcohol 96 per cent. strong to 84 per cent.?

635. To find the quantity of each ingredient when a relation between two or more quantities is given.

1. A grocer wishes to mix three grades of sugar worth respectively, 5, 7, and 8 cents per pound in such a manner as to form a mixture worth 7½ cents a pound. If the 5 cent grade and the 7 cent grade are placed in the mixture in the ratio of 3 to 4, what proportional quantities will the mixture contain?

Operation.

	A	B	C	D	E
5	$\frac{1}{2}$		3		3
$7\frac{1}{2}$ 7		$\frac{1}{2}$		4	4
8	$2\frac{1}{2}$	$\frac{1}{2}$	15	4	19

EXPLANATION. Solving by the linking method and placing the result of each couplet in columns, we obtain from the couplet 5 and 8, $\frac{1}{2}$ lb. of the 5 cent sugar and $2\frac{1}{2}$ lb. of the 8 cent sugar, which we record in column A. From the couplet 7 and 8 we obtain $\frac{1}{2}$ lb. of the 7 cent sugar and $\frac{1}{2}$ lb. of the 8 cent sugar, which we write in column B. Since the proportion of the lower grades are to be as 3 to 4, it is evident that column A must be multiplied by 6, and column B by 8. Multiplying column A by 6 gives 3 and 15, which are written in column C. The results of the multiplication of column B by 8 are written in column D. Combining columns C and D, we have the correct results, which appear in column E.

RULE. *Find the proportional quantities by principles formerly explained, and multiply or divide the columns containing the quantities which are to be of a given relation by any number or numbers that will produce such relation.*

EXAMPLES FOR PRACTICE.

2. A dealer mixes equal quantities of coffee worth respectively $12\frac{1}{2}¢$, $15¢$, and $20¢$ per pound with two other grades worth $30¢$ and $36¢$ per pound, and forms a mixture worth $25¢$ per pound. What proportional quantities does the mixture contain?

3. A miller mixes oats worth $25¢$, wheat worth $90¢$, corn worth $50¢$, and barley worth $42¢$ per bushel. If the quantity of corn contained in the mixture is to the quantity of wheat as 3 to 5, and the mixture is worth $48¢$ per bushel, what proportional quantities of grain does it contain?

4. A grocer formed a mixture of three grades of tea, worth respectively, \$.45, \$.60, and \$.65 per pound, and used 3 times as many pounds of the 60 cent tea as of the 65 cent tea. If the price of the mixture was \$.50, how many pounds of each grade were used?

5. A liquor dealer mixed 1 gal. of brandy worth \$4 *with* 2 gal. of wine worth \$3.50 per gallon, and added a

sufficient quantity of water to make the mixture worth \$3 per gallon. What quantity of water did the mixture contain?

636. To find the quantity of each ingredient when the total quantity of the mixture is limited.

1. A grain dealer sold wheat at 90¢, barley at 44¢, corn at 50¢, and oats at 24¢. The whole sale amounted to 342 bushels, and the average price was 60 cents a bushel. How many bushels of each kind of grain did he sell?

Operation.

		A	B	C	D	E	F	G	
60	24	$\frac{1}{3\,6}$			5			5 × 9 =	45
	44		$\frac{1}{1\,6}$			15		15 × 9 =	135
	50			$\frac{1}{1\,0}$			3	3 × 9 =	27
	90	$\frac{1}{3\,0}$	$\frac{1}{3\,0}$	$\frac{1}{3\,0}$	6	8	1	15 × 9 =	135
								38	342

EXPLANATION. Finding the proportionate quantities as previously explained, we have 5 bu. oats, 15 bu. barley, 3 bu. corn, and 15 bu. wheat, making 38 bu., as the sum of the proportional quantities; but the entire sales were 342 bu., which is 9 times the sum of the proportional quantities, hence each ingredient must be multiplied by 9.

RULE. *Find the proportional quantities as previously explained; divide the limited aggregate of the ingredients by the sum of the proportionals, and multiply each proportional by the quotient.*

EXAMPLES FOR PRACTICE.

2. A fruit dealer bought 216 barrels of apples at an average price of \$3 per barrel. The invoice consisted of 4 grades, billed respectively at \$2.50, \$2.80, \$3.25, and \$3.40 per barrel. How many barrels of each grade did the invoice contain?

3. A liquor dealer prepared a barrel of liquor, (36 gal.), by mixing water with brandy worth \$3.00 per gallon, and sold the same at \$3.30 per gallon, and realized a gain of 20%. How many gallons of brandy did the barrel contain?

4. The pay-roll of a certain factory contains the names of 250 laborers, consisting of boys, women, and two classes of men, ordinary laborers and skilled laborers. The wages paid each boy is $8 per week; each woman receives $10 per week; the men of the ordinary class receive $16 each per week; and each of the skilled workmen receives $20 per week. If the laborers are paid at the average rate of $15 per week, how many are there of each class?

GENERAL AVERAGE.

637. General Average is a method of apportioning among all the interests endangered, the loss sustained by the proprietors of the ship, freight, and cargo, respectively, when, in case of common peril at sea, any portion of the property has been voluntarily sacrificed or damaged to secure the safety of the rest.

638. The **Contributory Interests** are the kinds of property which are taxed in the adjustment of the loss. They include

1. The vessel, at its value before the loss.
2. The freight, less ⅓ as an allowance for seamen's wages.
3. The cargo, including the part sacrificed, at its market value in the port of destination.

REMARK. In a few states only one-half of the freight is made contributory to the loss.

639. Jettison is that portion of the cargo or equipment thrown overboard for the safety of the remainder.

640. Salvage is the remuneration allowed to persons who are instrumental in saving vessels or their cargoes from the perils of the sea.

641. An **Adjuster** of Averages is one who is employed to adjust and apportion the losses and expenses of a general average.

642. The loss which is subject to general average includes:

1. Jettison, including freight on the same, and damage done to the remainder of the cargo, in consequence thereof.

2. Repairs to the vessel, less $\frac{1}{3}$ on account of the enhanced value, by virtue of the new material which replaces the old.

3. The attending expenses, including salvage, port charges, adjuster's fees, etc.

REMARK. Any interest that is *fully* insured can recover from the insurance company the *full* amount of its contribution to the general average loss. If *partially* insured, such a part can be recovered as the amount insured sustains to the total value; provided that, in either case, the policy covers the class of risks which produced the general average loss.

1. The steamer Chesapeake, on a voyage from Cuba to New York, loaded with sugar, encountered a violent storm off the coast of Florida, making it necessary for the common safety to sacrifice a portion of the ship's rigging, which cost \$2400 to replace, and to jettison part of her cargo. Being thus disabled, she made the nearest port at a cost of detention amounting to \$220. The value of the ship, after undergoing repairs, was \$60000; and of her cargo, \$22800, of which A owned \$5900, B \$6500, and C \$10400, $\frac{1}{4}$ of which was jettisoned. The gross freight, including that on the jettison, was \$9000; freight on cargo jettisoned, \$280. What should have been the settlement reported by the adjuster, whose charges were \$140?

OPERATION.

Value of ship after undergoing repairs	\$60000
Increase of value ($\frac{1}{3}$ of \$2400) by virtue of repairs	800
Value of ship before damage was sustained	\$59200

Contributory interests.		*General average charges.*	
Contributory value of ship	\$59200	Jettison	\$2600
Net freight (9000 less $\frac{1}{3}$)	6000	Repairs ($\frac{2}{3}$ of \$2400)	1600
A's part of cargo	5900	Freight on jettison	280
B's " " "	6500	Cost of detention	220
C's " " "	10400	Adjuster's fee	140
Total	\$88000	Total loss	\$4840

\$4840 ÷ \$88000 = .055, or 5½%, rate of general average.

Adjustment.

$800 + (5½% of $59200) = $4056, Assessment on ship.
$6000 × 5½% = 330 " " freight.
Total, $4386

$4386 — $280 (freight paid on jettison) = $4106.00. paid by ship owners.
$5900 × 5½% = 324.50, " " A.
6500 × 5½% = 357.50, " " B.
$4788.00, total receipts.

$2600 — (5½% of $10400) = $2028, amount paid to C.
2400, " " for repairs.
220, " " for detention.
140, " " adjuster.
$4788, total disbursements.

EXPLANATION. Since the value of the ship *after* undergoing repairs was $60000, it was worth, *before* the disaster, ⅛ the cost of repairs less, or $59200. The other contributory interests are: freight, $6000; A's part of the cargo, $5900; B's part of the cargo, $6500; and C's part of the cargo, $10400; making $88000, as the total contributory interests.

The losses to be paid from the general average fund are: ⅜ the cost of the repairs, jettison, freight on jettison, cost of detention, and the adjuster's fees, making a sum total of $4840, which, divided by the sum of the contributory interests, gives 5½% as the rate of assessment.

The owners of the ship must pay ⅛ the cost of repairs, $800, in addition to the assessment of 5½% on $59200; they must also pay the assessment on ⅜ the freight, but are entitled to a rebate of $280, the freight on the part of the cargo jettisoned. Multiplying A's and B's interests by the rate of general average, and adding the results to the sum to be paid by the ship's owners, gives $4788, as the total receipts. In paying the claims, C must receive the value of his goods jettisoned, $2600, less 5½% assessment on his part of the cargo. All other disbursements are as stated in the example.

RULE I. *Divide the sum of the losses by the sum of the contributory interests; the quotient will be the rate of contribution.*

II. *Multiply each contributory interest by the rate; the products will represent the respective contributions to the loss.*

EXAMPLES FOR PRACTICE.

2. The ship Ocean Queen encountered a storm at sea, making it necessary, for the common safety, to cut away her mainmast, and cast overboard ¼ of her cargo; she then

put into the nearest port to refit; the repairs cost $1500, and the expenses of detention were $420. The ship was owned by the Western Transportation Co., and was valued at $25000; the cargo was owned by Geo. Appleton & Co., and consisted of 2800 barrels of flour, valued at $9 per barrel at the port of destination, the freight on same being $4200. In the adjustment of the loss by general average, how much was due from the Western Transportation Co. to Geo. Appleton & Co.

3. The steamer Norman, valued at $28000, having been disabled in a storm, entered port, and was refitted at an expense of $270 for repairs, and $120 for pilotage, dockage, and board of seamen. A's interest in the cargo was $2400; B's $1850; and the remainder, $750, belonged to C. The amount sacrificed for the ship's safety was $1400 of A's property, and $170 of B's. The gross charges for freight were $1500, $320 of which was on goods jettisoned. Required the balance payable or receivable, by each of the parties, the adjuster's fee being $102.

ADDENDA.

GOVERNMENT SURVEYS.

643. Government Lands are usually surveyed into rectangular tracts, bounded by lines conforming to the cardinal points of the compass.

REMARK I. A *Base-line* conforming to a parallel of latitude, and a *Principal Meridian* intersecting it, are first established. Other lines are then run *six miles* apart each way, as nearly as possible. The rectangular tracts thus formed are called *Government Townships*, and each contains, as nearly as may be, 23040 acres.

REM. 2. The townships are numbered north and south from the base-line, and a tier or row of these townships, extending north and south, is called a *Range*. The ranges are numbered east and west from the principal meridian.

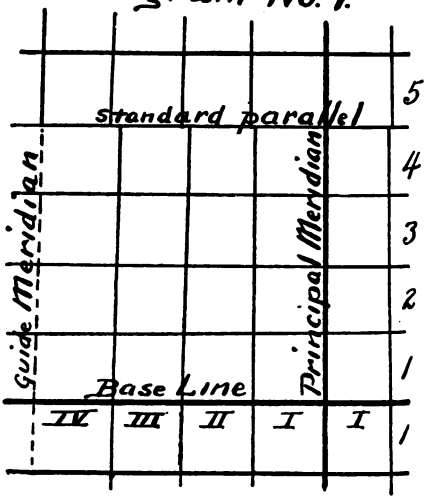
REM. 3. Since the earth's surface is that of a globe, it is evident that lines surveyed north from the base-line will *converge* and meet at the north pole if extended, and that those surveyed south from the base-line will constantly *diverge* until they reach the equator. This tends to throw the townships and subdivisions out of square, and necessitates occasional lines of offset, called *correction lines*. Correction lines, extending north and south, are usually termed "*guide meridians*". The *standard parallels*, although regularly established lines of the survey, are also called "*correction lines*", because distances lessened by convergency are corrected on these lines.

REM. 4. *Townships* are subdivided into *Sections*, and sections into *Half-Sections*, *Quarter-Sections*, *Half-Quarter-Sections*, *Quarter-Quarter-Sections*, and *Lots*.

EXPLANATION. The accompanying diagram is designed to show the first work of a government survey, which is to divide the tract into townships. The ranges, which are numbered at the lower part of the diagram, are 6 miles wide at the base line. The lines running east and west are six miles apart and are called parallels; every fourth one is called a *standard*

parallel, or *correction line*, for the reason that distances, lessened by convergency, as explained in Rem. 3, are corrected on these lines and made the same as at the base line. The range lines and parallels divide the surveyed territory into townships, designed to be 6 miles square. The numbering of the townships is shown on the right hand side of the diagram. Guide meridians are run as often as may be necessary for the purpose of straightening the standard meridian lines as they become irregular in consequence of the corrections made on the standard parallels.

Diagram No. 1.



The tract having been surveyed into townships, the next work is to divide each of the townships into 36 sections, each of which is *designed* to be 1 mile square. Diagram No. 2 contains one full township and adjacent parts of the townships bounding the same on the north and west. The subdivision of townships is usually performed by a different crew of surveyors than survey the tract into townships. The first work of subdivision is to run lines parallel with the east line of the township and one mile apart, and to locate stakes upon these lines a distance of one-half a mile apart. Lines one mile apart and parallel with *the south* line of the township are next run, thus dividing the townships *into 1-mile* squares, called sections, which are designated by number, as

Diagram No. 2.

A Township

N

W

E

S

36	31	32	33	34	35	36
1	6	5	4	3	2	1
12	7	8	9	10	11	12
13	18	17	16	15	14	13
24	19	20	21	22	23	24
25	30	29	28	27	26	25
36	31	32	33	34	35	36

hown in the diagram. After measuring 5½ miles north from the south line f the township, the distance remaining may be a few feet more or a few feet ess than one-half a mile, owing to the discrepancy of the two measure- nents; hence the quarter sections on the north are usually fractional; hose on the west of each township are also fractional, for a similar reason.

Diagram No. 3 is designed to illustrate the division of a section into parts formed by regularly surveyed lines, and to furnish a description of the various subdivisions.

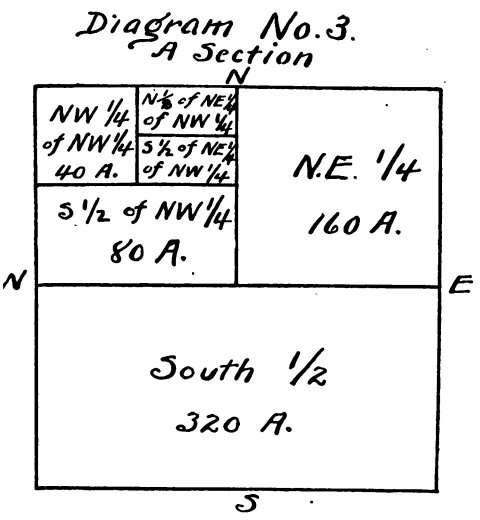


Diagram No. 4

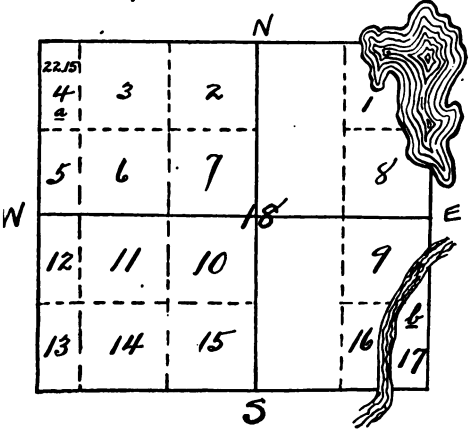


Diagram No. 4 is designed to illustrate the method of describing government lands bounded on the west by guide meridians, and also of lands bordering on meandered waters. Whenever a guide meridian is run, it is evident that all sections lying directly east must be extended to it, thus making all such sections more than a square mile; the north and south quarter line of each section is run as in other sections, and all that portion west of it is divided into lots as shown in the diagram.

Assuming the section, illustrated by the diagram, to belong to township 112 of range 17 west of the 5th principal meridian, the tract marked *a* should be described as follows: "Lot four (4) of the northwest quarter (¼) of section eighteen (18), of township one hundred twelve (112) north, of range seventeen (17) west

of the fifth (5th) principal meridian, containing twenty-two and fifteen-hundredths (22.15) acres, more or less, according to the government survey thereof." The tract marked *b* should be described in a similar manner; the "southeast quarter" of section 18 must be substituted for the "northwest quarter"; in other respects the descriptions would be the same except as to acreage.

REMARK. The following facts should be memorized:

1. The east tier of sections of a township, are, 1, 12, 13, 24, 25, and 36; and those forming the west tier are, 6, 7, 18, 19, 30, and 31.

2. If 5 be added to any one of the odd numbers designating sections belonging to the east or to the west tier of sections, or subtracted from any one of the even numbers, the result will be the number of the adjacent section belonging to the adjoining township.

3. If the number of any section belonging to the north or to the south tier of a township be subtracted from 37, the result will be the number of the adjacent section belonging to the adjoining township.

EXAMPLES FOR PRACTICE.

1. Make a plat of the following described lands: "The S. E. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ of Section 17." *2.* "The N. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ of Section 32." *3.* "The S. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ and the S. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ of Section 11."

2. James Roberts bought all of the S. E. $\frac{1}{4}$ of the section illustrated in diagram 4, except the tract marked *b*. Write the description as it should appear in his deed.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

644. The **Metric System** was adopted in France in 1795; its use was authorized in Great Britain in 1864, and in 1860 Congress passed a bill authorizing its use in the United States.

645. The **Metric System** of weights and measures is based upon the decimal scale.

646. The **Meter** is the *base* of the system, and is one ten-millionth part of the distance on the earth's surface from the equator to either pole, or 39.37079 inches.

647. From the **Meter** are made the **Are** (air), the **Stere** (stair), the **Liter** (leeter), and the **Gram**; these constitute the *primary* or *principal* units of the system, from which all the others are derived.

648. The **Multiple Units,** or higher denominations, are named by prefixing to the name of the *primary* units the Greek numerals, **Deka** (10), **Hecto** (100), **Kilo** (1000), and **Myria** (10000).

649. The **Sub-Multiple Units,** or lower denominations, are named by prefixing to the names of the *primary* units the Latin numerals, **Deci** ($\frac{1}{10}$), **Centi** ($\frac{1}{100}$), **Milli** ($\frac{1}{1000}$).

Hence, it is apparent from the *name* of a unit, whether it is *greater* or *less* than the standard unit, and also *how many times.*

MEASURES OF EXTENSION.

650. The **Meter** is the unit of **length,** and is equal to 39.37 in. nearly.

TABLE.

Metric Denominations.		*U. S. Value.*
	1 Millimeter	= .03937079 in.
10 Millimeter (mm.)	= 1 Centimeter	= .3937079 in.
10 Centimeters (cm.)	= 1 Decimeter	= 3.937079 in.
10 Decimeters (dm.)	= 1 *Meter*	= 39.37079 in.
10 Meters (M.)	= 1 Dekameter	= 32.808992 ft.
10 Dekameters (Dm.)	= 1 Hectometer	= 19.927817 rd.
10 Hectometers (Hm.)	= 1 Kilometer	= .6213824 mi.
10 Kilometers (Km.)	= 1 Myriameter (Mm.)	= 6.213824 mi.

The **Meter,** like our yard, is used in measuring cloths and short distances.

The **Kilometer** is commonly used for measuring long distances, and is about $\frac{5}{8}$ of a common mile.

651. The **Are** is the unit of **land measure.** It is a square, each side of which is 10 meters, and is therefore equal to 100 sq. meters, or 119.6 sq. yd.

TABLE.

100 centàrs (ca.)	= 1 are (A)	= 119.6 sq. yd.
100 ares	= 1 hektar (Ha.)	= 2.48 acres.

652. The primary unit for other surfaces than land is the **sq. meter.**

TABLE.

100 sq. centimeters (sq. cm.)	= 1 sq. decimeter (sq. dm.)	= .1076 sq. ft.
100 sq. decimeters	= 1 sq. meter (Sq. M.)	= 1.196 sq. yd.

653. The **Stere** is the unit of wood measure, and is equal to a **cubic meter,** or .2759 cord.

TABLE.

10 decisteres (dst.)	= 1 stere (S.)	= 35.316+ cu. ft.
10 steres	= 1 dekastere (Ds.)	= 13.079+ cu. yd.

654. The **Cubic Meter** is the primary unit for ordinary solids; as excavations, embankments, etc.

TABLE.

1000 cu. millimeters (cu. mm.)	=1 cu. centimeter (cu. cm.)	= .061+ cu. in.
1000 cu. centimeters	=1 cu. decimeter (cu. dm.)	=61.026+ cu. in.
1000 cu. decimeters	=1 cu. meter (Cu. M.)	=35.316+ cu. ft.

MEASURES OF CAPACITY.

655. The **Liter** is the *standard unit of capacity*, both of Liquid and Dry Measure, and is a vessel whose volume is equal to a cube whose edge is *one-tenth* of a *meter*, equal to 1.05673 qt. Liquid Measure, and .9081 qt. Dry Measure.

TABLE.

10 milliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 *Liter* (L.)
10 Liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 hectoliter (Hl.)
10 hectoliters	= 1 kiloliter (Kl.)
10 kiloliters	= 1 myrialiter (Ml).

656. The **Hectoliter** is the unit considered in measuring liquids, grain, fruits, and vegetables in large quantities.

657. **EQUIVALENTS IN U. S. MEASURE.**

Metric denominations.	*Cubic Measure.*	*Dry Measure.*	*Liquid Measure.*
1 myrialiter	= 10 cu. meters	= 283.72+ bu.	= 2641.4+ gal.
1 kiloliter	= 1 cu. meter	= 28.372+ bu.	= 264.17+ gal.
1 hectoliter	= $\frac{1}{10}$ cu. meter	= 2.8372+ bu.	= 26.417+ gal.
1 dekaliter	= 10 cu. decimeters	= 9.08 qt.	= 2.6417 gal.
1 *liter*	= 1 cu. decimeter	= .908 qt.	= 1.0567 qt.
1 deciliter	= $\frac{1}{10}$ cu. decimeter	= 6.1022 cu. in.	= .845 gi.
1 centiliter	= 10 cu. centimeters	= .6102 cu. in.	= .338 fl. oz.
1 milliliter	= 1 cu. centimeter	= .061 cu. in.	= .27 fl. oz.

MEASURES OF WEIGHT.

655. The **Gram** is the *unit of weight*, and equal to the weight of a cube of distilled water, the edge of which is *one-hundredth* of a *meter*, equal to 15.432 Troy grains.

TABLE.

10 Milligrams (mg.)	= 1 Centigram	=	.15432+ gr. Troy
10 Centigrams (cg.)	= 1 Decigram	=	1.54324+ gr. Troy
10 Decigrams (dg.)	= 1 *Gram*	=	15.43248+ gr. Troy
10 *Grams* (G.)	= 1 Dekagram	=	.35273+ oz. Avoir.
10 Dekagrams (Dg.)	= 1 Hectogram	=	3.52739+ oz. Avoir.
10 Hectograms (Hg.)	= 1 Kilogram or *Kilo*	=	2.20462+ lb. Avoir.
10 Kilograms (Kg.)	= 1 Myriagram	=	22.04621+ lb. Avoir.
10 Myriagrams (Mg.) or 100 Kilograms	= 1 Quintal	=	220.46212+ lb. Avoir.
10 Quintals or 1000 *Kilos*	= 1 Tonneau or Ton	=	2204.62125 lb. Avoir.

The **Kilogram,** or **Kilo,** is the *unit* of common weight in trade, and is a trifle less than $2\frac{1}{5}$ lb. avoirdupois.

The **Tonneau** is used for weighing very heavy articles, and is about 204 lb. more than a common ton.

656. *Units of the Common* System may be readily

changed to *units* of the *Metric* System by the aid of the following

TABLE.

1 Inch	= 2.54 Centimeters	1 Cu. inch	= 16.39 Cu. Centimet.
1 Foot	= 30.48 Centimeters	1 Cu. foot	= 28320 Cu. Centimet.
1 Yard	= .9144 Meters	1 Cu. yard	= .7646 Cu. Meter
1 Rod	= 5.029 Meters	1 Cord	= 3.625 Steres
1 Mile	= 1.6093 Kilometers	1 Fl. ounce	= 2.958 Centiliters
1 Sq. inch	= 6.4528 Sq. Centimeters	1 Gallon	= 3.786 Liters
1 Sq. foot	= 929 Sq. Centimeters	1 Bushel	= .3524 Hectoliter
1 Sq. yard	= .8361 Sq. Meters	1 Troy gr.	= 64.8 Milligrams
1 Sq. rod	= 25.29 Centiares	1 Troy lb.	= .373 Kilo.
1 Acre	= 40.47 Ares	1 Avoir. lb.	= .4536 Kilo.
1 Sq. mile	= 259 Hectares	1 Ton	= .907 Tonneau

657. The principles governing the reductions and computations of denominate numbers will apply with equal force to the Metric System.

POWERS AND ROOTS.

658. A **Root** is one of the equal factors whose unit value is repeated to produce a power.

For the definition of *a power* see (78). The process of raising a number to a given power is commonly termed *Involution*, and the reverse, extracting a required root of a number, is termed *Evolution*.

659. A **Rational Root** is a root whose exact value can be expressed by a finite number.

660. A **Surd** is an indicated root whose exact value can not be expressed by a finite number.

NOTE. Surds have exact roots, but their roots can not be expressed by finite numbers; approximate roots, however, can be obtained sufficiently near accuracy to meet all practical requirements. A square plat of ground may contain 10 square rods, in which case it is evident that the line representing one side (which can be easily described by geometry) is the exact square root of 10; but the exact length of the line can not be expressed by a *finite* number, as it contains an infinitesimal increment, or, in other words,

an increment less in value than the smallest fraction of which the human mind has any conception.

661. The **Radical Sign**, or sign of evolution, is the character $\surd$, which, placed before a number, indicates that its root is to be extracted.

662. The **Index** of the Root is the figure placed above the radical sign, to denote what root is to be taken.

NOTE. When no index is written, the index 2 is understood. $\surd 64$ denotes the square root of 64; $\sqrt[3]{125}$ denotes the cube root of 125, and $\sqrt[4]{81}$ denotes the 4th root of 81, etc.

663. Fractional exponents are frequently employed in powers and roots, the denominator indicating the root and the numerator the power; thus, $72^{\frac{1}{2}}$ indicates the square root of 72; $96^{\frac{2}{3}}$ indicates the cube root of the square of 96, etc.

664. The process of raising a root to a required power is fully set forth in multiplication, but for a better understanding of the extraction of roots, the student should possess a clear knowledge of unit relations as they combine in the computation.

1. What is the second power (square) of 32?

Operation.

$$\begin{array}{l} 30+2 \\ 30+2 \\ \hline 900+60 \\ \quad\ 60+4 \\ \hline 900+120+4=1024 \end{array}$$

EXPLANATION. If we had nothing in view but to obtain the square, we should simply multiply 32 by 32 as explained in simple multiplication, but we wish to have the student understand how these unit values combine in obtaining the result. By referring to the operation, we readily see that the first term, 900, is the square of the tens; 60, which occurs in each partial product, is the product of the tens by the units, and the sum, the second term of the product, is two times the tens by the units; the last term, 4, is evidently the square of the units.

NOTE. If we represent the tens of any number composed of two figures by t and units by u, the general formula for the square is $t^2+2tu+u^2$. In the literal notation, the writing of one letter after another with no sign between them signifies multiplication; thus, ab indicates the product of a by b; 2tu indicates two times the product of t by u, or, in the general formula, two times the product of the tens by the units.

665. Before considering the subject of roots, the student should become familiar with the elements of the literal notation. We believe that a careful study of the preceding note will render the following computations clear.

1. What is the third power (cube) of $t+u$?

Operation.

$$
\begin{array}{l}
t+u \\
t+u \\
\hline
t^2+tu \\
\quad tu+u^2 \\
\hline
t^2+2tu+u^2 \\
t+u \\
\hline
t^3+2t^2u+tu^2 \\
\quad t^2u+2tu^2+u^3 \\
\hline
t^3+3t^2u+3tu^2+u^3
\end{array}
$$

EXPLANATION. We first multiply the multiplicand by t and obtain t^2+tu, the first partial product; then multiplying by u, we obtain $tu+u^2$ for the second partial product; these partial products we combine by addition and obtain $t^2+2tu+u^2$; multiplying this square by $t+u$, we have the cube, $t^3+3t^2u+3tu^2+u^3$, which is read, the cube of the tens, plus 3 times the square of the tens by the units, plus 3 times the tens by the square of the units, plus the cube of the units.

666. The use of this formula is clearly shown by the following application, in determining the cube of 23:

The first term, $t^3=\overline{20}^3$	=	8000
The second term, $3t^2u=3\times\overline{20}^2\times 3$	=	3600
The third term, $3tu^2=3\times 20\times 3^2$	=	540
The fourth term, $u^3=3^3$	=	27
The cube of 23	=	12167

NOTE. The formulas, $t^2+2tu+u^2$ for the square, and $t^3+3t^2u+3tu^2+u^3$ for the cube, should be memorized.

EXAMPLES FOR PRACTICE.

2. Find, by multiplication, the square of $a+b$.

3. Find, by multiplication, the cube of $c+n$.

4. Find, by application of the formula, the square of 74.

5. Find, by application of the formula, the cube of 52.

6. Expand $t+u$ to the fifth power.

7. What is the value of $(a+b)^3$?

8. What is the value of $(x+y)^4$?

Find, from the formula, the expanded values of the following numbers: (*9*) $\overline{41}^2$. (*10*) $\overline{53}^2$. (*11*) $\overline{89}^2$. (*12*) $\overline{73}^2$. (*13*) $\overline{95}^3$. (*14*) $\overline{17}^2$. (*15*) $\overline{37}^3$. (*16*) $\overline{38}^2$. (*17*) $\overline{65}^3$. (*18*) $\overline{123}^2$.

NOTE. 123=12 tens +3 units, therefore $t^2=\overline{120}^2$.

667. Before proceeding with a demonstration of the subject of roots, it is necessary for the student to understand the relation between the number of unit orders in a power and the number of orders in its corresponding root. By referring to the table in Article 668, it will be seen that the square of any number represented by a single digit, contains either one or two orders; and that the cube of such a number contains either one, two, or three orders.

From the foregoing law of relation, we discover the following facts:

1. If we point off a number into two-figure periods, commencing at the right, the number of periods (the left one of which may be partial) will indicate the number of places in the square root.

2. If we point off a number into three-figure periods, commencing at the right, the number of periods will indicate the number of places in the cube root.

The foregoing facts suggest the following

GENERAL LAW.

The number of places belonging to any period, except the highest, is equal to the number of units contained in the root index.

SQUARE ROOT.

668. The **Square Root** of a number is one of two equal factors which, when multiplied together, will produce the given number. The square root of 81 is 9, since $9\times9=81$.

NOTE. The student should memorize the squares and the cubes of each of the roots contained in the following

TABLE.

Roots	Squares	Cubes	Roots	Squares	Cubes
1	1	1	6	36	216
2	4	8	7	49	343
3	9	27	8	64	512
4	16	64	9	81	729
5	25	125	10	100	1000

Analysis of the Formula.

$t^2+2tu+u^2$

t^2

$u)\ 2tu+u^2$

$2t+u$

EXPLANATION. A careful study of the formula will reveal the fact that it contains two parts, the first being t^2 (the square of the tens), and the second part that which remains from the subtraction of the first part, $2tu + u^2$ (2 times the tens by the units plus the square of the units). Since each term of the second part contains u, the units of the root, we divide this part, which is a dividend, by the root or quotient u, and obtain the divisor, which is $2t+u$ (2 times the tens plus the units). Since the formulas are general it follows that $2t + u$ is the formula for every divisor in square root.

1. What is the square root of 147456 ?

Operation.

14'74'56 (384
9

68	574
	544
764	3056
	3056

EXPLANATION. By pointing the number off into periods of two figures each, as previously explained, we find that the root will contain three figures, each of which is tens in its relation to the next lower order. Since the square of the first figure of the root is always contained wholly in the highest, or left-hand period of the power, we write the greatest perfect power contained in this period, which is 9, underneath, and its root 3 (which we regard as tens in relation to the next lower order) as the first figure of the root. We next subtract 9, the square of the first figure of the root, and bringing down the next period we have 574, the remainder, which is to be employed as a dividend, and is expressed by $2tu + u^2$ in the formula. The divisor, as shown by the analysis of the formula, is represented by $2t + u$ (2 times the tens plus the units). The value of t has now become known in the example, but as u, the units, is still unknown, we have not yet a complete divisor; but since the greater part of the divisor is included in 2t (2×30), we use this number, 6 tens or 60, as a trial divisor and thus *find* the units to be 8, which we write as the second figure of the root, and *add* the same to the trial divisor 60, thus making the complete divisor 68. *Multiplying* our completed divisor by the root figure, subtracting the result and bringing down the next period gives a remainder of 3056. We next

proceed with the formula $2t + u$ to find our new divisor. Since there is still another place in the root, the part already found, 38, must represent tens; hence the value of t becomes 38 tens, or 380. The new trial divisor $2t$ (380×2) is 760; dividing as in the former case and completing the divisor, we have 4 for the next and last figure of the root and 764 for a complete divisor. Performing the work of multiplication and subtraction as before, we have no remainder; hence 384 is the exact root.

669. From the foregoing principles and illustrations we deduce the following:

RULE I. *Point off the given number into periods of two figures each, counting in each direction from units' place.*

II. *Find the greatest square contained in the left-hand period, and write its root for the first figure of the required root; subtract the square from the left-hand period, and to the remainder bring down the next period, thus completing the dividend.*

III. *At the left of the dividend write twice the first figure of the root for a trial divisor; divide the dividend, exclusive of its right-hand figure, by the trial divisor, and write the quotient for the second figure of the root.*

IV. *Annex this root figure to the trial divisor, multiply the complete divisor by it, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *Write twice the root already obtained for a trial divisor, with which proceed as before until all the periods have been brought down.*

NOTE 1. If at any time, the product be greater than the dividend, the root figure is too large and must be diminished, and all other erroneous work corrected.

NOTE 2. Should a cipher occur in the root, annex a cipher to the trial divisor and another period to the dividend, and proceed as before.

NOTE 3. Should there be a remainder after all periods have been brought down, decimal periods of ciphers may be annexed, and the work extended to any desired limit, in which case, as many decimal places must be pointed off in the root as there have been decimal periods annexed.

NOTE 4. The square root of a common fraction, whose terms are perfect squares, may be obtained by extracting the square root of each term separately; *if the terms* are not perfect squares, the fraction should first be *reduced to a decimal.*

EXAMPLES FOR PRACTICE.

2. What is the square root of 192721?
3. What is the square root of 152399025?
4. What is the square root of 64368529?
5. Extract the square root of 49098049.
6. What is the square root of 10795.21?
7. What is the square root of 2878.32$\frac{1}{4}$?
8. What is the square root of 62.5?
9. Extract the square root of $\frac{7}{16}$.
10. Extract the square root of $\frac{169}{225}$.
11. Find the value of $\sqrt{611078400}$.
12. What is the length of one side of a square field that contains one acre?

CUBE ROOT.

670. The **Cube Root** of a number is one of three equal factors which, multiplied together, will produce the given number.

Analysis of the Formula.

$t^3+3t^2u+3tu^2+u^3$

t^3

$u)\,3t^2u+3tu^2+u^3$

$3t^2+3tu+u^2$

EXPLANATION. This formula, like the one in square root, we divide in two parts. Subtract the first term, which is the first part, and we have $3t^2u+3tu^2+u^3$, the second part. Since this quantity is a dividend having u for a quotient, we divide it by u and obtain $3t^2+3tu+u^2$, which is the formula for every divisor in cube root.

1. What is the cube root of 43614208?

Operation.

		43'614'208(352
		27
450	2700	16614
25	3175	15875
2100	367500	739208
4	369604	739208

EXPLANATION. We first point the number off into periods of three figures each and find that the root will contain 3 places. The greatest perfect cube contained in the highest period is 27; this we write in its place, and its root as the first figure of the required root. Performing the subtraction, and bringing down the next period, as explained in square root, gives 16614 as a dividend. We next employ the formula for the divisor, $3t^2+3tu+u^2$.

and determine the trial and the complete divisor. In this example the trial divisor $3t^2(3\times\overline{30}^2)$, is 2700, which we write at the left of the dividend, and find, by its use, that the next figure of the root (u) is 5; hence the first addition to the trial divisor, 3tu $(3\times30\times5)$ is 450, which we place to the left of the trial divisor. The second addition, u^2 (5^2), is 25, which we write under the first addition; combining these additions with the trial divisor, we have the complete divisor, 3175, which we place under the trial divisor. Multiplying the complete divisor by the root figure, subtracting the result from the dividend, and bringing down the next period, gives 739208 for the new dividend. The value of t is now 35 tens or 350; hence, the trial divisor, $\overline{350}^2\times3$, is 367500. The additions to the trial divisor, 3tu $(3\times350\times2)$, and u^2 (2^2) are 2100 and 4; completing the divisor by these additions, gives 369604, which is contained in the dividend 2 times with no remainder; therefore, 352 is the exact cube root of the given number.

671. From the foregoing principles and illustrations, we deduce the following:

RULE I. *Point off the given number into periods of three figures each, counting in each direction from units' place.*

II. *Find the greatest cube contained in the left hand period, and write its root for the first figure of the required root; subtract the cube from the left hand period, and, to the remainder, bring down the next period, thus completing the dividend.*

III. *At the left of the dividend, write three times the square of the first figure of the root, regarded as tens, for a trial divisor; divide the dividend by the trial divisor, and write the quotient for the second figure of the root.*

IV. *To the left of the trial divisor, write, for the first addition thereto, three times the product of the first figure of the root, regarded as tens, by the last root figure obtained. Under this first addition to the trial divisor write, as a second addition, the square of the last root figure. Add these additions to the trial divisor and place the sum underneath as the complete divisor.*

V. *Multiply the complete divisor by the last root figure, subtract the product from the dividend and to the remainder annex the next period.*

VI. Regarding that part of the root already obtained as

tens, find, from the formula, the trial divisor and additions thereto as before, and continue the work until the last period has been brought into computation.

NOTE 1. If, at any time, the product be greater than the dividend, diminish the root figure and correct erroneous work.

NOTE 2. Should a cipher occur in the root, annex two ciphers to the trial divisor and a period to the remainder, and proceed as before.

EXAMPLES FOR PRACTICE.

2. What is the cube root of 551368?

3. What is the cube root of 10460353203?

4. What is the cube root of 127263527?

5. What is the cube root of 35937000000?

6. What is the cube root of .30625?

7. What is the cube root of 5.1872?

8. What is the value of $\sqrt[3]{1367631}$?

9. What is the value of $\sqrt[3]{\sqrt{640000 \times 10}}$?

10. What is the value of $\sqrt[3]{\sqrt[3]{10077696}}$?

11. What is the value of $\sqrt[3]{\frac{125}{729}} \div \sqrt[3]{\frac{8}{27}}$?

12. What is the cube root of 5, correct to three decimal places?

ANY ROOT.

1. What is the 5th root of 33554432?

Operation.

$5t^4$	$=4050000$	$335'54432(32$
$10t^3u$	$= 540000$	243
$10t^2u^2$	$= 36000$	$\overline{9254432}$
$5tu^3$	$= 1200$	9254432
u^4	$= 16$	$\overline{00}$
	4627216	

EXPLANATION. We first point off the number into periods of 5 figures as previously explained, and find that the root will contain 2 figures. The highest 5th power contained in the left hand period is 243, whose 5th root is 3. Proceeding as in the lower roots we obtain 9254432 for a dividend. The next step is to obtain a trial divisor, which will require the formula for the 5th root. This formula is easily obtained by raising $t+u$ to the 5th power. The operation gives $t^5+5t^4u+10t^3u^2+10t^2u^3+5tu^4+u^5$. Removing *the first term* and dividing by u, we have $5t^4+10t^3u+10t^2u^2+5tu^3+u^4$, *which is the formula* for the complete divisor. The first term is the trial

divisor; and the other terms represent the additions. Dividing the dividend by the numeral value of $5t^4$ gives 2 for the value of u. The trial divisor and the numeral values of the additions added together make our complete divisor, which is contained in the dividend twice with no remainder; hence, 32 is the required 5th root.

672. From the foregoing illustration we deduce the following

RULE I. *Point off the given number into periods of as many places each, counting from units, as there are units in the root index.*

II. *Under the left hand period write the highest power, corresponding to the root, that the period contains, and the root thereof as the first figure of the required root; subtract the power as in the lower roots, and to the remainder annex the next period, thus forming a dividend.*

III. *Raise the root formula, $t+u$, to the power corresponding to the required root, remove the first term of the result, and divide the remainder by u; the quotient will be the formula for the complete divisor.*

IV. *Write the numeral value of the first term of the formula for a trial divisor; divide the dividend by it and thus obtain the value of u, which is the second figure of the root.*

V. *Under the trial divisor, write the numeral value of each term of the formula, except the first, and add these several sums to the trial divisor, thus obtaining the complete divisor.*

VI. *Multiply the complete divisor by the last root figure, subtract the product from the dividend, and, to the remainder, annex the next period, and continue the work in the same manner until all periods shall have entered into the computation.*

NOTE 1. The 4th root of a number may be obtained by extracting the square root of the square root.

NOTE 2. The sixth root may be obtained by extracting the cube root of the square root, or by extracting the square root of the cube root.

NOTE 3. The 8th root of a number is equal to the square root of the square root of the square root.

NOTE 4. The 9th root of a number is equal to the cube root of the cube root.

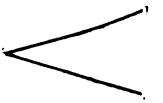
EXAMPLES FOR PRACTICE.

2. What is the 4th root of 6250000 ?
3. What is the 5th root of 1889568 ?
4. What is the 6th root of 8303765625 ?
5. What is the 8th root of 248155780267521 ?
6. What is the 9th root of 20661046784 ?

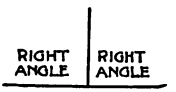
MENSURATION.

673. Mensuration is properly a branch of applied geometry, which furnishes rules for ascertaining the different elements belonging to measurement of extension.

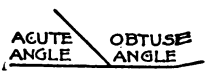
NOTE. This treatise is designed as a continuation of the subject of Measurement of Extension, which begins with Article 347. Definitions therein contained will not appear here.



674. An **Angle** is the space contained between two lines which meet at a common point.

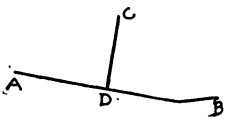


675. A **Right Angle** is an angle formed by the meeting of two lines in such a way as to make the adjacent angles equal.



676. An **Obtuse Angle** is an angle greater than a right angle.

677. An **Acute Angle** is an angle less than a right angle.



A line is perpendicular to another when it forms with it a right angle. The line CD is perpendicular to the line AB; AB is also perpendicular to CD.

PLANE FIGURES.

678. A **Plane Figure** is a portion of a plane surface *limited* by straight or curved lines.

679. A **Polygon** is a plane figure bounded by straight lines.

680. The **Perimeter** of a **polygon** is the sum of its sides.

A *regular polygon* has all its sides and all its angles equal.

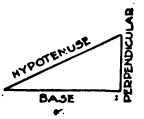
A polygon of three sides is a *triangle;* one of four sides is a *quadrilateral;* one of five sides is a *pentagon*, etc.

TRIANGLES.

681. A **Triangle** is a plane figure having three sides and three angles.

682. A **Right-angled Triangle** is a triangle having one right angle.

683. The **Hypotenuse** of a right-angled triangle is the side opposite the right angle.



684. The **Base** of a triangle, or of any plane figure, is the side upon which it is supposed to stand.

685. The **Perpendicular** of a right-angled triangle is the side which unites with the base to form the right angle.

686. The **Altitude** of a triangle is a line drawn perpendicular to the base from the angle opposite.

Triangles are named from the relation of their sides, and also from a relation of their angles.

687. An **Equilateral Triangle** is one whose three sides are equal. (Fig. 1.)

688. An **Isosceles Triangle** is one which has two of its sides equal. (Fig 2.)

689. A **Scalene Triangle** is one which has no two of its sides equal. (Fig. 3.)

690. An **Equiangular Triangle** has three equal angles. (Fig. 1.)

691. An **Acute-angled Triangle** has three acute angles. (Fig. 2.)

692. *An* ***Obtuse*-angled Triangle** has one *obtuse angle. (Fig.* 3.)

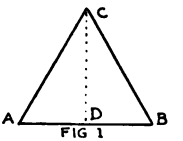


FIG 1

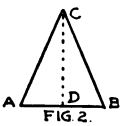


FIG 2.

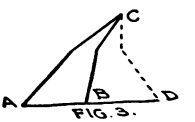


FIG. 3.

The line AB in each of the three figures is the base, and the line CD in each figure is the altitude.

693. The base and altitude of a triangle being given to find its area.

1. What is the area of a triangle whose base is 18 in. and altitude 10 in.?

Operation.

$18 \times 5 = 90$ sq. in.

or

$18 \times 10 = 180$ sq. in.

$180 \div 2 = 90$ sq. in.

EXPLANATION. Since a triangle contains one-half the area of a square of corresponding dimensions, it is evident that one dimension multiplied by one-half the other will give the area; also that the product of the two dimensions will give twice the area.

RULE. *Multiply one dimension by one-half the other; or divide the product of the two dimensions by 2.*

2. Find the area of a triangle whose base is 32 ft. and altitude 23 ft. 6 in.

3. What is the area of a triangular plat of ground whose base is 36 rds. and altitude 28 rds.?

4. How many square feet of boards will be required for the gable end of a house that is 26 ft. wide, and the ridge of the roof 14 ft. higher than the foot of the rafters, there being a waste of $\frac{1}{6}$ for matching?

5. What is the value, at \$60 per acre, of a triangular piece of ground whose base is 18 ch. 52 l., and altitude 23 ch. 12 l.?

694. The area and one dimension being given, to find the other dimension.

1. What is the base of a triangle whose area is 96 sq. ft. and altitude 16 ft.?

Operation.

96 sq. ft. $\div$ 8 sq. ft. $= 12 \therefore$ 12 ft.

RULE. *Divide the area by one-half the given dimension.*

Find the other dimension of the triangle.

2. When the area is 64 sq. in. and the altitude 9 in.

3. When the area is 18 sq. yd. and the base 13 ft.

4. When the base is 12 ch. and the area $4\frac{1}{2}$ acres.

5. If I pay \$120 for a triangular piece of ground, at the rate of \$4 per square rod, what is the base, the altitude being 10 rds.?

695. The three sides of a triangle being given, to find the area.

1. Find the area of a triangle whose sides are 15, 20, and 25 ft.

Operation.

$(15+20+25)\div 2=30;\ 30-15=15;\ 30-20=10;$
$30-25=5.\ \sqrt{30\times 15\times 10\times 5}=150$ sq. ft.

RULE. *From half the perimeter subtract each side separately; multiply the half-sum and the three remainders together, and extract the square root of the product.*

NOTE. The facts contained in the foregoing and following rules have been established by higher mathematics; a demonstration of these principles would be out of place in a work of this character.

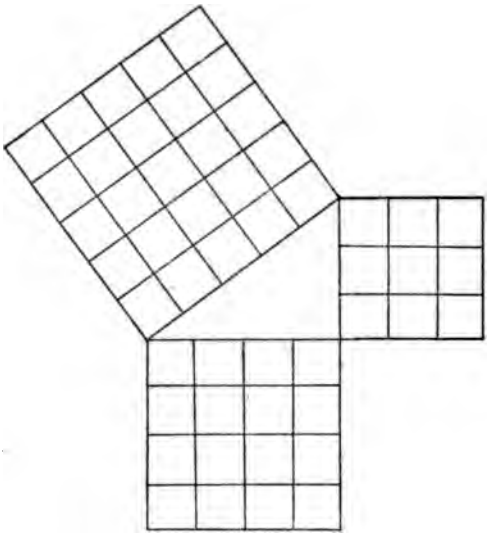
2. What is the area of an equilateral triangle, each of whose sides measures 20 ft.?

3. How many acres in a tract of land in the form of a triangle whose sides are respectively 40, 50, and 60 rods?

696. To find one side of a right-angled triangle when the other two sides are given.

THEOREMS. 1. *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

2. *The square of either the base or perpendicular of a right-angled triangle is equal to the difference of the squares of the hypotenuse and the other side.*



1. The base of a right-angled triangle is 21 in. and the perpendicular 28 in. What is the hypotenuse?

Operation.

$\overline{21}^2+\overline{28}^2=1225$. $\sqrt{1225}=35$, hypotenuse.

2. The base of a right-angled triangle is 9 ft., and the hypotenuse 15 ft. What is the perpendicular?

Operation.

$\overline{15}^2-\overline{9}^2=144$. $\sqrt{144}=12$, perpendicular.

RULE I. To find the hypotenuse: *Extract the square root of the sum of the squares of the base and perpendicular.*

II. To find base or perpendicular: *Extract the square root of the difference of the squares of the hypotenuse and the other given side.*

3. If the base of a right-angled triangle is 20, and the perpendicular is 24, what is the hypotenuse?

4. The perpendicular of a right-angled triangle is 15, and the hypotenuse is 28; find the base.

5. What is the distance between the opposite corners of a room which is sixteen ft. square?

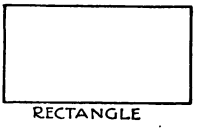
QUADRILATERALS.

697. A **Quadrilateral** is a plane figure bounded by four straight lines.

There are three kinds of quadrilaterals, the *Parallelogram*, the *Trapezoid*, and the *Trapezium*.

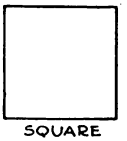
698. A **Parallelogram** is a quadrilateral which has its opposite sides parallel.

There are two kinds of parallelograms, the *Rectangle* and the *Rhomboid*.



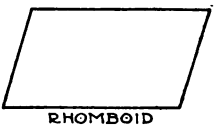
699. A **Rectangle** is a parallelogram whose angles are all right angles.

700. A **Square** is a rectangle whose sides are equal.



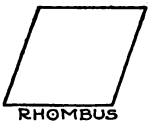
701. A **Rhomboid** is a parallelogram whose angles are not right angles.

702. A **Rhombus** is a rhomboid whose sides are equal.



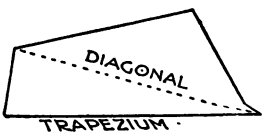
703. A **Trapezoid** is a quadilateral, two of whose sides are parallel and two oblique.

704. A **Trapezium** is a quadrilateral having no two sides parallel.



705. The **Altitude** of a parallelogram or trapezoid is the perpendicular distance between its parallel sides.

706. A **Diagonal** of a polygon is a straight line connecting the vertices of two angles not adjacent.



707. To find the area of any parallelogram.

1. Find the area of a parallelogram whose base is 12.5 ft. and altitude 7 ft.

Operation.

$12.5 \times 7 = 87.5$ sq. ft., area.

RULE. *Multiply the base by the altitude.*

2. I have a piece of land in the form of a rhomboid, the base being 10 ch. 8 l., and the altitude 9 ch. What is its value at $90 per acre?

3. What is the area of a rhombus whose base is 2 yds. 1 ft. and altitude 5 ft.?

708. To find the area of a trapezoid.

1. What is the area of a trapezoid whose parallel sides are 18 and 14 ft., and the altitude 11 ft.?

Operation.

$(18+14) \div 2 = 16$ ft. $\quad 16$ ft. $\times 11 = 176$ sq. ft., area.

RULE. *Multiply half the sum of the parallel sides by the altitude.*

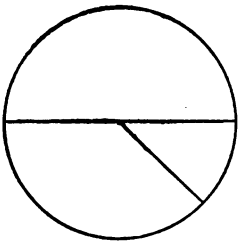
2. How many square feet in a board 12 ft. long, 16 in. wide at one end and 10 in. wide at the other end?

3. Find the area of a trapezoid whose parallel sides are 40 and 33 ft., and the distance between the parallel sides 20 ft.

To find the area of a trapezium, or any other irregular polygon, divide the figure into triangles, find the area of each separately, and combine the results.

THE CIRCLE.

709. A Circle is a plane figure bounded by a curved line, called the circumference, every point of which is *equally distant* from a point within, called the center.



710. The **Diameter** of a circle is a straight line passing through its center and terminated at both ends by the circumference.

711. The **Radius** of a circle is a straight line extending from its center to any point in the circumference.

712. To find the circumference or the diameter of a circle when the other dimension is given.

The ratio between the circumference and diameter of a circle is expressed by the number 3.1416. Hence when either of these dimensions is given, the other may be found by the following

RULE I. *To find the circumference, multiply the diameter by 3.1416.*

II. *To find the diameter, divide the circumference by 3.1416.*

1. What is the circumference of a circle whose diameter is 15 in.?

2. What is the diameter of a circle whose circumference is 6 ft. 8 in.?

3. The circumference of a circle containing an inscribed square is 40 in. Find a side of the square.

4. A circular race course is 101.859 rods in diameter; what is the length of the track?

713. To find the area of a circle.

RULE I. *Multiply the circumference by $\frac{1}{4}$ of the diameter.*

II. *Multiply the square of the radius by 3.1416.*

1. What is the area of a circular plat of ground whose diameter is 40 rods?

2. What is the area of a circle whose circumference is 2 yds. 1 ft. 6 in.?

3. What is the area of a circle which has a radius of *30 inches?*

714. To find any dimension of a circle whose area is known.

RULE. *Divide the area by 3.1416 and extract the square root of the quotient; the result is the radius.*

NOTE. The radius being known, the other dimensions may be easily found from the preceding rules.

1. The area of a circle is 19.2423 square rods. What is its diameter?

2. The area of a circular plat of ground is 2 acres. What is the circumference?

715. Similar figures are such as have the same form. *Squares*, *equi-angular triangles*, *regular polygons* of the same number of sides, and *circles*, are examples of similar figures.

PRINCIPLES. 1. *The like dimensions of similar plane figures are proportional.*

2. *The areas of similar plane figures are to each other as the squares of their like dimensions.* And conversely,

3. *The like dimensions of similar plane figures are to each other as the square roots of their areas.*

1. A triangle having a base of 16 inches, contains 96 square inches. What is the area of a similar triangle whose base is 21 inches?

Operation.

$\overline{16}^2 : \overline{21}^2 :: 96 : (?) = 165.375$ sq. in. (Prin. 2.)

2. The areas of two circles are to each other as 4 to 9; the diameter of the smaller being 65 inches, what is the diameter of the larger?

3. If a pipe two inches in diameter will fill a barrel in 50 seconds, how long a time will be required for a pipe $1\frac{1}{2}$ inches in diameter to fill the same barrel?

4. If a board 12 feet long, 16 inches wide at one end, and tapering regularly to a point, be cut crosswise into two pieces equal in area, what will be the length of the longer piece?

SOLIDS.

716. A **Prism** is a solid whose ends are equal and parallel polygons, and its sides parallelograms.

The number of sides a prism contains determines its name. One with three sides is a *triangular* prism; one with four sides is *quadrangular*, etc.

717. A **Cube** is a quadrangular prism bounded by six equal square faces.

718. A **Cylinder** is a body bounded by a uniformly curved surface, its ends being equal and parallel circles.

The line joining the centers of the bases of a cylinder, is called the *axis*.

719. To find the convex surface of a prism or a cylinder

1. Find the convex surface of a triangular prism, whose altitude is 6 feet, and the sides of the base 4, 3, and 2 feet respectively.

Operation.

4 ft.+3 ft.+2 ft.=9 ft., perimeter.
9 ft.×7=63 sq. ft. convex surface.

RULE. *Multiply the perimeter of the base by the altitude.*

2. Find the area of the convex surface of a cylinder whose altitude is 5 ft. 6 in., and the circumference of its base 8 ft. 4 in.

Operation.

$8\frac{1}{3}$ ft.×$5\frac{1}{2}$=45 sq. ft. 120 sq. in.

RULE. *Multiply the circumference of the base by the altitude.*

To find the entire surface, add the area of the bases.

3. Find the convex surface of a prism whose base is 5 ft. square, and altitude 4 ft. 5 in.

4. Find the convex surface of a prism whose base is a regular pentagon (5 feet on each side), and its altitude 5 ft. 6 in.

5. *What is the* convex surface of a cylinder formed by

the revolution about one of its sides of a rectangle that is 4 ft. 3 in. long and 3 feet wide?

6. What is the entire surface of a cylinder whose base is 4 ft. and altitude 6 ft. 2 in.?

720. To find the volume of any prism or cylinder.

1. What is the volume of a square prism whose altitude is 18 inches, and each side of the base 8 inches?

Operation.

64 (area of base) $\times$ 18 (altitude) = 1152 cu. in.

2. Find the volume of a cylinder whose altitude is 5 feet, and the diameter of its base 4 feet.

Operation.

$2^2 \times 3.1416 = 12.5664$, area of base. (713 II.)
$12.5664 \times 5 = 62.832$, volume.

RULE. *Multiply the area of the base by the altitude.*

3. What is the volume of a prism whose altitude is 27 inches, and its base a scalene triangle whose sides are 10, 11, and 12 inches?

4. What is the volume of a shaft 5 inches in diameter and 14 feet long?

5. At 32 cents a cubic foot, what is the value of a stick of timber 14 inches square, and 36 feet long?

6. Find the volume of an iron rod 3 inches in diameter and 12 feet long.

721. A **Pyramid** is a solid whose base is a polygon and its other faces, called *lateral faces*, are triangles having a common vertex.

The intersections of the lateral faces are called the *lateral edges of the pyramid.*

The sum of the areas of the lateral faces is called the *lateral area of the pyramid.*

Pyramids, like prisms, take their names from their bases, and are *triangular*, quadrangular, pentagonal, etc.

722. A **Cone** is a body having a circular base, and a curved lateral surface, tapering uniformly to a vertex.

723. The **Altitude** of a pyramid or of a cone is the perpendicular distance from its vertex to the plane of its base.

724. The **Slant Height** of a pyramid is the perpendicular distance from its vertex to one of the sides of the base. The slant height of a cone is measured from the vertex to the circumference of the base.

725. The **Frustum of a pyramid** or cone is the part remaining after cutting off the top by a plane parallel to the base.

726. A **Sphere** is a body bounded by a uniformly curved surface, all points of which are equally distant from a point within, called the center.

727. The **Diameter** of a sphere is a straight line passing through the center, and terminated at both ends by its surface.

728. The **Radius** of a sphere is a straight line drawn from the center to any point in the surface.

729. To find the lateral area of a pyramid or cone.

1. Find the lateral area of a square pyramid, the slant height being 12 feet, and each side of the base 5 feet.

Operation.

$$(5 \text{ ft.} \times 4) \times (12 \div 2) = 120 \text{ sq. ft.}$$

2. Find the lateral area of a cone whose diameter is 7 ft. 6 in. and the slant height 10 ft.

Operation.

$7.5 \times 3.1416 = 23.562$, circumference.

$23.562 \times (10 \div 2) = 117.81$ sq. ft.

RULE. *Multiply the perimeter* or circumference by one-half *the slant height.*

To find the entire surface, the area of the base must be added to the product.

3. Find the entire surface of a cone, whose diameter at the base is 16 inches, and the slant height 2 yds. 1 ft.

4. Find the lateral area of a pyramid whose slant height is 20 feet, and its base an octagon, each of its eight sides being 6 feet.

5. Find the lateral area of a cone whose altitude is 16 feet, and the radius of its base 3 feet.

730. To find the volume of a pyramid or of a cone.

1. What is the volume of a square pyramid whose base is 8 feet on each side, and its altitude 15 feet?

Operation.

$$8^2 \times (15 \div 3) = 320 \text{ cu. ft.}$$

2. Find the volume of a cone, the radius of whose base is 3 feet, and its altitude 18 feet.

Operation.

$$3^2 \times 3.1416 \times (18 \div 3) = 169.6464 \text{ cu. ft.}$$

RULE. *Multiply the area of the base by one-third the altitude.*

3. Find the solid contents of a cone whose altitude is 7 ft. 4 in., and diameter at the base 4 feet.

4. At $3 per cubic foot, what is the value of a pyramid whose base is 6 feet square, and altitude 10 feet?

5. How many cubic feet of stone in the walls of a pyramidal structure whose altitude is 24 feet, and its base, measured on the outside, 20 feet square; the thickness of the walls, measured horizontally being 3 feet?

731. To find the lateral area of a frustum of a pyramid or a cone.

1. Find the lateral area of a frustum of a pyramid whose slant height is 30 inches, its upper base 5 inches square, and its lower base 8 inches square.

Operation.

The perimeter of the greater base is 32 in., of the less 20 in.

$(32 \text{ in.} + 20 \text{ in.}) \times (30 \div 2) = 780 \text{ sq. in.} = 5 \text{ sq. ft. } 60 \text{ sq. in.}$

2. The diameter of the upper base of a frustum of a cone is 6 feet and of the lower base 10 feet; if the slant height is 9 feet, what is the lateral area?

Operation.

6 ft. $\times$ 3.1416 = 18.8496 ft., circumference of upper base.
10 ft. $\times$ 3.1416 = 31.416 ft., circumference of lower base.
$(18.8496 + 31.416) \times (9 \div 2) = 226.1952$ sq. ft.

RULE. *Multiply the sum of the perimeters, or circumferences, of the bases by one-half the slant height.*

3. Find the difference, in lateral area, between the frustum of a pyramid whose greater base is 9 feet square, and the less 6 feet square, and the frustum of a cone whose greater base has a diameter of 9 feet and the less a diameter of 6 feet, the slant height of each being 12 feet.

732. To find the volume of a frustum of a pyramid or a cone.

1. Find the volume of a frustum of a pyramid whose altitude is 12 feet, and whose lower base is 8 feet square, and upper base 6 feet square.

Operation.

$8^2 + 6^2 = 100$; $(100 + \sqrt{64 \times 36}) \times (12 \div 3) = 592$ cu. ft.

2. How many cubic feet in the frustum of a cone whose altitude is 9 feet, and the diameters of its bases 5 feet and 4 feet?

Operation.

$\overline{2.5}^2 \times 3.1416 = 19.635$, area of lower base.
$2^2 \times 3.1416 = 12.5664$, area of upper base.
$(\sqrt{19.635 \times 12.5664} + 19.635 + 12.5664) \times (9 \div 3) =$ 143.7282 cu. ft.

RULE. *To the sum of the areas* of the bases add the square

root of their product, and multiply this sum by one-third of the altitude.

3. Find the volume of a triangular frustum of a pyramid having the following dimensions: Altitude, 40 feet; lower base, 30 feet on each side; and upper base 20 feet on each side.

4. How many gallons of water will a vessel hold, the inside dimensions being: depth, 9 inches; upper diameter, 11 inches; and lower diameter, 8 inches?

733. To find the surface of a sphere.

1. What is the surface of a globe 16 inches in diameter?

Operation.

16 in. $\times$ 3.1416 = 50.2656, circumference of great circle.
50.2656 $\times$ 16 = 804.2496 sq. in.

RULE I. *Multiply the circumference of a great circle by the diameter;* or

II. *Multiply the square of the diameter by 3.1416.*

2. What is the surface of a globe whose circumference is 15.73 inches?

3. Find the surface of a globe having a radius of one-half a yard.

734. To find the volume of a sphere.

1. Find the volume of a sphere whose diameter is 12 inches.

Operation.

$\overline{12}^2 \times 3.1416 = 452.3904$ sq. in., surface.
$452.3904 \times (6 \div 3) = 918.7808$ cu. in.

RULE. *Multiply the surface by one-third of the radius, or by one sixth of the diameter.*

2. Find the volume of a globe whose diameter is 2 feet.

3. Find the volume of a globe whose circumference is 15 inches.

4. Find the volume of a globe whose radius is 20 inches.

735. To find the volume of any irregular solid.

1. Wishing to determine the volume of a piece of copper of irregular shape, I placed it in a water-tight box which was uniformly 8 inches square; I then poured sufficient water into the box to cause the copper to become completely submerged, and found by measurement that the water was 3 inches deep. If the water in the box was but $2\frac{1}{4}$ inches deep after the copper was removed, what was the volume of the copper?

Operation.

$8\times8\times3=192$ cu. in., volume of water and copper.
$8\times8\times2\frac{1}{4}=144$ cu. in., volume of water.
192 cu. in.$-$144 cu. in.$=$48 cu. in., volume of copper.

RULE. *Immerse in water and measure the volume of water displaced.*

2. A jeweler, wishing to know the volume of a gold chain, placed the same in a vessel having a uniform diameter of 4 inches; he then poured water into the vessel to the depth of 2 inches, which was sufficient to completely cover the chain. After removing the chain the depth of the water was 1.9 inches. What was the volume of the chain?

SIMILAR SOLIDS.

736. Similar Solids are such as are alike in form.

From the law of relation regarding similar solids the following principles are deduced:

1. The volumes of similar solids are to each other as the cubes of their like dimensions.

2. The like dimensions of similar solids are to each other as the cube roots of their volumes.

1. If a ball 3 inches in diameter weighs 5 po what is the weight of a ball of the same material, w is 6 inches in diameter?

Operation.

$3^3 : 6^3 :: 5 : (?) = 40$ pounds. (Prin. 1.)

2. If a stack of hay containing 8 tons is 18 feet in di eter, what is the diameter of a stack of the same form c taining 15.625 tons?

Operation.

$\sqrt[3]{8} : \sqrt[3]{15.625} :: 18 : (?) = 22.5$ ft. (Prin. 2.)

3. If a cone having an altitude of 30 inches and a ba diameter of 18 inches, be cut in two pieces, equal in v ume, by a plane parallel to the base, what will be t altitude of the frustum?